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Network-constrained Cournot models of liberalized electricity markets: the devil is in the details

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Abstract

Numerical models of transmission-constrained electricity markets are used to inform regulatory decisions. How robust are their results? Three research groups used the same data set for the northwest Europe power market as input for their models. Under competitive conditions, the results coincide, but in the Cournot case, the predicted prices differed significantly. The Cournot equilibria are highly sensitive to assumptions about market design (whether timing of generation and transmission decisions is sequential or integrated) and expectations of generators regarding how their decisions affect transmission prices and fringe

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generation. These sensitivities are qualitatively similar to those predicted by a simple two-node model.

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Keywords: Electric power; Electric transmission; Cournot; Oligopoly models; Numerical market models; Optimization

1. Introduction

Due to advances in mathematical modeling capabilities, numerical models of strategic behavior in power networks are gaining increasing attention. Such models are being used to support policy decisions on market design, merger analysis, market coupling and other regulatory tasks (e.g., Nordic Competition Authorities, 2003).

Several research groups have developed mathematical models for calculating short-term Cournot equilibria among oligopolistic generators. This paper is the result of workshops in which three of these models were compared. We wanted to understand the representation and implications of different assumptions. These assumptions include the role of fringe generators, the timing of the energy and transmission markets, different levels of bounded rationality of strategic generators, and energy contract coverage by generators. These assumptions can alter the results of policy analyses; for example, two of the models considered here (Cambridge II and COMPETES) have been used in cooperation with the Dutch regulator to examine proposals for coupling the Belgian and Dutch markets, and they arrived at opposite conclusions about whether prices would decrease for Dutch consumers (Neuhoff, 2003a; Newbery et al., 2003). In contrast, these assumptions should not matter if producers are price takers; therefore all models should (and indeed do) predict the same equilibrium prices in the competitive mode. The focus was on short-term output decisions, and therefore long-term contracting, investment decisions, threat of entry and regulatory threats are only briefly discussed.

In strategic models, the main issue to be considered is the causal relationships between different components of the market. For example, do strategic generators assume that their output decisions directly influence the output of fringe generators? The models show that such additional responsiveness can reduce equilibrium market prices. However, the inclusion of power network constraints complicates the results in sometimes surprising ways. For instance, even when average prices fall, differences in locational prices imply changes in the allocation of transmission capacity such that net imports into some nodes can be reduced, inducing local price increases.

Another issue is the sequence of energy and transmission markets. In the highly meshed European electricity network, two designs for the allocation of scarce transmission capacity are currently debated (Boucher and Smeers, 2002). In the separated transmission and energy markets design (which now prevails for between-country transactions), transmission capacity is first allocated in an auction, and then local energy spot markets clear. It can be argued that generators are likely to consider only the elasticity of local demand when making output decisions. In contrast, in the integrated energy and transmission market (also called nodal/zonal pricing, market splitting, or market coupling), the system operator accepts energy bids at all locations and clears the energy markets using available transmission capacity. The integrated market design implies that transmission capacity is allocated in reaction to the energy bids of strategic generators, which can increase effective demand elasticity (Neuhoff, 2003b), assuming that generators correctly anticipate how their actions will affect transmission prices. This additional responsiveness can mitigate market power and, in the example of this paper, reduces prices in all zones compared to the market design with separate transmission and energy markets, confirming previous results (Ehrenmann and Neuhoff, 2003).

A third strategic modeling issue concerns bounded rationality. One could assume that strategic generators simplify the world when deciding on their optimal outputs (Barquin and Vazquez, 2003). This bounded rationality could imply, for instance, that a generator always presumes that the last period's transmission constraints will remain binding and that no other transmission constraints will limit the solution when calculating the equilibrium Nash output decisions of his fellow strategic generators and himself. If the anticipated constraints coincide with the realized constraints then the beliefs of the agents are consistent, and the algorithm converges to a Nash equilibrium. In our calculations, the approach resulted in similar prices to the full rationality model of an integrated market design, although the solution procedure often yielded oscillations.

Unfortunately, the mathematical model for an integrated market is inherently nonconvex and difficult to solve. The structure of that model is an "equilibrium problem with equilibrium constraints" (EPEC), and may have either no or several pure strategy equilibria (Daxhelet and Smeers, 2001; Ehrenmann, 2004; Hobbs and Helman, 2004). One way to avoid the related difficulties is to assume that each producer is Bertrand with respect to transmission prices while still playing a Cournot game against other producers (Metzler et al., 2003). The Bertrand assumption can be interpreted as a kind of bounded rationality: strategic generators do not anticipate that they will influence transmission prices. Under this assumption, their output decision is less affected by transmission constraints and therefore they bid more competitively. In our data set with a highly meshed network, the Bertrand approach gave lower prices than if generators correctly predict how their outputs affect transmission prices.

Joskow and Tirole (2000) show that strategic generator at an importing node will withhold more output if they own import transmission contracts (both financial or physical). For two reasons, we assume in our models that strategic generators do not own or acquire transmission contracts. First, we perceive it to be extremely difficult to solve a three-stage model that represents both the allocation of transmission contracts and the interactions in the spot markets. We therefore focus on understanding the implications of different approaches to design and model spot markets. Second, Gilbert et al. (2004) show that strategic generators will not obtain market power enhancing contracts in arbitraged uniform price auctions, and then suggest methods to restrict ownership of market power enhancing transmission contracts. The Netherlands already have rules in place to restrict the ownership of transmission contracts by strategic generators.

The purpose of this paper is to document differences in the results of these various Cournot model approaches, and to relate those differences to the assumptions made. In Section 2, we review the literature on models of oligopoly on power networks, and then summarize the data set in Section 3. In Section 4, a Cournot model of strategic generators in networks is introduced. We then expand the model to allow comparisons of different cost function approximations (Section 5), and to include more complex interactions such as generator anticipation of the reaction of fringe suppliers (Section 6) and generator expectations about the reaction of the system operator and transmission prices (Section 7). Section 8 discusses how simplifications concerning those expectations can facilitate computation, but also might change the results in systematic ways. Additional features that enhance model realism, such as long-term contracts, are presented in Section 9. Conclusions in Section 10 close the paper, including results of a poll of modelers and model users on additional features they would like included in future models. Appendix A illustrates some of the basic differences among the models using a simplified analytical framework and a two-node example, while Appendix B provides additional information on the assumptions and solution procedures for each of the models. Appendix C presents the methodology and detailed results of the poll on additional features.

2. Literature review

Because transmission constraints can isolate markets and enhance market power, a number of models of strategic interaction on networks have been developed (see reviews by Daxhelet and Smeers, 2001; Day et al., 2002; Ventosa et al., 2005). Most models of generator competition take a general approach of defining a market equilibrium as a set of prices, generation amounts, transmission flows, and consumption that satisfy each market participant's first-order conditions for maximizing their net benefits while clearing the market. If a market solution exists that satisfies this set of conditions, it will have the property that no participant will want to alter their decisions unilaterally (as in a Nash equilibrium). Although it is recognized that no modeling approach can precisely predict prices in oligopolistic markets, there appears to be agreement that equilibrium models are valuable for gaining insights on modes of behavior and relative differences in efficiency, prices, and other outcomes of different market structures and designs (Smeers, 1997).[2]

Equilibrium market models differ in many ways, including the market mechanisms modeled, the type of game assumed, fidelity to the physics of power transmission, and computational methods. Regarding market clearing mechanisms, most studies of generation markets implicitly or explicitly assume a single buyer or "pool"-type

[2] There are, of course, other ways to project the extent and impacts of market power, including empirical studies based on past behavior in the market of interest or similar markets (e.g., Borenstein et al., 2002), experiments with live subjects (e.g., Schuler et al., 2001), and simulations using artificial automata (e.g., Bower and Bunn, 2000). The approaches have complementary advantages. The main advantages of equilibrium models are verifiability and replicability, the ability to prove general results, ease of computation, and their grounding in accepted economic and game theoretic concepts.

centralized bidding process supervised by an Independent System Operator (ISO) (e.g., Cardell et al., 1997). This process results in a set of publicly disclosed market clearing prices. Other studies model bilateral trading with or without the presence of traders/arbitragers (Metzler et al., 2003; Wei and Smeers, 1999). Some studies assume that transmission services and energy markets are cleared simultaneously or are well arbitraged, while others assume a sequential process. The practical differences between these formulations are one focus of the comparisons we report in this paper.

Turning to the type of game represented, most models assume some type of Nash game. Cournot models are the most popular, and are the focus of this paper. Network constraints are treated in a variety of ways in Cournot models. Some assume that generators are price takers (Bertrand) relative to the cost of transmission (Blake, 2003; Metzler et al., 2003; see Section 8, infra), while more sophisticated (but computationally more difficult) models represent generators as being Stackelberg leaders with respect to transmission pricing and allocation decisions by transmission system operators (TSOs) (Borenstein et al., 2000; Cardell et al., 1997; see the 2-stage model of Section 7, infra). Each generator's problem is a MPEC (mathematical program with equilibrium constraints, the constraints being the TSO's first-order conditions), and the market equilibrium problem is an EPEC. The quantity strategies in the Cournot models are usually the amounts generated and sold by existing power plants. However, a few multistage dynamic Cournot models have been formulated that also include investments in new plants as strategic variables, although transmission constraints are disregarded. An open-loop dynamic model is presented by Chuang et al. (2001), while Murphy and Smeers (forthcoming) propose a closed-loop model.

Nash games in other types of strategies have also been modeled in transmission constrained markets, including games in prices (Hobbs and Schuler, 1985) and games in supply functions, in which each firm submits a schedule of the quantities it is willing to deliver under different prices. As an example of the latter, system operators in Denmark and California have applied models in which each company decides a fixed amount or percentage by which it will mark up all its marginal costs when constructing bids (Kristoffersen et al., 2003; London Economics, 2003). As further examples, Hobbs et al. (2000) and Weber and Overbye (1999) have represented bidding games among competitive generators who are also Stackelberg leaders with respect to TSO decisions about transmission. Research is also being done on other symmetric games among generators, such as conjectural variations (Garcia-Alcalde et al., 2002), conjectured rival supply functions (Day et al., 2002), and "supergames" in which collusive solutions are bounded by incentive compatibility constraints (Harrington et al., 2003). In addition, there are a few models of asymmetric games – in particular, Stackelberg games – in which larger generators act as Stackelberg leaders with respect to a set of smaller generators who are either Cournot players or price takers (e.g., Chen et al., 2003). One of the models compared in this paper is of the latter type, in which the competitive fringe is modeled as a Stackelberg follower (Section 6, infra.).

Early research in power markets usually disregarded transmission constraints or considered only Kirchhoff's current law, disregarding the voltage law that forces power to flow in parallel paths. However, because the voltage law results in tighter constraints in flows and can yield surprising pricing results, more recent transmission-constrained models have included both of Kirchhoff's laws.[3] This is usually accomplished using the linearized "DC" load flow model (Schweppe et al., 1988), in which constant "power transmission distribution factors" PTDF_{ijk} describe how many MW of flow occur on a particular line k in response to an assumed injection of 1 MW at node (or "bus") i and a matching withdrawal of MW at bus j. The DC model's linearity allows use of the principle of superposition, which simplifies load flow calculations for market models relative to the complete nonlinear AC load flow model. The DC model disregards reactive power flows and voltage constraints, and usually excludes calculations of resistance losses. Only very few oligopoly models have either included nonlinear resistance losses in the DC model (Chen et al., 2003) or a full AC representation (Bai et al., 1997). Most models that represent both of Kirchhoff's laws are formulated as Cournot games.

Finally, turning to solution methodology, a variety of approaches have been used to solve equilibrium models that are sufficiently complex that closed form solutions are not possible. One basic approach is to discretize the strategy space, and then either examine all possible combinations for possible Nash equilibria (Bai et al., 1997), or heuristically search for an equilibrium (Kristoffersen et al., 2003; London Economics, 2003). The former is possible only with very small models and limited numbers of strategies. In order to make the computational burden reasonable, discretization of the decision space has been used for models that represent complex features such as AC load flow-based models or network-constrained supply function equilibrium models. The alternative is to retain the continuous strategy space. If first-order (Karush–Kuhn–Tucker, KKT) conditions can be defined for each market player's optimization problem, then an equilibrium can often be found by simultaneously solving n equilibrium conditions (including each player's KKT conditions and market clearing) for n variables representing the decisions and market quantities and prices.[4]

Equilibria for models with continuous strategy spaces can also be sought using diagonalization (a Gauss–Seidel-type algorithm) (Cardell et al., 1997; Ehrenmann and Neuhoff, 2003; Hobbs et al., 2000; Weber and Overbye, 1999). In each iteration of diagonalization, one agent chooses the optimal value of its strategic variables taking as fixed the decisions made by other firms in previous iterations. Once no firm wants to change its decisions, a Nash equilibrium has been found. In terms of the above classification, the models compared in this paper are Nash–Cournot equilibrium models of generator competition on linearized DC networks, in which continuous strategy spaces are considered and solutions are obtained either by diagonalization or simultaneous solutions of equilibrium conditions. A variety of market-clearing mechanisms are considered. The

[3] As an example of the counterintuitive effects of Kirchhoff's Voltage Law, an addition of a transmission line can lower the transmission capacity of a system (Wu et al., 1996); an optimal strategy of a Cournot generator can be to increase output in order to congest lines and keep out competition (Cardell et al., 1997); and decreased market concentration can increase prices by worsening transmission congestion at critical locations (Berry et al., 1999; Hobbs et al., 2004).

[4] In general, the complete set of KKT and market clearing conditions defines a *mixed complementarity problem* (MCP). The general form of a MCP is as follows: find vectors $\boldsymbol{x}$, $\boldsymbol{y}$ that satisfy the conditions $\boldsymbol{x}\geq 0$, $\boldsymbol{f}(\boldsymbol{x},\boldsymbol{y})\leq 0$, $\boldsymbol{x}^{\mathrm{T}}\boldsymbol{f}(\boldsymbol{x},\boldsymbol{y})=0$, and $\boldsymbol{g}(\boldsymbol{x},\boldsymbol{y})=0$, where $\boldsymbol{f}$ and $\boldsymbol{g}$ are vector valued functions. There should be exactly as many conditions as variables.

models considered here and their distinguishing features are summarized in Table 1. A simplified version of their mathematics is given in Appendix A, while more detail on their assumptions and solution procedures is provided in Appendix B.

- Cambridge I (Ehrenmann and Neuhoff, 2003): Cournot generators assume that all of their output is sold at the location where generated. By the time energy bids are submitted the TSO or traders can no longer alter the amount of imported or exported electricity. Thus, generators consider only local demand response. As an option, behavior of the fringe generators can be chosen as non-responsive (1-stage game) or responsive (2-stage game) to the generators' output choices.
- Cambridge II (Ehrenmann and Neuhoff, 2003): Cournot generators anticipate the TSO will arbitrage price differences by changing the amounts of power it buys and sells at different nodes. The TSO allocates transmission rights to traders as suggested by Chao and Peck (1996)such that the allocation would maximize social welfare if the bids submitted by all participants were competitive. This standard assumption for the TSO will be retained throughout the paper. Cambridge II is a 2-stage model in which the optimality conditions of the TSO are included as constraints in the generators' problems. These optimality conditions are generally nonconvex, converting the generator's problem into an MPEC. The fringe generators' output quantities are modeled as being responsive to the strategic generator's output decisions.
- Madrid (Barquin and Vazquez, 2003): like Cambridge II, Cournot producers anticipate how their output decisions will affect transmission prices, but under the further assumption that those decisions will not change which transmission constraints are binding. This convexifies the TSO optimality conditions in the producers' constraint set, facilitating computation. An iterative procedure is proposed in order to obtain a set of binding transmission constraints consistent with both generator and TSO actions.
- Netherlands Energy Research Center (ECN) COMPETES (Cournot generators version) (Hobbs et al., 2004: Cournot generators are Bertrand (price-taking) with respect to the price of transmission services, which is a further simplification (but a computationally

Table 1
Summary of models in comparison

Model assumption	ECN (COMPETES)	Madrid	Cambridge I, II
Cournot or conjectured supply function	Both	Both	Cournot
2-Stage implementation (generation-transmission)	No	Yes	Yes
Transmission reaction to generation	No	Approximated (fixed set of binding constraints)	Yes
Transmission contracts	Both	No	No
Strategic generation exceptions of fringe generation	Cournot (fixed output)	Cournot	Cournot or actual response
Cost functions	Piecewise linear (PL), or step	Continuous or PL	PL

convenient one) of the full 2-stage Cambridge II model (a further optional simplification is to constrain generators to sell only in their local market, like Cambridge I). COMPETES also offers the option to use linear or stepwise cost functions for generators.

3. Test system

3.1. Transmission network

A realistic case study was created for the purposes of this model comparison.[5] Fig. 1 shows the representation of the electricity grid that we applied. This representation contains fifteen nodes and is derived from a more detailed representation of the network that was already used in COMPETES. Demand for electricity and production capacity is allocated to seven of these nodes, while the other nodes are used as intermediate nodes of the linearized DC network. In the Netherlands, demand and production is allocated to three nodes (Zwol, Krim, Maas), in Belgium demand and production is split between two nodes (Merc, Gram), and in France and Germany all demand and production is allocated to one node (respectively F and D).

Fig. 1 shows nodes being interconnected by 28 different transmission links. Each link has a flow constraint (in MW) in each direction. All links are characterized by power transmission distribution factors (PTDF_{ijk}) for all nodes.

In operating an electricity network, the operator should take into account the possibility of line and other equipment outages in order to avoid possible interruptions of supply. For this purpose, TSOs have developed the so-called "$n-1$ principle" as a security criterion to ensure a reasonable level of network reliability. It means that a network must be able to cope with a sudden out-age of one of the network components without interrupting the supply of electricity. For the purposes of our study, when calculating the PTDFs and the upper limit of the links, one critical contingency – an outage of one of the two "Selfkant" tie line circuits between the nodes Maas and Romm (Haubrich et al., 2001) – is considered. Therefore instead of two circuits, only one circuit is available on this link. The capacity of the link is reduced and the increased reactance changes the values of PTDF_{ijk} relative to a non-security constrained world.

3.2. Generators and their capacity

Eight firms are considered as strategic generators with production in one or several countries. These strategic generators are shown in Table 2 in the countries where they generate. Production units in Belgium and Netherlands are divided between the nodes in each country according to location. In Germany and France all units are allocated to the national nodes *D* and *F*. Generation plants not owned by one of the strategic players are assumed to bid competitively.

[5] For a detailed listing of the problem data, please refer to www.electricitymarkets.info/modelcomp/index.html. Data sources are summarized in Hobbs et al. (2004).

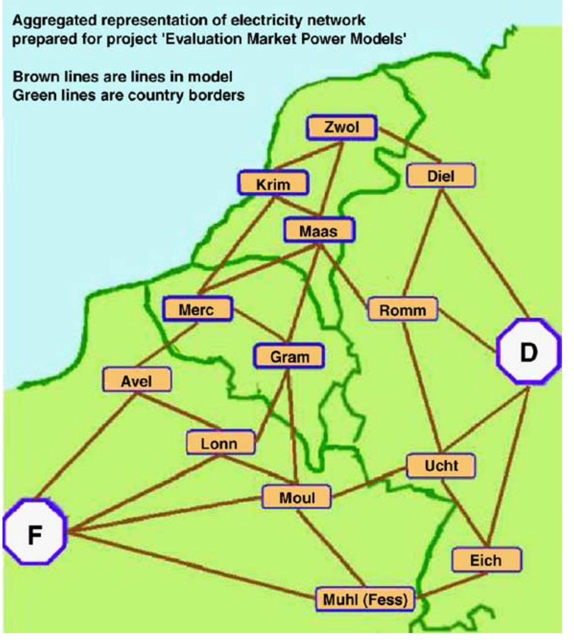

Fig. 1. Network used in the model comparison.

Variable costs of generators for each company and each node are represented in two distinct ways: first, as a two-part linear cost curve and second, as a four-step cost function. These functions are derived from cost and capacity data on the 5272 generating units in the study area.

3.3. Demand data

To create ten different demand scenarios, actual loads for the summer and winter peak have been used as the first two scenarios. Then an additional four summer and four winter scenarios were produced by reducing the peak with a scenario- and location-specific random factor. We assume that these values represent the quantities demanded at a price of 30 Euro/MWh. Their ranges are shown in Fig. 2. An affine demand curve is then calibrated for each node in each period by assuming that the demand elasticity is 0.1 at the 30 Euro/MWh price.

Table 2
Strategic players by country

Countries	Strategic companies
Germany	EOn, ENBW, RWE, Vattenfall, EDF
Belgium	Electrabel
France	EdF, ENBW
Netherlands	Essent, Nuon, E.ON, Electrabel

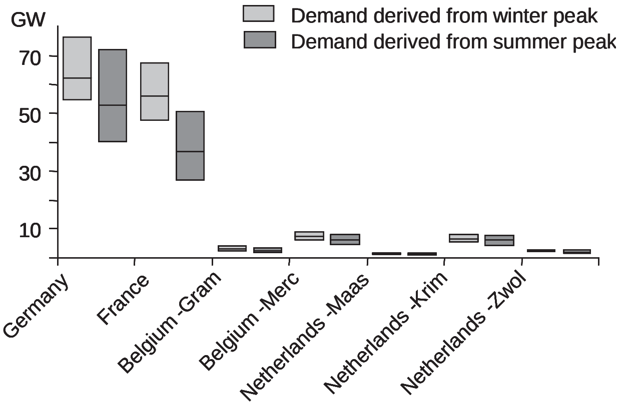

Fig. 2. Ranges of demand levels in MW at the nodes for five summer and five winter scenarios.

All participating models calculated the competitive case with this data set assuming price-taking behavior by generators, obtaining the same equilibrium (Fig. 3).

4. Separate energy and transmission markets

The model discussed in this section represents strategic generators that decide on their outputs assuming that their decisions will not affect the output choices of competitors (Nash–Cournot) or fringe generators, nor the amounts of power moved by traders from one location to another. Generators only sell where they produce their electricity. Traders move power from one location to another to arbitrage price differences. A key implication of these assumptions is that each producer believes that

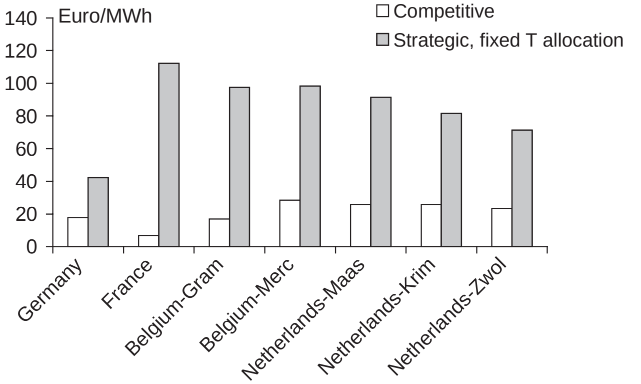

Fig. 3. Prices in competitive solution and strategic solution with non-responsive transmission allocation and fringe generation (averaged over 5 summer scenarios) (competitive solutions calculated by price-taking versions of all models; strategic solutions calculated by Cambridge I and restricted version of COMPETES).

any change in output by any of its generation units must be absorbed by local demand; this yields a relatively inelastic residual demand curve at each node (and, as we shall see, higher prices than under alternative assumptions). Obviously the approach of separate energy and transmission markets is only viable if a 'node' in this abstract model does in reality contain various generators and load to provide some net-demand elasticity. We refrain from using the term zonal pricing in this paper, because zonal pricing can create intrazonal transmission congestion, which this paper does not address.

Under these assumption, an equilibrium is achieved if each generator chooses its profit maximizing outputs for all their plants, the TSO achieves the welfare maximizing allocation of transmission rights assuming bids for transmission are competitive, and the competitive fringe produces until its marginal cost equals the locational price or it faces capacity constraints. This model has the nice feature that with convex costs it becomes a convex problem for each player. Players each have a compact set of output choices, which does not depend on choices by other players; therefore existence of a solution is guaranteed (Harker and Pang, 1990).

Two approaches can be used to find an equilibrium. First, the KKT conditions of all agents can be compiled together with market clearing conditions into one complementarity problem (e.g., Wei and Smeers, 1999; Kemfert and Tol, 2000) and solved by algorithms such as PATH (Dirkse and Ferris, 1995). Second, using a diagonalization algorithm, the optimization problems for each strategic generator, the system operator, and each fringe generator are solved sequentially. Here, a version of COMPETES consistent with the above assumptions was solved using the MCP approach, while Cambridge I applied diagonalization (this is a restricted version of COMPETES, since normally it is instead formulated assuming that generators can export power to other nodes). For all scenarios, both models yielded the same equilibria, as would be expected since, under these assumptions, their formulations are equivalent.

Fig. 3 gives the equilibrium prices for this case for all nodes averaged over five summer scenarios.[6] The calculated strategic prices exceed competitive prices by a large margin. These prices are high because of the low price elasticity for each location's demand curve. There is no demand response from other locations, because transfers to these locations are not adjusted in response to deviations of the generator's output choice from the expected equilibrium.

5. Step vs. linear cost function

The previous calculations used continuous, linearized costs functions with two segments as illustrated in Fig. 4. However, in general, power market simulation models use a variety of cost function approximations, and it is important to understand whether choice of approximation is important. To see whether linearization affects the results, the

[6] The complete numerical results for all models are available at www.econ.cam.ac.uk/electricity.

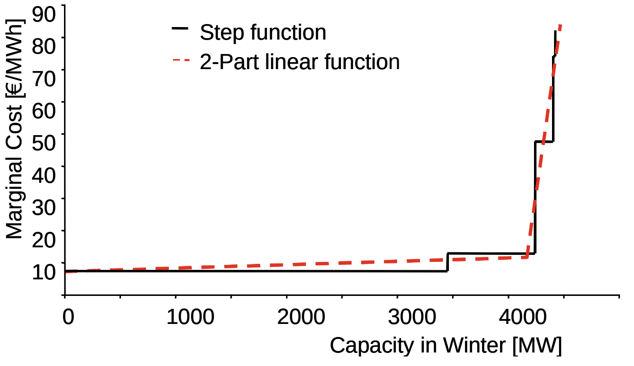

Fig. 4. Marginal cost curves for Electrabel generators at node Gram.

two models from Section 4 were also run with step cost functions having up to four steps. The mean price increase was 1 Euro/MWh (standard deviation=1.6 Euro/MWh). This impact is small relative to price levels.[7]

6. Strategic generator anticipates fringe reaction

The 1-stage Cournot model in Section 4 assumes that each strategic generator does not anticipate that fringe generation will react to their strategic output decisions. This can be justified as bounded rationality in the form of generators paying limited attention to the detailed market structure. However, typical spot market arrangements allow fringe generators to submit several bids to the spot market, such that they are called to produce if the day-ahead price exceeds their marginal cost. If strategic generators understand this mechanism, then they will anticipate a change in fringe supply when calculating the profits they could earn with different output choices. In real power markets, demand actually has little short-run elasticity, so that the fringe (including imports) provides most of the elasticity in the effective demand curves facing generators (Bushnell, 2003).

One approach to representing the reaction of the fringe is to approximate its effect on strategic producers by increasing the elasticity of the demand functions (ibid.). However, this would distort estimates of flows and total production; for example, higher prices would then be modeled as decreases in quantity demanded, rather than as increases in fringe output. Another approach is to explicitly represent the reaction of fringe generators to the output of strategic generators in a 2-stage (Stackelberg) model. In the second stage, the fringe generators (Stackelberg followers) choose their outputs such that their marginal costs equal the local spot price. Then in the first stage, the strategic generators (Stackelberg leaders) choose output to maximize their profits and the TSO decides on transmission

[7] The same calculations were performed for responsive transmission allocation with Bertrand assumption (see Section 8). The stepped cost function increased average prices by 0.7 Euro/MWh with a standard deviation of 2 Euro/MWh.

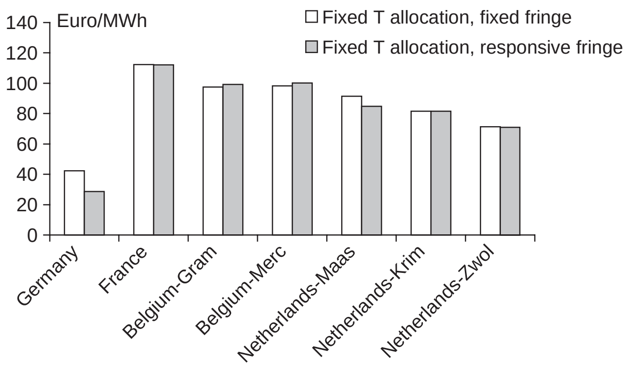

Fig. 5. Effect of allowing generators to assume that fringe output reacts to their output decision (averaged over five summer scenarios) (two-stage Cambridge I model).

allocation to maximize welfare. Both decisions are made in anticipation of the reaction of demand and fringe generation in the second stage.

However, the 2-stage approach destroys the nice properties of the 1-stage model that its solution exists and is usually unique. This is because capacity constraints or kinks in cost functions of fringe generators in the second stage translate into non-concavities in the profit functions in the first-stage model, which is a mathematical program with equilibrium constraints (MPEC). Algorithms commonly used to solve such problems can only ensure local stationarity of a solution.[8] It is difficult to prove that the local MPEC solution is global because of the large strategy space. The problem of calculating a Cournot equilibrium among such MPECs is an equilibrium problem with equilibrium constraints (EPEC); in general, uniqueness (and even existence) of equilibria cannot be guaranteed. Nevertheless, a 2-stage version of the Cambridge I model succeeded in calculating an equilibrium set of output choices and prices, although multiple solutions were found for some of the scenarios. However, differences between the solutions were small relative to differences between those results and solutions found for other market designs (such as the integrated transmission-energy models in Section 7) (see Ehrenmann and Neuhoff, 2003).

Fig. 5 shows that the additional demand elasticity provided by fringe generators induces generators to increase their outputs. The result is lower prices at most nodes, broadly consistent with the simple two-node example in Appendix A. The effect is particularly strong in Germany because of the large amount of fringe generation.

Also, Netherlands–Maas has more fringe generation than other Benelux nodes, which causes a greater price decrease at that location than elsewhere in Belgium and the Netherlands. Due to changes in prices, the equilibrium allocation of transmission capacity by the TSO also differs, with smaller net imports at some nodes. If the reduction of net imports exceeds output increases by generators, as in Belgium–Merc, then local equilibrium prices can actually be higher.

[8] For a maximization problem we call a point x^* B-stationary for a function $f: R^n \rightarrow R$ if its directional derivative $f'(x^*, d) \leq 0$ for all feasible directions d.

7. Integrated transmission-energy market design

In the models of the previous sections, strategic generators assume that net imports by traders (or the TSO) to each of the nodes do not change as result of their production decisions. However, in some market designs, TSOs integrate the energy and transmission markets, and their allocation of transmission capacity depends on the bids submitted by generators, e.g., as in PJM or Nordpool. If local generators submit less energy to one node, then the higher price at the node makes energy imports into that node more valuable. As a result, more of the scarce transmission capacity will be used to transmit energy to that node, effectively increasing the net demand elasticity generators face at any one node (Neuhoff, 2003b).

In Cambridge II, the strategic generators not only anticipate the reaction of fringe generators, but also the reaction of the TSO to their output decisions. This is represented by using the KKT conditions of the TSO and the fringe generators as constraints in the optimization problems of strategic generators (Ehrenmann and Neuhoff, 2003), which thereby become MPECs. Calculating an equilibrium among the generators in this case is a nonconvex EPEC. As mentioned in Section 2, some previous Cournot models have represented this Stackelberg game between generators and TSO in a similar manner (Borenstein et al., 2000; Cardell et al., 1997).

Cambridge II uses a diagonalization approach to solve the EPEC. The diagonalization converged for six scenarios. For two scenarios the diagonalization did not converge but created a sequence of prices with deviations of less than 10^{-5}, which might be attributed to numerical difficulties. In one case the sequence cycled with variations of up to 20%, indicating that a pure strategy Nash equilibrium either does not exist or was not found by the algorithm.

Fig. 6 shows that increased demand elasticity that generators anticipate because of the energy-transmission market integration increases competitiveness and therefore yields lower prices in all locations, relative to the base Cournot solution (Fig. 3) (this result is generally consistent with Appendix A's two-node example.) The basic reason is that

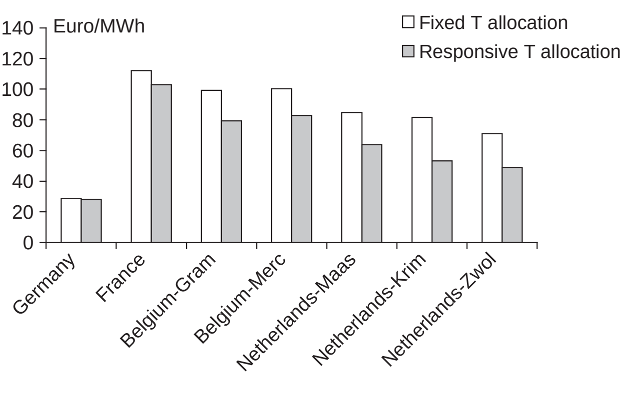

Fig. 6. Generators anticipate TSO transmission allocation decisions as response to their output decision (averaged over five summer scenarios) (Cambridge II model).

producers now recognize that they can compete not only for load at their own locations, but also elsewhere, subject to transmission limitations; further, they are also competing with generators located throughout the system. However, the prices are still well above competitive levels (cf. Fig. 3).

8. Simplified model of transmission response to output decisions

Solving 2-stage models is an active field of research in part because of their analytical and computational difficulty. First, individual MPECs may have multiple local solutions. Second, one can easily construct a network that has no pure strategy equilibrium solution to the EPEC (e.g., Berry et al., 1999). Madrid and COMPETES use alternative ways to simplify the integrated market design, thereby avoiding the difficulties of solving a 2-stage model.

The Madrid model makes the assumption that each generator believes that the set of constrained transmission lines will not change with deviations of its output decisions from the equilibrium. This gives a closed-form expression for net demand at all nodes, which can be substituted into the profit function of generators. Then the generators' optimization problems can be solved in a single stage.

The Madrid group complemented the approach with an iterative algorithm to search for a set of consistent beliefs, which can oscillate between different sets of binding constraints (see Appendix B). Unfortunately, such an oscillation means that a full equilibrium has not been found, although it does not rule out the existence of such an equilibrium. Fig. 7 illustrates such an oscillation for the summer peak scenario; fortunately, the upper and lower prices reached during one of these oscillations are typically close to an equilibrium calculated with the full 2-stage approach (Cambridge II).

A second simplification approach is used in the version of COMPETES applied here, and is based upon Metzler et al. (2003). This version uses the Bertrand

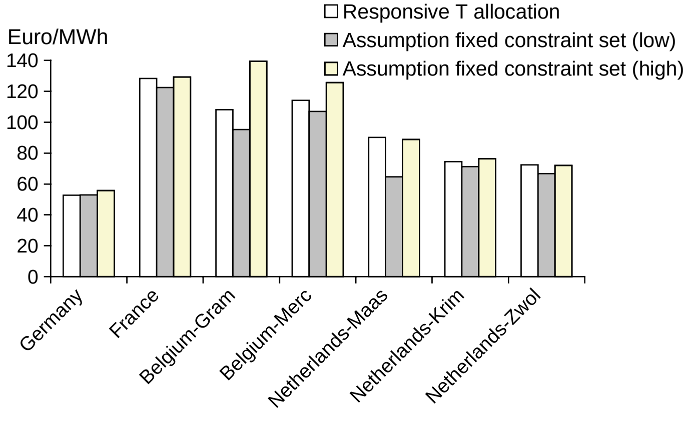

Fig. 7. Lowest and highest price during oscillations of Madrid model with generators choosing output based on fixed set of transmission constraints (for load scenario Summer 1).

assumption for transmission rights: generators assume that the TSO sets prices for moving power from one location to another and offers, in effect, an unlimited amount of rights at this price.[9] Generators therefore assume that transmission prices are exogenous to their optimization problem and are not influenced by their output decisions. This can be interpreted as bounded rationality on the part of strategic generators; such an assumption is more likely to be credible if the transmission system is a grid rather than radial in nature, and if the generation market is not too concentrated. In equilibrium, the TSO will set prices such that transmission constraints are satisfied. To represent an integrated market with the Bertrand assumption, the transmission operator or traders have to determine their transmission flows in reaction to the output decisions of strategic generators. This would still require a 2-stage implementation. However, Metzler et al. (2003) show that if generators are allowed to sell their output at several locations using virtual transmission rights, then the 2-stage problem with a Bertrand assumption for transmission can be implemented as a mixed complementarity problem.[10]

The results of the integrated Cournot market with a Bertrand assumption for transmission prices (COMPETES) are compared to the results of the integrated market with perfect rationality (full 2-stage model, Cambridge II). Fig. 8 shows that for our set-up, the Bertrand assumption results in lower prices than the other strategic models (just as in the two-node case of Appendix A), although those prices are still well in excess of competitive levels (Fig. 3). The two applied models furthermore differ, because in Cambridge implementation, strategic generators anticipate that fringe generation reacts to their output changes. As noted in Section 6, this leads to slightly lower prices for

[9] However, generators are still Cournot players with respect to each others' production and sales.

[10] Virtual transmission rights are sold simultaneously with the energy market. They do not share the typical property of transmission rights of being sold in a contracting stage preceding the energy market. This additional contracting stage usually creates pre-commitment and therefore changes generators' incentives to exercise market power in the spot market (Joskow and Tirole, 2000). Virtual transmission rights do not provide additional contracting incentives and do not influence the exercise of market power. The easiest proof of this is that under nodal pricing the first order conditions for strategic generators are identical with and without virtual transmission contracts.

The profit function of a generator producing at node r and selling quantity s_i at node i consists of the sales revenue minus costs for transmission contracts (which under nodal pricing equal the price difference between the selling node i and the production node r) and production costs:

$$\pi = \sum_i s_i p_i - \sum_i s_i(p_i - p_r) - C\left(\sum_i s_i\right).$$

The optimal sales quantity at any one node is given by the first order condition with respect to s_k:

$$\frac{\partial \pi}{\partial s_k} = p_k + \sum_i s_i \frac{\partial p_i}{\partial s_k} - p_k + p_r - \sum_i s_i\left(\frac{\partial p_i}{\partial s_k} - \frac{\partial p_r}{\partial s_k}\right) - C'\left(\sum_i s_i\right) = p_r + \frac{\partial p_r}{\partial s_k}\sum_i s_i - C'\left(\sum_i s_i\right).$$

This corresponds to the first order condition of the generator only selling at his home node.

The construction of virtual transmission contracts that do not distort the exercise of market power ensures that the COMPETES model can represent an integrated energy and transmission market based on the Bertrand assumption as a 1-stage model. This representation as a 1-stage model facilitates the identification of a solution. Generic local uniqueness and existence of a solution is guaranteed under very mild assumptions (Metzler et al., 2003).

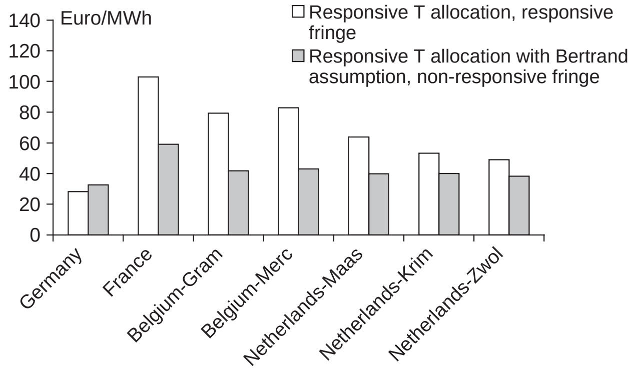

Fig. 8. Comparison of full 2-stage model (Cambridge II) with price-taking behavior with respect to transmission prices (COMPETES).

Cambridge II than would have been obtained if strategic generators instead assumed that the fringe is non-price responsive.

9. Long-term contracts

The above Cournot models showed prices above the levels that are actually observed in those markets (with the exception of the competitive and, possibly, COMPETES results of Figs. 3 and 8 respectively). As a modeler, one is tempted to calibrate the model to match the observed prices. One of several possible approaches is to increase demand elasticities, thereby reducing the extent to which players in Cournot models exercise market power. However, we believe that there are other economic reasons why actual prices are not at the modeled Cournot level. One reason frequently suggested (e.g., Newbery, 1998) is that generators sign long-term contracts for their electricity output. If generators only sell a fraction of their output in the spot market, then they benefit less from pushing up prices by withholding output. At the same time they incur the same costs of forgone revenue on the withheld output, and so are less inclined to withhold output. The gray bars in Fig. 9 represent the results when EDF is modeled in Cambridge II as selling 20 GW forward in France, while Electrabel in Belgium contracts 2 GW each at the nodes Gram and Merc. This reduces prices at all three nodes relative to the situation with only nodal pricing. The higher aggregate production also decreases prices in the Netherlands.

An alternative approach is to assume that an implied or explicit threat of regulatory intervention prevents monopolists from exercising market power. This can be represented by assuming that EDF bids competitively in the COMPETES model.[11] The dark striped bars in Fig. 9 show the resulting prices, and illustrate a dramatic decrease in France and smaller increases elsewhere.

[11] An alternative assumption would be some sort of average-cost-based regulation, but that would require further model development.

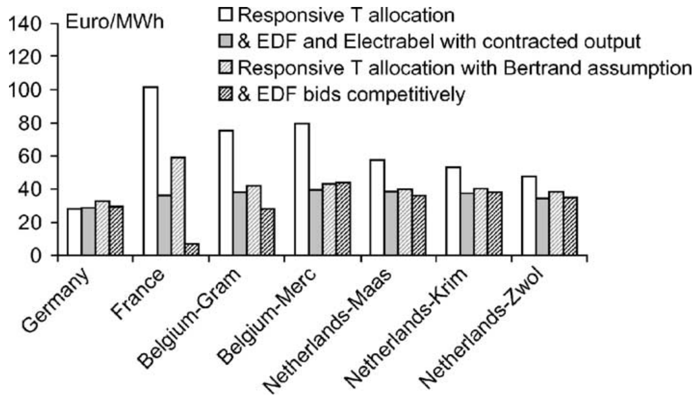

Fig. 9. Prices resulting from changes in Cambridge II and COMPETES assumptions that yield prices closer to observed levels (averaged over five summer scenarios).

Transmission contracts are a further group of instruments that can reduce the market power of generators. Gilbert et al. (2004) show that allocating transmission contracts in uniform price auctions always mitigates the market power of generation companies. The representation of the separate allocation of such contracts before the energy market in numerical models is an out-standing research issue.

Clever choice of reasonable assumptions can be used to replicate any observed price; this is a generic problem with over-parameterized models. For this reason, we based the model comparisons in this paper on models that did not include assumptions about forward contracting.

10. Closing comments

Since the days of Cournot and Bertrand, it has been well known that structural and behavioral assumptions in oligopoly models affect the results. We have shown that even within a family of models (Cournot), assumptions concerning transmission can dramatically affect the solutions of electricity market models. The results of models developed by three research groups have been compared, representing a range of market design and behavioral assumptions.

In the competitive mode, all models predict the same price when using the same cost functions, even though they represent distinct market designs (e.g., separate vs. integrated energy and transmission markets). This confirms that market design is of less relevance if generation companies behave competitively. Furthermore, comparing different cost function approximations, the results for piecewise linear and step-functions are surprisingly similar.

Turning to the strategic models, strategic generators are assumed to be Cournot with respect to each other, and can either anticipate the reaction of the competitive (price-taking) fringe to price changes or adopt the Cournot (fixed output) conjecture regarding fringe output. Cambridge implemented both options and showed that at nodes with significant fringe generation, anticipating the responsiveness of the fringe to price changes reduces the market power of strategic generators and lowers prices.

The just mentioned simulations assumed that generators optimize only against local demand elasticity, because they believe that transmission allocations are fixed (separate and sequential transmission and energy markets). This is, for example, how the Belgian–Dutch–German power markets work, where traders bid in transmission auctions before they know what energy prices will prevail in the region's day-ahead spot market markets. However, there are also integrated market designs that allow the system operator to determine the optimal use of the transmission network after receiving energy bids. All models were used to represent this more demanding modeling task. The Cambridge group uses a 2-stage implementation to represent the complete causality (Cambridge II). Solutions are neither guaranteed to exist nor to be unique. In 6 out of 10 demand scenarios at least one solution could be identified, while in two further scenarios the prices cycled within a small range. In the two final scenarios some nodal prices cycle by 4%, indicating that a pure strategy Nash equilibrium could not be found by the algorithm.

The integrated market was modeled by the Madrid group by making the simplifying assumption that generators have a shared expectation on which transmission lines will be congested. However, their subsequent output decisions could (and often did) result in congestion patterns that differ from their expectations. As a result, their iterative calculation procedure can yield nonconvergent and oscillating congestion patterns. Nonetheless, it is reassuring that the Madrid and Cambridge II model predicted generally similar prices. Meanwhile, COMPETES avoids the numerical difficulties of the 2-stage implementation by introducing a computationally convenient Bertrand assumption for transmission prices: generators assume that transmission prices will not change in reaction to their output decisions. The model provides the additional feature that generators can sell at other locations, simulating bilateral contracts with consumers elsewhere made possible by purchasing transmission services from TSOs. Even though generators in COMPETES are still Cournot players with respect to each others' quantities, prices are significantly lower than in the other two implementations. This shows that the Bertrand assumption for the transmission rights market makes generators bid more competitively than if we assume that they can anticipate their impact on transmission prices.

The results of these alternative model designs and features were discussed at a workshop held in October 2003 in Berlin by eleven experts in power market modeling and regulation, who considered their implications for future model development and comparisons. The experts voted on policy questions that need to be addressed by power market equilibrium models, shortcomings of present models, prerequisites for models to be useful in public forums, and desirable enhancements.

Details on the voting procedures and results are provided in Appendix C. In general, the experts concluded that the most important questions that models should address concern the effect of market power, how it can be mitigated, and generation and transmission investment. Thus, the focus of the models of this paper on strategic behavior is well placed; however, the models have a short-run perspective, and do not represent the dynamics of investment, which are of concern to policy makers. Therefore, the workshop participants would like to see explicit representations of long-run decisions, including allocation of forward contracts, in the models. Although the absence of long-run decisions in present models was of concern to the workshop participants, the survey revealed that an even greater concern is the robustness, assumptions, transparency, and possible misuse of present models. A goal of this paper has been to partially address the latter concern by clarifying the assumptions of various models and assessing their impact on the results. This comparison can help make models more useful in regulation by illustrating what can – and cannot – be learned from them. It is interesting to note that a requirement to match observed market behavior was ranked very low by the participants; insight rather than specific predictions is their desire.

Acknowledgements

We would like to thank David Newbery and the referees for valuable comments, ECN for providing the test data set, the French, Dutch and Belgium Regulators for funding workshops to compare numerical models, and Madrid University and DIW Berlin for hosting these workshops.

Appendix A. Modelling alternative market design assumptions

This appendix presents in mathematical form the basic assumptions of the various models, and analyzes the impact upon equilibrium prices of responsive fringe allocation and the Bertrand assumption regarding transmission pricing (drawing upon Neuhoff, 2003b).

Assume two nodes i=1,2 are connected by one link with transmission capacity K (Fig. 10). For the first calculations we assume that transmission capacity K is so large so that the transmission constraint does not bind. At the exporting node 1, there is both a strategic generator q_s and fringe generators with aggregate production q_f, each with production costs $C(q)$=1/2 q^2. There is demand at both nodes, characterized by a linear demand function:

$$p_i(d_i) = 1 - d_i, \quad i = 1, 2.$$

A.1. Separate transmission, energy (Cambridge I; COMPETES, generators sell locally)

We first calculate the equilibrium of a separate energy and transmission market for the case that strategic generators hold the Cournot conjecture relative to fringe output. We label this as “non-responsive fringe,” in contrast to the next subsection, where the fringe’s response to prices is anticipated by strategic generators. In the non-responsive case, fringe generation sets output q_f and traders determine the amount of energy t that they buy at

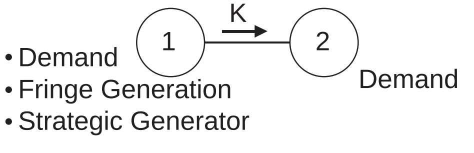

Fig. 10. Two-node network used in the models of this appendix.

node 1 and sell at node 2. The strategic generator sets output to maximize his profits at node one:

$$\pi = p_1 q_s - \frac{1}{2} q_s^2 = \left(1 - q_s - q_f + t\right) q_s - \frac{1}{2} q_s^2$$

The first-order condition gives:

$$q_s = \frac{1 - q_f + t}{3}. \tag{1}$$

The competitive arbitrageurs buy power at one node and sell it at the other, setting t such that prices at both nodes coincide:

$$\left(1 - q_s - q_f + t\right) = p_1 = p_2 = (1 - t), \quad t = \frac{q_s + q_f}{2}, \tag{2}$$

and the competitive fringe chooses its output bid such that marginal cost equals price:

$$p_1 = \frac{\partial C\left(q_f\right)}{\partial q_f}, \quad q_f = \frac{1 - q_s + t}{2}. \tag{3}$$

Solving Eqs. (1)–(3) simultaneously gives the Nash equilibrium output and trade choices and price:

$$q_f = \frac{4}{7}, q_s = \frac{2}{7}, t = \frac{3}{7}, p_i = \frac{4}{7}. \tag{4}$$

A.2. Strategic generators anticipate fringe reaction (Cambridge I)

Now assume that the strategic generators act as Stackelberg leaders with respect to the fringe. They know that the fringe generators submit a price dependent bid to the spot market; therefore the quantities of the fringe generators depend on their output decision. In our model this is represented by defining the fringe production as a function of the output choice of the strategic generator. Substituting Eq. (3) into the generators profit function (1) gives:

$$\pi = p_1 q_s - \frac{1}{2} q_s^2 = \left(1 - q_s - \frac{1 - q_s + t}{2} + t\right) q_s - \frac{1}{2} q_s^2, \quad q_s = \frac{1 + t}{4}. \tag{5}$$

Combining Eqs. (5) with (2) and (3) shows that the responsive fringe induces the strategic generator to increase output. Hence prices fall:

$$q_f = \frac{6}{11}, q_s = \frac{4}{11}, t = \frac{5}{11}, p_i = \frac{6}{11}. \tag{6}$$

A.3. Integrated transmission and energy markets (Cambridge II, similar to Madrid)

What happens if we now allow the strategic generator not only to anticipate the reaction of the fringe but also how the system operator sets transmission in reaction to the observed prices? Substituting Eq. (2) for the responsive transmission allocation and Eq. (3) for the responsive fringe into the generators' profit function (5) gives:

$$\pi = p_1 q_s - \frac{1}{2}q_s^2 = \left(1 - q_s - \frac{2-q_s}{3} + \frac{1+q_s}{3}\right)q_s - \frac{1}{2}q_s^2, \quad q_s = \frac{2}{5}. \tag{7}$$

Using Eqs. (2) and (3) shows that the imported demand elasticity induces strategic generators to further increase output relative to only responsive fringe generation (Cambridge I) and hence price falls:

$$q_f = \frac{8}{15}, q_s = \frac{2}{5}, t = \frac{7}{15}, p_i = \frac{8}{15}. \tag{8}$$

A.4. Bertrand with respect to transmission (COMPETES, generators can sell anywhere)

The Bertrand assumption on transmission implies that generators assume they can buy unlimited transmission capacity at a price that is independent of their output decision. This is reflected in the following profit function of the strategic generator selling output at both locations $q_{s,i}$. Generators assume that transmission prices do not change with their strategic output choice, hence the transmission price in their profit function is denoted as $\overline{p_2} - \overline{p_1}$:

$$\pi = p_1 q_{s,1} + p_2 q_{s,2} - \frac{1}{2}\left(q_{s,1} + q_{s,2}\right)^2 - q_{s,2}\left(\overline{p_2} - \overline{p_1}\right) \tag{9}$$

Substituting again prices p_1=1−$q_{s,1}$−q_f+t and p_2=1−$q_{s,2}$−t and the responsive fringe (3) gives:[12]

$$\pi = \left(1 - q_{s,1} - \frac{1 - q_{s,1} + t}{2} + t\right)q_{s,1} + \left(1 - q_{s,2} - t\right)q_{s,2} - \frac{1}{2}\left(q_{s,1+q_{s,2}}\right)^2$$
$$- q_{s,2}\left(\overline{p_2} - \overline{p_1}\right),$$

The first-order conditions gives:

$$q_{s,1} = \frac{1 + t - 2q_{s,2}}{4}, q_{s,2} = \frac{1 - t - q_{s,1} - \left(\overline{p_2} - \overline{p_1}\right)}{3} = \frac{1 - 3q_{s,1} + t}{4} \tag{10}$$

With the strategic generator selling at both nodes (2) turns into

$$t = \frac{q_{s,1} - q_{s,2} + q_f}{2}, \tag{11}$$

and combining Eqs. (3), (10) and (11) gives:

$$q_f = \frac{8}{15}, q_{s,1} = \frac{4}{15}, q_{s,2} = \frac{2}{15}, t = \frac{1}{3}, p_i = \frac{8}{15}. \tag{12}$$

The identical prices and output choice in Eqs. (8) and (12) of fringe and strategic generator q_s=$q_{s,1}$+$q_{s,2}$ shows that in a situation without binding transmission constraints,

[12] Note however that the actual implementation of COMPETES instead has a non-responsive fringe.

the Bertrand assumption with generators selling at several locations can mimic the calculations for the integrated market design (Cambridge II).

A.5. Cournot and Bertrand assumptions about transmission (Cambridge I, COMPETES)

This is no longer the case if the transmission constraint binds. Returning momentarily to the case where the strategic generation assumes that the quantity of power transmitted is fixed (Cambridge I), we can derive that equilibrium as follows. The amount of energy transmitted by the system operator in (7) is $t=K$ Substituting $t=K$ in Eqs. (1) and (3) gives

$$\pi = p_1 q_s - \frac{1}{2} q_s^2 = \left(1 - q_s - \frac{1 - q_{s,1} + K}{2} + K\right) q_s - \frac{1}{2} q_s^2$$

The first-order condition gives the Cambridge I solution under constrained transmission:

$$q_s = \frac{1+K}{4}, q_f = 3\frac{1+K}{8}, p_1 = 3\frac{1+K}{8}, p_2 = 1 - K. \tag{13}$$

In contrast, if the strategic generator sells at both locations and assumes that unlimited transmission capacity is available at the prevailing transmission price, then he assumes the same profit function (10) as without the binding constraint (COMPETES). The only difference is that now the equilibrium is reached if $t+q_{s,2}=K$. Substituting in Eqs. (10) and (3) gives:

$$\begin{aligned} &q_f = \frac{4}{11}(1+K), q_{s,1} = \frac{2}{11}(1+K), q_{s,2} = \frac{1}{11}(1+K), t = \frac{10K-1}{11}, \\ &p_1 = \frac{4}{11}(1+K), p_2 = 1 - K. \end{aligned} \tag{14}$$

For K=7/15, Eq. (14) coincides with Eqs. (8) and (12). This shows that the strategic generator bids as if no transmission constraint were binding, even so the transmission price is p_2-p_1=7/15–15/11 K. Hence, if the constraint is binding, output of the strategic generator $q_{s,1}+q_{s,2}$=3/11 (1+K) under Bertrand assumption (14) (COMPETES) is larger than if he instead recognizes that the quantity of energy transmitted is fixed (i.e., q_s=1/4 (1+K) in Eq. (13)) (Cambridge I).

A.6. Conclusion of simple comparison

The models of this appendix assume extremely high demand responsiveness, hence the results should only be used for qualitative comparisons of the modelling approaches. When there are no transmission constraints, prices are equalised at both nodes in all models. The equilibrium price in the non-responsive fringe model is 0.57 (4), falls to 0.54 if the fringe is responsive (6) and is further reduced to 0.53 (8) if the transmission and energy markets are integrated. The same results, 0.53, follows if fringe is responsive and generators assume that their output choice does not influence transmission costs (Bertrand assumption).

But if the transmission line is constrained, then the Bertrand assumption matters. Assume that the transmission capacity K=2.0, then the equilibrium price at node one with non-responsive fringe is 0.45 (13) irrespective of whether transmission and energy markets are integrated (Cambridge I, II). If the strategic generator assumes that transmission prices are independent of his output choice (Bertrand assumption, COMPETES), then his output quantity is increased and the equilibrium price at node 1 drops slightly to 0.44. The price at node 2 is only determined by the available import capacity K and hence is model independent.

Appendix B. Description of modelling approaches

B.1. COMPETES

This model (Hobbs et al., 2004) is a generalization of previous complementarity-based models for simulation of Cournot and conjectured supply function (CSF) models of competition among electricity generators on a transmission network. Generating firms compete to sell power at the local price in each market (each node of the network, most generally), and they pay a TSO for transmission services from their generators to the point of sale. COMPETES is a static equilibrium model in which power producers either play a Cournot game at each node in the network (as in Metzler et al., 2003) or a more sophisticated CSF game in which producers anticipate that rival supply will respond in a locally linear manner to perturbations in energy prices (as in Day et al., 2002).

Regarding transmission, the model assumes that generators are Bertrand price takers with respect to the transmission services market, which is represented as being constrained by a linearized DC load flow. The price charged to generators consists only of congestion-based fees needed to clear the market for transmission services. COMPETES has also implemented two options that generalize this transmission representation. First, an individual generator can conjecture that transmission prices will change if it alters its demand for energy services, as opposed to the Bertrand assumption. Thus, a large generator upstream of a transmission constraint might recognize that if it decreases the amount of transmission services it requests, the price for services will decline. Just as for the CSF model, the degree of price response is an exogenous parameter.

The second option concerning transmission allows for inefficient pricing mechanisms, reflecting actual institutions. These inefficient mechanisms include: lack of netting of transmission flows (no credit for congestion relief by counterflows), export taxes, and path-based transmission pricing (disregarding of parallel flows). Neither these features, nor the conjectured supply and transmission price responses, are considered in the model comparisons of this paper.

The model is formulated as a linear mixed complementarity problem. The steps involved in constructing and solving a linear MCP for a power market are as follows:

1. Define a linear or quadratic optimization problem for each market participant (generating firm, TSO, consumer, purveyor of emissions allowances, arbitrager/trader, etc.), in which each participant maximizes its profit or other objective subject to internal constraints, prices, and conjectures about how those prices are affected by its decisions.

2. Derive the first-order conditions for optimization of each problem (KKT conditions).
3. Define market clearing conditions, and collect the first-order and market clearing conditions into a single complementarity problem, which should be "square" (as many conditions as variables).
4. Implement the conditions using a matrix generation language for optimization problems, and then use a numerical complementarity solver to obtain a solution to the problem.

Formulation as a MCP makes possible the solution of large-scale models using efficient complementarity solvers (Dirkse and Ferris, 1995). COMPETES and other MCP-based models have been formulated and solved for very large systems with thousands or tens of thousands of variables. For instance, such models have been used to assess the relative competitiveness of different regions within the North American Eastern Interconnection (Hobbs and Helman, 2004). The MCP formulation also facilitates the establishment of existence and uniqueness properties.

Further description of an application of COMPETES to a more detailed network of north-west Europe is documented in Hobbs et al. (2004). There, the effects of transmission pricing policies, generation market structure, and expansion of transmission capacity are evaluated assuming both competitive and oligopolistic market conditions.

B.2. Cambridge II model—integrated markets for energy and transmission

This static bilevel equilibrium model (Ehrenmann and Neuhoff, 2003) is motivated by Cardell et al. (1997) and allows the study of market power and transmission pricing issues in a deregulated electricity market under an independent system operator regime. Generators submit quantity bids to an independent TSO. Following the Nash equilibrium assumption, each generator assumes that the bids of his competitors stay fixed. The TSO clears the market by deciding about dispatching generation and consumption based on submitted quantity bids by generators, demand curves of generators, and marginal cost curves of competitive generators. The TSO dispatches such that social welfare would be maximized, were the bid curves to truthfully represent marginal costs.

Each strategic generator's problem is an MPEC; the overall problem is therefore an EPEC. The steps involved in construct and solving such an EPEC for a power market are as follows:

1. Define linear or quadratic optimization problem for the TSO in which the TSO maximizes social welfare with respect to internal constraints (network constraints, Kirchhoff's Laws).
2. Derive the first-order conditions for the optimization problem of the ISO and express them as constraints in the optimization problem of each strategic generator.
3. Define the quadratic optimization problems for the generating firms in which the participants optimize profit with respect to internal constraints and the optimality conditions of the TSO.

4. Implement the conditions using a modeling language (GAMS), and then use an MPEC solver to solve the generators optimization problems sequentially (diagonalization).
5. If the diagonalization procedure converges, check for optimality for all generators (existence of solution).

Nonconvexities created by transmission constraints can prevent the uniqueness or existence of a solution (Weber and Overbye, 1999) and the fact that all the generators share the same complementarity constraints can imply that a continuum of solutions exists.

Analytical models show that the ability of generators to exercise market power depends on the market design. The two basic design options are separate transmission auctions with subsequent energy spot markets or combined energy and transmission markets (nodal pricing, market splitting). Neuhoff (2003b) shows that the second approach is preferable from a welfare perspective, but could not provide analytic solutions for situations with generation companies owning generation assets at several locations. More detailed descriptions regarding the economic implications and different implementations are available in Ehrenmann and Neuhoff (2003).

B.3. Cambridge I model—separate markets for energy and transmission

The Cambridge II model implements combined energy and transmission markets, therefore a second model was developed to represent a market design that separates these markets (Ehrenmann and Neuhoff, 2003). Cambridge I assumes that a competitive group of traders, not the system operator, would arbitrage the markets. These traders buy transmission rights from an auction for transmission rights and then arbitrage the energy spot markets. The difference compared to Cambridge II is that traders have to submit their quantity bids to the energy spot markets at the same time as the strategic generators submit theirs, and generators do not anticipate that their output decisions will affect trader actions. This is done by moving the TSO from the second stage – where his scheduling decisions are contingent on generator output choices – to the first stage.

B.4. Madrid

While the above models were designed to be solved using standard optimization methods, the Madrid model (Barquin and Vazquez, 2003) instead evolved from models with a more detailed representation of generation structure. To allow use of any differentiable cost function, the model requires consistent beliefs among generators and the TSO about which transmission constraints bind.

The Madrid model iterates two different submodels. The first submodel simulates the behavior of the TSO when receiving a set of quantity offers from the different generators. The TSO is assumed to maximize the apparent social welfare implied by offers and bids, taking into account network constraints. This submodel's main output is a matrix that relates the price sensitivity in every bus to power injections in each bus. Mathematically, a solution of a linear system is all that is required. The second submodel computes the

Table 3
Summary of responses to questionnaire

Numbers in brackets represent importance attributed to issue by participants
Q1: What sort of questions would you like answers for?
A. Market power impacts and mitigation (30)
–Give insights about impacts of market structure and design on market power, collusion, strategic gaming
–What are alternatives for mitigating market power?
–What is the impact of market transparency (e.g., information on binding transmission constraints in real time) on the exercise of market power? Is it different for radial and meshed systems?
B. Generation investment (16)
–When should we invest in new production capacity?
–How do market uncertainties and market power interact to affect capacity addition decisions? Will there be enough capacity?
–How does market power mitigation affect incentives for capacity additions?
–How effective is long-term contracting in encouraging entry and controlling market power?
C. Effects of transmission investment on market (12)
D. What criteria to use to choose the right market equilibrium in case of multiple equilibria/learning (2)
Q2: What are your concerns about present market equilibrium models?
A. Assumptions, robustness, calibration (15)
–Robustness of simulations—sensitivity analyses with respect to assumptions made
–Too many behavioral assumptions are made when modelling strategies
–What difference does different behavioral assumptions (type of game) make? Is it really, e.g., Cournot? What approach is most appropriate to actual markets? How are long-term considerations (like entry deterrence) relevant to bidding behavior?
B. Appropriate use (13)
–People/policy makers take the exact numerical results too seriously
–Numbers/outcomes take on life of their own, unless model inputs/outputs and restrictions/possibilities are well explained
C. Transparency of assumptions and algorithms (12)
D. Inclusion of market features (10)
–Absence of capacity expansion capabilities
–Interface with other (possibility non-electrical) agents' concerns
–They do not account for liquidity problems
–Use of market-based methods when there is no market and when one player is better informed than others
E. Data: the availability and correctness of data, as producers dislike revealing good, realistic data, adding errors to the assumptions and results of the model (8)
F. Ability to anticipate the consequences of market design decisions such as choice of capacity allocation method, coupling of geographic markets (3)
Q3: What would we require to use models to defend regulatory decisions?
A. Appropriate use and cautions (17)
–Understanding of what models can and cannot tell us
–Scientific "agreement" or best practice—no technical discussion in court
–Caution with numerical results—could ruin trust

(continued on next page)

Table 3 (*continued*)

Numbers in brackets represent importance attributed to issue by participants
B. Robustness and testing of results (15)
C. Transparency about model assumptions and limitations (13)
D. Not an appropriate use (11)
–(Our agency) does not intend to use models to defend its decisions. It rather wants to use them to prepare its decisions. Therefore, models should be simple but realistic and include a "parameterisation" of most possible solutions: (a) it is better to admit model limitations; and (b) we are interested in qualitative behavior, not absolute levels.
E. Realistic inputs/outputs, match observed behavior (1)
–Strong empirical evidence to support decisions made
F. Inclusion of certain market features (1)
–Integration of gas/electricity markets
Q4: What capabilities do we need next to answer the questions identified above?
A. Long-run decisions: capacity investments, forward contracts (22)
–Endogenous long-run decisions
–Represent different instruments for capacity markets
–Consider initial position (initial degree of forward contracting) in modeling day-ahead/contracting decisions
–Include entry deterrence and other long-term considerations in bidding behavior
B. Interactions among multiple markets (emissions trading, gas/electricity markets, ancillary services) (8)
C. Risk (7)
–Uncertainty modeling (demand, fuel prices, contingencies...)
–What affects liquidity; how does liquidity affect market power?
D. Transmission issues (6)
–Define quantities of transmission capacity allocated for spot markets and longer term
–Model the role of the system operator as a strategic agent
–In-depth evaluation of "netting" of inter-connector flows (as opposed to "no netting": constraining transactions in each direction separately)
E. Rules (6)
–Effect of bidding rules and market power mitigation on equilibria
–Model flexibility: it should be easy to include new rules and markets features
F. Market dynamics (multisettlement markets, dynamic market power models (6)

oligopolistic market equilibrium when every generator conjectures the same price sensitivities, based on the binding transmission constraints that are input from the first submodel. Generators behave as Cournot oligopolists. Even though this equilibrium problem is not per se an optimization problem, it can be transformed into one. Consequently, standard techniques for solving large systems can be applied. If quadratic cost functions (linear marginal cost functions) are used, then the optimization problem is a quadratic program with linear constraints. The output of this submodel is the quantity offer submitted by each generator that is required by first submodel. Full consistency requires that the generators' conjectures used in the second submodel have the same values as sensitivities obtained from market clearing performed by the TSO, as simulated by the first submodel. Therefore, there is a need to iterate between the submodels.

If the model converges, a pure strategy Nash–Cournot equilibrium has been computed. However, there are circumstances when no equilibrium exists. In this case, the algorithm oscillates between two or more states, defined by different sets of binding interconnections. The algorithm stops when detecting these oscillations.

Appendix C. Survey on modeling priorities

Eleven modeling experts were surveyed about priorities for future model development during the October 2003 workshop. The experts included four staff from French, Belgian, and Dutch regulatory agencies and seven researchers representing a Danish system operator, universities from Spain, the UK, and the US, and the Netherlands Energy Research Center. The four questions they addressed concerned policy questions that need to be addressed by power market equilibrium models, shortcomings of present models, prerequisites for models to be useful in public forums, and desirable enhancements. A Nominal Group procedure was used to generate ideas (Delbecq et al., 1975). This was done in four steps: (1) ideas were written down silently and anonymously, followed by (2) a group discussion and (3) anonymous vote, and (4) concluding with a group review of the results. The votes consisted of a 1-2-3 ranking by each person of the most preferred or important responses to each question. Table 3 groups those responses that received at least one first place vote, and tallies the points received by each category of responses. First place votes received three points, while second and third place votes were assigned two and one points each, respectively.

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