

# A research survey of induction motor operation with non-sinusoidal supply wave forms

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## Abstract

The developments in the power electronics field have lead to an ever-increasing use of static switching devices to control the torque and speed of ac motors. Invariably, the output voltage and current waveforms of these devices contain numerous harmonics and these harmonics have detrimental effects on the motor performance in form of derating and torque pulsation, especially at low speed. The order and magnitude of these harmonics depend on the design as well as nature of load being supplied. The extensive research has been underway for a long time in order to assess the effects these harmonics have on induction motor performance, and to investigate the various issues related to the use of induction motor to improve the drive performance and reduce the losses. This paper, therefore, presents a comprehensive review of research and developments in the induction motor operation with non-sinusoidal supply waveforms since its inception. Attempts are made to highlight the current and future issues involved in the development of induction motor drive technology to impart good dynamic stability with improved performance. A list of 167 research publications on the subject is also appended for a quick reference.

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## 1. Introduction

Prior to the advent of solid-state controllers for the speed control of induction machines, the supply voltage used to be sinusoidal in nature, being practically free from the time harmonics. The standard integral slot windings, having similar pattern of conductor distribution for all the phases, have been used with these machines giving reasonably good performance [1,2]. At present time, the induction motors are widely supplied from several types of solid-state adjustable voltage–frequency controllers with a wide range of operating features. However, in any case, the motor has to be derated for the harmonic effects due to the non-sinusoidal nature of the voltage supply. The magnitude and the distribution of the additional losses and the related motor derating, in steady state, depends on the harmonic contents of the applied voltage and, in some way, on the motor design. The output voltage of the present day static controllers deviates substantially from the sinusoidal form and contains

wide spectrum of time harmonics of which the lower order time harmonics in general, having frequency closer to the wanted output frequency and the sub-harmonics in particular, are found to be potentially objectionable in practice [3–5] and are at the same time found difficult to be filtered off [6,7]. Besides directly reducing the rating of the machine, these time harmonics produce other undesirable effects on the performance. Attempts have, however, been constantly made to modify the design of controller circuit for improving waveform of the output voltage [8–11]. The success has been achieved to a limited extent at the cost of added complications, which has reduced the reliability and increased the cost of controller unit besides an increase in switching losses [12–16]. This paper, therefore, deals with a state-of-the-art discussion on performance of induction motor operating on non-sinusoidal supply waveforms, highlighting the analytical and technical considerations as well as various issues addressed in the literature towards the practical realization of this technology for better drive stability with improved performance. One hundred and sixty-seven publications [1–167] are reviewed and classified in 11 parts.

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## 2. Modeling and analysis

A basic paper for induction machine analysis has been reported by Stanley [17]. Later on, many researchers made their contributions on the basis of the machine model given by Stanley with some modifications. Hughes and Alderd [18] have suggested a general model for transient and unbalanced operation of two- and three-phase induction machine. The machine equations are expressed with respect to  $\alpha\beta$ - $dq$  coordinate. The model is simulated using analogue computer and the obtained results are compared with practical ones. Sarkar and Berg [19] have presented a direct three-phase model using phase variables and two axis models for three-phase induction machine. The models are simulated using digital computer. The machine performance under rectangular waveform supply is also included. Jacobides [20] has reported the model of a drive consisting of three-phase induction motor fed from a full bridge cycloconverter. The model based on phase variables and Gaussian elimination method is used to reduce the order of the connecting matrix. Murthy and Berg [21] have presented a dynamic model based on instantaneous symmetrical components theory. The model is used to obtain the transient behavior of thyristor-controlled three-phase induction motor. The model covers most of the operating modes and the analysis is done without reference to the rotor position. Krause and Lipo [22–25] and many other researchers have reported different analytical techniques based on phase variables and reference frame transformations for three-phase induction machine. Different machine and supply conditions have been included.

Fallside and Wortley [26] and Ueda et al. [27] have studied the effect of supplying the machine from variable frequency source from the point of view of stability. The effect of machine parameters variation on the machine stability has been discussed. Levi [28,29] have reported the modeling of induction motor and synchronous machine considering sinusoidal and non-sinusoidal supply. The models are based on  $d$ - $q$  axes and the analysis includes the effect of machine saturation and core loss. Different control strategies of induction motor are also considered. Neto et al. [30] have presented a mathematical model of three-phase induction motor using phase variables. The model is based on the harmonic impedance concept and accordingly the voltage and torque equations are derived. Two cases are included in the analysis, supplying the machine from sinusoidal supply and from non-sinusoidal supply using PWM inverter. Toliyat et al. [31–33] have developed analysis method for modeling the multiphase cage induction motor. The model considers the mmf harmonics using winding function approach. The model is used effectively to simulate different types of machine faults such as asymmetry in the stator winding, air gap eccentricity and rotor bar fault. Joksimovic and Penman [34] have presented an induction motor model based on winding function approach to investigate the relation between the machine faults and the current harmonics. One of the common methods used to simulate the supply faults and unbalance in

stator and rotor circuit is the symmetrical component method [35].

The formulation of machine dynamic and steady state equations and the role of transformations in the analysis of  $m$ - $n$  winding induction machine with space harmonics have been reported by Fudeh and Ong [36,37]. In another study [38], the authors have derived the voltage equations in terms of  $\alpha$ - $\beta$  and  $dq$  components for a three-phase cage rotor induction machine with space harmonics. In the same study, skin effect, skewing of rotor bars and the influence of slot opening on the mmf are all taken into account in the derivation of the parameters for the machine. Krause and Lipo [39] have presented the analysis and simplified representation of a rectifier-inverter induction motor drive. A semi-empirical induction motor loss model as a function of motor power rating has been given by Buck et al. [40]. Oguchi [41] has reported a closed-form, analytical solution for the current and torque waveform for a three-phase induction motor, supplied with multiple phase-shifted voltage source inverter system. Motor performance, such as the torque ripple factor and the peak stator current ratio, has been calculated for the inverter output voltage waveforms with 6–48 steps. A guideline for the design of multiple inverter systems has been given and a comparison between motor performances of multi-stepped voltage-fed and a PWM voltage-fed design has been made. Belmans et al. [42] have presented the analysis of the audible noise of three-phase squirrel-cage induction motors supplied by inverters. In the study, a way for predicting the spectrum components produced by the motor and for relating it to the air gap flux density distribution time harmonics caused by the non-sinusoidal supply have been given. Enjeti and Lindsay [43] have developed a direct three-phase model for analytical investigation on steady state and transient behavior of an induction motor fed from non-sinusoidal supply. An analytical procedure for computing the exact nature obtained from PWM inverter has been described.

In another study, a simulation procedure for investigating problems concerning road traction drives based on three-phase induction machines supplied by modulated inverters has been presented by Denegri et al. [44]. Bonnet [45] has reported the analysis of the impact of the PWM inverter voltage waveforms on ac induction motors. The effects of the maximum voltage, rate of rise, switching frequencies, capacitors, resonance and harmonics have been considered in the study. Wang and Liu [46] have discussed the steady state harmonic modeling and simulation of a cycloconverter drive system (CSD). The operation and control of a cycloconverter drive and a synchronous motor load were modeled in time domain. The theme of this paper is to understand harmonic problem associated with a CDS from an integrated point of view, with special attention given to harmonics filtering and cancellation effect of converter coupling transformers. Frequency domain methods useful for traction drives design and harmonic generation mechanism understanding have been given by Lordache et al. [47]. An iterative solving algorithm is used for load flow and converter operating point calcula-

Table 1  
Comparison between induction machine models

Machine model	Complexity	Characteristics
Phase variable model	(6 × 6) Matrix	Suitable for all machines conditions Direct presentation of machine variables
Stationary reference frame ( $\omega = 0$ )	(4 × 4) Matrix	Better suited for unbalanced condition of supply and stator Transformations needed of machine variable
Rotor reference frame ( $\omega_a = \omega_r$ )	(4 × 4) Matrix	Better suited for unbalanced condition of rotor faults Transformations needed of machine variable
Synchronously rotating reference frame ( $\omega_a = \omega_s$ )	(4 × 4) Matrix	Better suited for non-sinusoidal supply Transformations needed of machine variable

tion. A numerical application is developed and results are compared with time domain simulation results. Dell'Aquila et al. [48] have presented an analytical method to model and calculate the line side currents produced by variable speed induction motor drives. In a paper by Gersen et al. [49], time-harmonic finite-element simulation, as commonly applied to the three-phase induction machine model, is used to simulate a single-phase capacitor start/run motor by decomposing the air gap field in two revolving fields in the opposite direction. A comparison of various reference frame models with phase variable based model is depicted in Table 1. Although, the complexity of phase variable model and time needed for simulation of the machine behavior is high as compared to the other models, it can be used to cover all the machine conditions with ease of implementation.

### 3. Induction motor losses on non-sinusoidal supply

Doggett and Queer [3] for first time in 1929 have presented the preliminary investigation of an induction motor operation with non-sinusoidal impressed voltages and suggested that for a known pattern of non-sinusoidal supply, the voltage profile can be analyzed into a fundamental component and a series of time harmonics. If magnetic saturation is neglected, a motor operating on such a supply system may be regarded as a linear device and the principle of superposition can be applied. The overall response to the non-sinusoidal voltage is then obtained as aggregation of responses to the individual components. Klingshirn and Jordan proposed a three-phase induction motor performance under a non-sinusoidal voltage source [5]. In 1968, Chalmer and Sarkar studied the induction motor losses due to non-sinusoidal supply waveforms [50]. In 1972 and 1979, Linders investigated the effects of poor quality power sources on ac motors and proposed the hidden costs and containment due to the electric wave distortion [51,52]. Raphael discussed the additional losses and torque pulsations in PWM inverter-fed squirrel-cage induction motors [53]. In 1986, Cummings simplified the harmonic equivalent circuit and proposed a method to estimate motor loss and temperature rise [54]. In 1987, Fuchs et al. investigated the sensitivity of appliances of harmonics [55]. Sen and Landa in 1990, according to material of induction mo-

tor, proposed derating operation under waveform distortion [56]. In 1985 and 1993, Ortmeier et al. and Wagner et al., respectively, presented a summary of the state-of-knowledge about the effects on power system equipment and load under harmonics [57,58]. Lee and Lee in 1999, reported a paper on effects of non-sinusoidal voltage on the operating performance of a three-phase induction motor [59].

There have been many studies on iron losses in induction machines. The well-known equivalent circuit has been modified in various ways to improve the accuracy of the loss estimate for wide ranges of operating conditions such as in Ref. [1]. In case of distorted voltage waveform supply, it is not possible to use, in general, such a model due to the iron non-linear behavior and the complexity of the phenomena involved. One such modification of the standard equivalent circuit representation of the motor for the  $k$ th harmonic of the voltage waveform proposed by Vamvakari et al. [60] is shown in Fig. 1. In this figure,  $V_k$  denotes the voltage harmonic of order  $k$ ,  $R_s$  the stator winding resistance,  $R_{rk}$  the corresponding rotor resistance,  $X_{ls}$ ,  $X_{lr}$  and  $X_m$  the stator, rotor leakage reactances and magnetizing reactances at fundamental frequency, respectively,  $R_{mk}$  the core loss resistance.  $R_{lsk}$  and  $R_{lrk}$  are resistors representing harmonic iron losses associated with stator and rotor leakage fluxes, respectively, placed in parallel with corresponding leakage reactance terms. Iron loss models suitable for use in  $dq$ -models of the induction machine have also been studied [61,62]. These lumped models of iron loss are useful but do not allow to detail of iron loss to be studied. Some studies have specifically addressed losses arising from inverter supply [63,64] but are limited in nature. There are models for iron loss in thin laminations [1,60,61,65]. These models provide loss density results for a

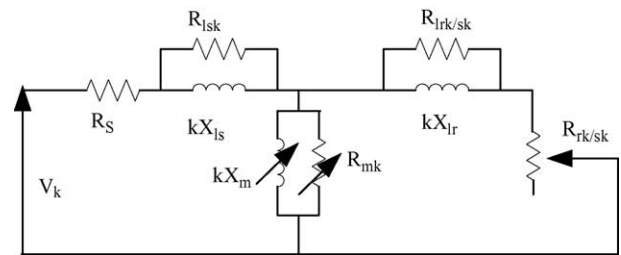


Fig. 1. Modified equivalent circuit of induction motor for the voltage harmonic component of  $k$ th order.

localized flux density expressed as a function of time. Boglietti et al. [66] have reported the effect of inverter characteristics on the iron loss increment in induction motors fed by PWM-controlled converters. The inverter parameters like modulation index, modulation waveform and switching frequency are considered in the study. In a study [67], Green et al. have examined the extent and nature of additional losses in induction machine over those occurring with grid supply for a switching frequency representative of current practice.

A comparative study of the losses in voltage and current source inverter-fed induction motor has been presented by Venkatesan and Lindsay [68]. In this study, the equivalent circuit that includes the effects of space harmonics and corrected for the skin effect in rotor bars has been used for calculation of main and stray copper loss. Boglietti [69] has given a method for evaluation of the steady state loss in medium power industrial induction motors supplied by PWM inverters. The method is based on the no-load and short circuit tests with PWM supply and is quite simple to realize. Gerlando and Perini [70] have reported a methodology for the calculation of the extra iron losses occurring in the core of the inverter-fed electromagnetic devices. In another study, Boglietti et al. [71] have presented a simple method based on no-load and short circuit tests in order to get the useful machine parameters to adopt in equivalent circuit model for the loss evaluation. In a recent study [72], authors have presented a methodology for a convenient modification of induction motor equivalent circuit parameters, taking into consideration switching frequency iron losses in case of inverter supply. The authors have also presented a modified two axes equivalent circuit as shown in Fig. 2. The various studies [73–76] made on harmonic losses reveal that:

- Presence of harmonic currents in the stator winding causes an increased copper loss. When skin effect is negligible, the stator copper loss on a non-sinusoidal supply is proportional to the square of the total rms current.

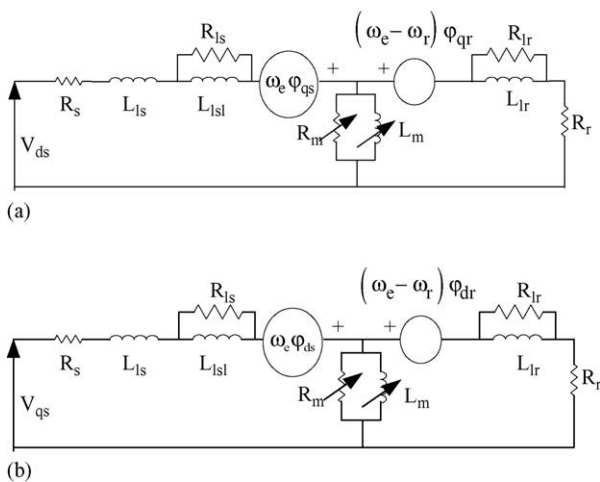


Fig. 2. Modified equivalent circuits for induction motor dynamic model: (a)  $d$ -axis equivalent circuit and (b)  $q$ -axis equivalent circuit.

- Presence of harmonic contents also increases the fundamental component of current slightly, due to an increased magnetizing current.
- The assumption of a constant resistance at harmonic frequencies is reasonably justified for the stator windings of the wire-wound machines. For large ac motors, there is an increase in stator resistance with frequency that depends on the shape, size and disposition of the conductor in the stator slots.
- The skin effect is much more pronounced in the cage rotor, which exhibits a significant increase in resistance at harmonic frequencies, particularly in case of deep bar rotors. Since the rotor resistance is a function of harmonic frequency, the rotor copper loss is calculated independently for each harmonic.
- It is appropriate to use a reduced value of per unit reactance because the rotor leakage inductance is reduced significantly as a result of skin effect.
- The core loss in the machine is also increased by the presence of harmonics in the supply voltage and current.
- The core loss due to space harmonic air gap flux is negligible, but the end-leakage and skew-leakage fluxes, which normally contribute to the stray load loss, may produce an appreciable core loss at harmonic frequencies. Consequently, these effects must be taken into consideration for motor operation on non-sinusoidal supply.
- The magnitude of harmonic loss obviously depends on the harmonic content of the motor voltage and current. Large harmonic voltage at low-harmonic frequencies cause significantly increased machine loss and reduced efficiency.
- Higher order harmonic currents usually have small magnitudes. For such waveforms, the reduction in full load motor efficiency is not excessive.

#### 4. Harmonic reduction techniques

The increasing application of power electronic equipment, especially adjustable-speed drives (ASDs) in the industrial environment has led to a growing concern for harmonic distortion and the resulting impacts on system equipment and operations. Possible problems include transformer overheating, motor failures, fuse blowing, capacitor failures and malfunctioning of control. Harmonic currents are generated by the operation of non-linear loads and equipment on power system. Voltage distortion results from the interaction of these currents with the system impedance versus frequency characteristics. The characteristics of the input current for ASDs depend on the drive type, drive loading and characteristics of the system supplying the drive. The harmonic distortion in these currents can vary over a wide range. In the last two decades, major focus has been on harmonic reduction techniques. Some summaries on three-phase harmonic reduction technique can be found in Refs. [77–81]. The third harmonic injection schemes for the three-phase diode rectifier for reducing the harmonic currents has drawn some promis-

ing results and has been presented among others in Refs. [82–86].

McGranaghan and Mueller [87] have discussed the application of IEEE 519 harmonics standard to typical industrial facilities employing ASDs. In a study [88], authors have proposed a robust three-phase active power-factor-correction (PFC) and harmonic reduction scheme suitable for higher power applications. The proposed system is unique combination of a low-kilovolt ampere 12-pulse rectifier system with a single-phase boost PFC scheme to shape the input current to near sinusoidal wave shape. Hansen et al. in [89] have given an integrated single-switch approach to improve harmonic performance of standard pulse width modulation adjustable-speed drives. The approach is essentially an add-on solution to a standard ASD topology and is based on circulating a third harmonic current to reduce the harmonics of line current. A space-vector-based PWM strategy that closely approximates in ‘real time’ the switching, angles of the selective harmonic elimination PWM strategy [90]. Alexa and Sirbu [91] have presented a thorough analysis of the operation for the combined filtering system consisting of a passive filter with diodes connected in parallel with the capacitors and a low-power inverter. In the paper, authors have also discussed the advantages and shortcomings of the passive, active and combined filtering systems and have suggested essential modifications. In a paper [92], a harmonic elimination and suppression scheme for a dual-inverter-fed open-end winding induction motor drive is presented. In Ref. [93], a method is presented by which the harmonic current of a two-level, network-connected inverter can be controlled for use as an active filter in addition to supplying real power via the intermediate dc link to a motor inverter. In a recent study [14], Sundareswara and Kumar have reported a voltage harmonic elimination technique in PWM ac chopper using genetic algorithm. Based on the findings of the study, it is found that:

- The simplest method to provide some level of harmonic control and also accomplish power factor correction requirement is to add the power factor correction in the form of tuned capacitor banks. This prevents magnification of any characteristic harmonic components from the drives.
- A significant harmonic reduction can be obtained from PWM type adjustable-speed drives just by adding a choke inductance at the input. Some drives manufacturers include this choke inductance in the dc link of the drive.
- Two isolated dc link sources with voltage ratio of approximately 1:0.366 feeding an open-end induction motor are capable of eliminating Triplen harmonic currents from the motor phase.
- A reduction of harmonic content by increasing the pulse number does not necessarily offer a substantial improvement. The application of a PWM inverter within a broad time-fundamental frequency range thus offers disadvantages, unless it is possible and intended to filter the output.
- Fixed frequency or on/off PWM current-controlled techniques can provide a high-quality, controlled-current source.
- A high-carrier ratio improves waveform quality by raising the order of the principle harmonics. At low fundamental frequencies, very large carrier ratios are feasible and resulting in near-sinusoidal output current waveforms account for one of the main attributes of the sine wave PWM inverter drive—the extremely smooth rotation at low speeds.

## 5. Noise and vibration in induction motor due to non-sinusoidality

Induction motors are used in many industrial applications and their use has been expanding in variable speed applications. However, environmental concerns have increased and there have been an increasing demand for a quiet induction motor. Acoustic noise sources from electrical machines have been generally attributed to electromagnetic, aerodynamic and mechanical origin [94]. The acoustic noise radiated by induction motors increases when they are operated from non-sinusoidal power supplies, such as quasi-square waveform and pulse width modulation converters [95,96]. This can be of considerable significance in certain applications, for example when they are employed as traction machines in the drive train of electric vehicles. The effect of non-sinusoidal supplies on the acoustic noise has been addressed extensively [97–103] and various PWM strategies have been developed primarily to reduce harmonic distortion and improve efficiency. In these studies, the effect of various PWM strategies on noise has been largely restricted to a consideration of the switching noise of the drive with relatively less emphasis on their interaction with machines. Lo et al. [104] have developed a theory for analyzing the noise spectrum further to account for the interaction between the motor and the drive. In the study, it is shown that manufacturing tolerances can result in significant differences in the noise level emitted from nominally identical motors and that the mechanical resonance can result in extremely high-noise emissions. In Ref. [105], the authors have introduced a statistical method for the prediction of the acoustic noise radiation from variable speed induction motors and compared the results with traditional deterministic method [106–108]. A general method for reducing noise due to the coupling of an electric machine and a power converter is reported in Ref. [109]. The method presented is semi-analytical based on rotating field theory, while the fast Fourier transform permits a global approach to the problem. This method may be used at the design stage of the electric machine (slot number, slot aperture and stator yoke thickness) or for the choice of PWM strategy and parameters for power converter. In Ref. [110], authors have proposed a new random position space vector PWM (RPPWM) for reduction of acoustic noise in inverter drive system. It is shown that the voltage and current harmonics are spread to the wide-band area and audible switching noise is reduced by RPPWM

method. Xu et al. [111] have studied the acoustic noise emissions from a direct torque-controlled induction motor drive. The investigation on the influence of load and flux level on the emitted noise has shown that the noise level increases with the load, but there is an optimal flux level for minimal noise. In a recent study [112], electromagnetic noise has been calculated by boundary element method program using the electromagnetic force of Maxwell stress. The natural frequency and behavior of stator have been calculated by mechanical finite-element method considering the contact between stator core and frame. The various findings can be summarized as:

- The radial forces in a machine are directly proportional to the square of the flux density in the air gap. One of the obvious ways to reduce noise would be to reduce flux density by enlarging the air gap.
- Skewing of the stator and rotor slots reduces the average radial forces in small machines, leading to a considerable reduction in noise, whereas in large machines, it may lead to Torsional vibrations and, hence, an increase in acoustic noise.
- Skewing is also helpful in eliminating standstill locking and crawling in induction machines. Skewing of rotor by about one stator slot produce best results with regards to reducing noise and vibrations.
- Magnetic noise produced by rotor eccentricity can be reduced by using parallel paths in stator windings.
- For controller, there are two strategies to reduce noise: using high-switching frequencies (above 15 kHz), which is a very efficient method but impose high stress on the switches and using spread-spectrum strategies, for example random strategies reduce acoustic noise but may coincide with mechanical resonance.

## 6. Detection of rotor slot harmonics and speed estimation

Detection of rotor slot and other eccentricity related harmonics in the line current of a three-phase induction motor is important both for sensor less speed estimation as well as for eccentricity related fault detection. However, it is now clear that not all three-phase induction motors are capable of generating such harmonics in the line current. Recent research has shown that the presence of these harmonics is primarily dependent on the number of rotor slots and the number of fundamental pole pairs. The detection of slot harmonics and speed/position estimation in adjustable-speed drive has formed the subject of investigation of several studies [113–118]. Blasco-Gimenez et al. [113] have reported some results relating to real-time speed measurements obtained from the spectral estimation of rotor slot harmonic frequencies present in the stator currents of a cage induction motor. Derivation of the measurement accuracy and a method for attaining the maximum accuracy are presented. The expressions for predicting the performance under low-motor

loads and the region of signal indeterminacy in the torque-speed plane arising from interference by PWM harmonics have been obtained.

The aim of the study [114] is to determine the line current distortion under consideration of the actual drive loading. The study on several converter-fed drives has shown that the reduction of total harmonic currents can be further achieved by cancellation effects based on the knowledge of the phase angles of the current harmonics. In Ref. [115], a simple and robust way of utilizing harmonic saliencies created by rotor and stator slotting, present in some induction machine designs for the estimation of rotor position, has been described. Authors in Ref. [116] have proposed a new transfer function for the estimation of harmonic distortion in 6-pulse ac/dc-controlled converters working in continuous mode. Use of transfer function has been extended for the case of distorted and/or unbalanced supply. Nandi et al. [117] have carried out a detailed study about the detection of rotor slot and other eccentricity related harmonics in a three-phase induction motor with different cages. In a recent study [118], investigation of the distribution pattern of harmonic waves in ac electric motors is presented. On the basis of this, a generalized approach for calculating differential leakage related to the harmonic waves has been developed. It is stated that this approach is applicable to any ac electric motors, regardless of winding type and phases, regular or non-regular, integral-slot or fractional slot and symmetrical or asymmetrical.

## 7. Harmonic current and torque

### 7.1. Harmonic current

When the induction motor is running near fundamental synchronous speed, the harmonic equivalent circuit is quite similar to the locked rotor equivalent circuit for the particular harmonic being considered. The magnetizing branch may be neglected since the magnetizing reactance for the  $n$ th order harmonic ( $n \times X_m$ ) is much greater than the rotor leakage impedance. For the similar reason, the resistances representing the core (mechanical) losses for the fundamental and the different harmonics are neglected [33]. The  $n$ th harmonic current is then given as:

$$I_n = \frac{V_n}{[(R_{sn} + R_{rn}/S_n)^2 + (X_{lsn} + X_{lrn})^2]^{1/2}} \quad (7.1)$$

where  $V_n$  is the voltage due to  $n$ th harmonic,  $R_{sn}$  and  $R_{rn}$  the stator and rotor resistances and  $X_{lsn}$  and  $X_{lrn}$  are stator and rotor leakage reactance for the  $n$ th order harmonic. The total harmonic current is:

$$I_h = \left[ \sum_{n=2}^n I_n^2 \right]^{1/2} \quad (7.2)$$

Various studies conducted on induction machine operation on non-sinusoidal supply reveal that:

- When an induction motor is supplied by a voltage source inverter with a specific output waveform at a particular frequency, the harmonic current remains constant for all operating conditions of the motor from no-load to full load and even down to stand still.
- The fundamental stator current is determined by the motor loading and as a result, the relative harmonic content of the machine current is considerably greater for light load operation than for full load condition.
- The greater harmonic content will cause a significant increase in the no-load losses of the machine compared with normal sine wave operation.
- The harmonic distortion not only increases the rms value of the stator current but also produces large current peaks, which increase the required current rating of the inverter transistors or commutating duty imposed on the inverter thyristors.

### 7.2. Harmonic torque

The presence of time-harmonic mmf waves in the air gap results in additional harmonic torques on the rotor. These torques are of two types: steady harmonic torques and pulsating harmonic torques. A steady component of torque is produced by the interaction of each rotor harmonic mmf with a harmonic air gap flux of the same order. The pulsating torque components are produced by interaction of harmonic rotor mmf with harmonic rotating flux of different order. At very low speeds, motor rotation takes place in series of jerks or steps and this irregular cogging motion sets a lower limit to the useful speed range of the motors [119]. Revankar and Havanur [120] have developed the expression for calculation of average and pulsating torque for five different current source inverter configurations. A systematic procedure for optimum design of the current source inverter drive has been reported by Pavithran et al. [121]. The optimization is carried out using the sequential unconstrained minimization technique. The study also takes into account the inherent slow commutation process, the torque pulsation and the peak voltage stresses in the CSI drive. A comparative study of the optimum CSI-fed motor and a cost optimal mains-fed CSI supply has also been included in the study. Buck [122] has discussed some aspects of the theoretical performance of an electric motor, supplied by unfiltered PWM inverters. In the study, an induction motor system is assumed and the creation of low-frequency parasitic torques is described. Chreighton [123] has reported a detailed study on torque pulsation in induction motor supplied by current source inverter. Lipo [124] has presented a systematic analysis and control of torque pulsations in current-fed induction motor drive. Some guidelines for eliminating the pulsating torque in CSI-fed induction motor have been reported by Chin and Tomita [125]. Jubek [126] has given a comparative study on the techniques for reducing the shaft cogging in current-fed ac drives. Lienau [127] has reported a study on torque oscillations in traction drives with current-fed asynchronous

machines. Observations made from various studies are [119,128–130]:

- The low-order torque pulsations can be avoided by supplying the motor with an improved voltage or current waveform, such as PWM waveform employing a sinusoidal modulation strategy.
- Increasing the number of stator phase has also been suggested as a means of reducing the amplitude of torque pulsations.
- Amplitude of pulsating torques increases with lower value of rotor slip frequency. As the slip frequency increases, pulsating torque ratio becomes smaller and tends to become constant in CSI drives.
- For low-speed operation with a high-switching frequency, sinusoidal PWM gives near-optimum torque smoothness with a simpler modulation strategy.
- Distortion minimization PWM cannot be seriously considered for low-frequency operation. The overall harmonic distortion is minimized, but no specific attention is paid to the lower order harmonics, so that large low-frequency pulsating torques are developed.
- Harmonic elimination PWM seeks to suppress the specific lower order torque harmonics that cause speed fluctuation at reduced speeds. Elimination of the 5th, 7th, 11th and 13th harmonic voltages removes the 6th and 12th harmonic pulsating torques, but higher order torques may be significant.

### 8. Effect of space harmonic on induction machine performance

Space harmonic fields are produced by the distributed type of windings, slotting of stator and rotor, magnetic saturation and inequalities in air gap length. The space harmonics caused by the variation of air gap reluctance are called slot harmonics. The effect of these harmonics in the air gap flux wave is to give birth to unwanted parasitic torques, vibration and noise. Parasitic torques is of two types: asynchronous or induction torques and synchronous torques. These torques are responsible for generation of asynchronous crawling and synchronous crawling or cogging, respectively, in induction machine particularly cage type. Harmonic asynchronous torques cannot be avoided but can be reduced by proper choice of coil span, skewing the stator or rotor and using symmetrized winding [1,131,132]. The synchronous harmonic torques can be avoided entirely by a proper slot combination of stator and rotor slots.

### 9. Stability analysis

When ac motors are operated on adjustable-frequency supplies, system instability may occur for certain critical frequency ranges and loading conditions. Machines that are per-

fectly stable on an ac utility network may become unstable with an inverter supply, and machines that are stable when operated individually may become unstable when several of the motors are operated simultaneously as a group drive. Investigation of sources of this instability shows that the two causes are inherent low-frequency instability in the motor and instability due to interaction between the motor and inverter.

It was recognized first by Rogers [133] that an induction machine operating quite satisfactorily at normal speed might display an oscillatory response that is frequency dependent. The analysis was based on root locus technique. Fallside and Wortley [134] have also analyzed the instability of the induction motor fed by variable frequency inverter, neglecting the effect of harmonics. The effect of machine parameters on the stability of the system was also reported. Lipo and Krause [135] have performed the stability study of a rectifier-inverter induction motor drive system neglecting stator voltage harmonics and using Nyquist stability criterion. Cornell and Lipo [136] have used transfer function techniques for the development of controlled current induction motor drives and study its stability. Macdonald and Sen [137] have developed a linearized small signal model of the current source inverter-induction motor drive to study the stability of the drive and provide a transfer function for different control strategies. Tan and Richards [138] have calculated the eigenvalues of double-cage induction motor using decoupled boundary layer model. In these works, only the fundamental component of the inverter voltage has been taken into account neglecting the effect of harmonics. There is much evidence in the historic literature [139–142] dealing with the transient analysis that the transient behavior predicted using the conventional linearized model, although agrees qualitatively, deviates from the actual quantitatively due to non-inclusion of the saturation effect. Some schemes [143,144] have included the saturation effect in the induction motor fed by variable frequency source. These models are adaptive to digital simulation and time domain analysis and successfully predict the instability region precisely. Ahmed et al. [145] have analyzed the stability of linear time invariant systems based on the Liapunov's first method using the placement of eigenvalues. A linearized model for six-phase machine with arbitrary displacement between the two three-phase windings in  $d$ - $q$  variables of the machine in synchronously rotating reference frame were developed by Singh et al. [146]. The eigenvalues under different operating conditions were calculated and analyzed. The correlations between different machine parameters and pairs of eigenvalues have been established. Based on the findings of the study [119,147–150], it is found that:

- The transient response of the induction motor becomes more oscillatory as the supply frequency is reduced, but the normal machine does not usually become unstable on an infinite system. However, small motors with a low-inertia constant may be unstable. Reducing the magnetizing reactance and increasing the stator and rotor resistances may improve induction motor stability.

- Stability of the inverter-fed induction motor drive is generally enhanced by: (a) increasing the load torque and inertia, (b) reducing the stator voltage and (c) increasing filter capacitance and reducing filter inductance and resistance.
- Appropriate modification of machine parameters can also help to eliminate instability in an open-loop adjustable-frequency drive, but the provision of special machine designs may be uneconomical, and steady state performance and efficiency may not be as good as in normal motor.
- An inverter-fed induction motor may be stabilized by controlling inverter frequency with motor emf or a derivative of dc link current. Alternative feedback methods have also given satisfactory stabilization of adjustable-frequency drives.

## 10. Control scheme for loss minimization in induction motor drives

Selection of the level of flux in the induction motor remains an open problem from the perspective of maximizing motor efficiency and many researchers continue to work on this problem. Numerous operation schemes have been proposed by many researchers concerning the optimal choice of excitation current or flux level for a given operating point [151–163]. The techniques allowing efficiency improvement can be divided into two categories. The first category is the so called loss model-based approach [151–154], which consists of computing losses by using the machine model and selecting a flux level that minimizes these losses. The second category is the power-measure-based approach, also known as search controllers (SCs) [155–160], in which the flux (or its equivalent variables) is decreased until the electrical input power settles down to lowest value for a given torque and speed.

Different loss models for loss minimization can be found in the literature [151–154]. Lorenz and Yang [151] took copper loss and iron loss into account to formulate the loss model. Using an objective function that depends on the drive's loss and includes constraints, they calculated the optimal flux trajectories for the vector control online. Garcia et al. [152] obtained a loss model after simplifying the induction motor equivalent circuit by deleting leakage inductance in  $d$ - $q$  coordinates. The loss model consists of resistors reflecting iron loss, rotor and stator copper losses as a function of stator current  $i_{ds}^e$  and  $i_{qs}^e$  in the  $d$ - $q$  frame. Kioskeridis and Margaritis [153] calculated the total iron loss, copper loss and stray loss and derived an optimal flux level that minimizes the total loss. In model-based loss-minimization algorithms, the leakage inductance of stator and rotor are usually neglected to simplify the loss model and minimization algorithm [152,154]. However, with this simplified model, the exact loss minimization cannot be achieved, especially for high-speed operation of EV motors, since a large voltage drop across the leakage inductance is neglected.



Search controllers necessitate the use of input power measurement. Kirschen et al. [155] proposed a solution of minimizing the input power by decreasing the flux command in steps. This is a very simple technique, but torque pulsation is unavoidable. Sousa et al. [156] improved the work of Kirschen et al. [155] by adaptively reducing the reference flux current with the aid of fuzzy logic. They solved the torque pulsation problem by applying feed forward compensation. Kim et al. [157] adjusted the squared rotor flux according to a minimum power algorithm based on the Fibonacci search method. The torque ripple is not generated in this configuration, since the speed and rotor flux are decoupled by means of non-linear control. Moreira et al. [158] used the information of third harmonic components of air gap flux to reduce the  $d$ -axis current. Sul and Park [159] proposed an efficiency-maximizing technique by defining an optimal slip. The optimal slip is firstly searched by trial and error, and stored in the microprocessor memory. Then, the control system is forced to track the optimal slip presented in the lookup table. The important findings can be summarized as:

- In commercial applications, a fixed switching frequency is preferred because acoustic noise is less annoying and inverter switching losses are more predictable.
- The modulation index plays an important role on the motor iron losses. As a consequence, in order to reduce the iron losses, it is better to operate with highest allowed modulation index. In particular, for applications where the energetic performances are important in comparison with dynamic ones, the use of inverter with fixed and high-modulation index and variable dc bus voltage can be considered as suitable solution.
- The switching frequency is not so important from the iron loss point of view. In particular, with the switching frequency increase, a small reduction of the core losses can be obtained. Obviously, it is important to take into account the loss increase inside the power switches to have a correct evaluation of the system efficiency (motor and inverter) with increment of the switching frequency.
- Modulation function waveform is not an important factor on the iron losses increase. In fact, the three usually adopted modulation waveforms (sinusoidal, sinusoidal plus third harmonic and space vector) do not change the motor iron loss.
- With the increase in carrier ratio, the extra iron losses due to the PWM inverter supply decrease because the hysteresis losses are not too different from those in sinusoidal operation, and they practically coincide with the last ones when there are no minor hysteresis loops with winding delta connections. The extra eddy current losses that represent just a portion of the iron losses decreases because of reducing effect of the loss coefficient  $K_{fe}$ .
- The behavior of eddy current losses when the modulation ratio increases is similar to that of the first harmonic contribution (that is similar to the contribution in sinusoidal

operation), but it is nearer to that of the limit losses as lower is carrier ratio.

- The PWM-induced copper losses can increase with even more than 50% depending on the winding design, the fundamental frequency and the degree of filtering.
- Iron plus stray loss contributions in total loss may become very important as soon as the frequency surpasses 250–2500 Hz (depending on motor power rating).
- The iron losses due to the possible presence of the sub-harmonics are always negligible compared with contributions of the fundamental and of higher harmonics.
- Dip proof machines are found to be less affected by harmonic distortion than the totally enclosed machines. Efficiency plays an important role in the degree of derating. Less efficient machines would require a higher derating. It is also clear that smaller machines (less than 5 kW) are affected more than are larger machines.

## 11. Utilization of undesirable time- and space-harmonics

The output waveform of the present day static controllers deviates substantially from the sinusoidal form and contains wide spectrum of time harmonics. Present trends in the research indicate that efforts are exclusively directed to modify the design of the controller unit in order to eliminate, suppress or filter-off the objectionable time harmonics from the output supply waveform. Success has been achieved to a limited extent at the cost of the added complications, which has reduced the reliability and increased the cost of controller unit. Bhattacharyya [164] and Bhattacharyya and Singh [165] have suggested an alternative approach to the above problem. This methodology termed as ‘Dual Convergence Approach’, (DCA) contemplates a gainful use of the till-believed undesirable time harmonics in the supply system for production of ‘aiding’ torque through interactive effects of space harmonics deliberately introduced in conductor distribution through careful design of the winding. A detailed theoretical and experimental study is included in Ref. [166,167]. The conclusions drawn from these studies are:

- The deliberately introduced space-harmonics in the winding distribution lead to a marginal increase in no-load current and total iron losses when compared to a standard machine.
- The overall power factor of the machine may be only marginally inferior to that displayed by a standard machine working at the same level of load with similar terminal conditions.
- Machine designed with DCA draws lesser current at rated load as compared to a standard machine.
- The torque produced by the machine with specially designed armature using DCA show marked improvement over the entire motoring range.

- The increase in torque leads to a reduced slip of operation of such a machine, thereby increasing the output of the machine for a given speed. With careful design, it is possible to realize a proportionally higher increase in output than the corresponding increase in losses, thus resulting in more efficient energy utilization.

## 12. Conclusion

The modern methods of static frequency conversion have liberated the induction motor from its historical role as a fixed speed machine, but the inherent advantages of the adjustable-frequency operation cannot be fully realized unless a suitable control technique is employed. The choice of control strategy is vital in determining the overall characteristics and performance of the drive system. Also, the power converter has little excess current capability; during normal operation, the control strategy must ensure that the motor operation is restricted to the regions of high torque per ampere, thereby matching the motor and inverter ratings and minimizing the system losses. The intensive investigations spread over last three decades indicate the technical and economic viability of adjustable-speed induction machine drives. The technology of adjustable-speed induction machine drives has many advantages to offer over the conventional dc and synchronous drives in terms of precise and continuous control of speed, torque or position with long term stability, good transient performance and high efficiency. The simple and robust rotor construction permits reliable maintenance-free operation at high speed and results in a lower cost motor and higher power/weight ratio. Substantial progress has been made in the induction motor operation with non-sinusoidal supply waveforms research covering analysis, simulation, and hardware development and testing. However, many problems and issues, especially those related to the development of protection schemes, efficient fault analysis tools, etc., still need to be addressed to brighten the prospects for broad industrial applications of this technology.

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