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A new analytical procedure for the derivation of displacement-based vulnerability curves for populations of RC structures

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Abstract

A new procedure is proposed for the derivation of analytical displacement-based vulnerability curves for the seismic assessment of populations of reinforced concrete structures. The methodology represents an optimum solution compromising between reliability and computational efficiency. Adaptive pushover analysis is employed within a capacity spectrum framework of assessment, to determine the performance of a population of building models for increasing ground motion intensity. The building model population is generated from a single design through consideration of material parameter uncertainty, with design of experiment techniques used to optimise the population size. Uncertainty in ground motion is accounted for through the use of suites of accelerograms with characteristics that are representative of the hazard level associated with the performance level assessed in each vulnerability curve. The new homogeneous reinforced concrete damage scale, which is experimentally calibrated to maximum inter-storey drift for different structural systems, is used to determine the damage state of the building at the performance point. The results of the assessments are used to construct response surfaces from which the damage statistics forming the basis of the vulnerability curves are generated through re-sampling. The proposed methodology is illustrated for the case of low-rise, infilled RC frames with inadequate seismic provisions. The derived curves show good correlation with observational post-earthquake damage statistics.

Keywords: Vulnerability curves; Capacity spectrum assessment; Adaptive pushover analysis

1. Introduction

Vulnerability curves relate the probability of exceedence of multiple damage states to a parameter of ground motion severity, and can therefore be regarded as a graphical representation of seismic risk. At any given ground motion value, the vertical distance between adjacent damage state curves represents the probability of a building being within the lower of the two damage states considered. In the case of building populations, use of vulnerability curves yields a prediction of the proportion of the exposed stock in each damage state after an earthquake. Several vulnerability relationships for reinforced concrete (RC) buildings have been proposed in the past which are based on analytically simulated building damage statistics. No unique methodology exists for the derivation of these relationships, with a variety of analysis techniques, structural idealisations, seismic hazard and damage models being used. These factors strongly influence the derived vulnerability curve shapes, and different choices have been seen to result in significant discrepancies between the seismic risk assessments made by different authorities for the same location, structure type and seismicity [1]. Regardless of these choices, all existing methods for analytical fragility function derivation are computationally very intensive, as a large number of analyses are required to fully represent the

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Table 1
Threshold $ISD_{max\%}$ values of the HRC damage scale for infilled RC frame structures

HRC damage state	None	Slight damage	Light damage	Moderate damage	Extensive damage	Partial collapse	Collapse
ISD_{max} (%)	0.00	0.05	0.08	0.30	1.15	2.80	>4.36

structural and ground motion uncertainties involved in the seismic assessment of building populations. For example, Singhal and Kiremidjian [2] carry out non-linear time-history analyses on finite-element bare reinforced concrete frame models of varied material properties, using 100 artificial acceleration records, scaled to 40 seismic intensity levels (4000 analyses in total). The repetition of such a study for many different structure classes is impractical due to the time involved and specialised analysis tools required. In order to reduce the analysis time, existing analytical fragility curve derivation methods commonly make compromises, either as regards the number of structural variations and earthquake records used, or as regards the accuracy of the structural modelling, analysis and assessment technique. For example, Mosalam et al. [3] adopt 800 earthquake records covering a wide range of intensities to test 200 material variations of their structural model, but adopt a single-degree-of-freedom system idealisation for the analyses. Compromises such as these may affect the reliability of the final vulnerability curves. Therefore, within this paper a new displacement-based procedure for the generation of vulnerability curves for RC building populations is proposed. The procedure adopts a response surface methodology in the generation of the population damage statistics, which allows both a reduced number of analyses and a reliable representation of the population response uncertainty to be achieved. The consequent reduction in analysis time allows accurate models and analysis tools to be used. Furthermore, the combined use of an adaptive pushover analysis and capacity spectrum method of assessment in the proposed curve derivation procedure avoids the repetition of analyses for increasing ground motions and further reduces the computational effort.

Problems associated with the choice of parameters for ground motion and damage characterisation can be identified in almost all existing vulnerability relationships [4]. The parameter chosen to represent ground motion in the construction of vulnerability curves must be both representative of the damage potential of earthquakes and easily quantifiable from knowledge of the earthquake characteristics. Peak ground values or intensity values are therefore unsuitable for this purpose. It is widely recognised that a closer relationship exists between observed damage and structural deformations than applied forces, due to the ability of the former to account for non-linear structure behaviour. This observation is confirmed by Rossetto and Elnashai [4], where spectral displacement ($S_d(T)$) is shown to give a better correlation to the damage observed in 99 populations of buildings after 19 worldwide earthquakes than spectral acceleration. In view of these observations, of the development of displacement-based techniques in earthquake engineering which have rendered possible the use of spectral displacement as a design parameter, and of the recent derivation of reliable attenuation relationships for $S_d(T)$ by Bommer et al. [5], elastic spectral displacement (at 5% critical damping, $S_{d5\%}(T)$) is chosen to represent the seismic demand in the proposed vulnerability curves. The homogenised reinforced concrete (HRC) damage scale presented in [4] is used to evaluate the structural performance from the analyses and to define the damage limit states associated with the developed curves. The scale is subdivided into seven damage states ranging from “No-damage” to “Collapse”, each of which is clearly defined in terms of the typical structural and non-structural damage expected in the four main types of RC structure found in Europe. The scale is experimentally calibrated to the parameter of maximum inter-storey drift response ($ISD_{max\%}$) for the different structure types. An example of the $ISD_{max\%}$ values obtained for the class of infilled RC frames is presented in Table 1. As the damage state and ground motion are assessed using measures of deformation and displacement, respectively, the proposed curves are appropriate for use in a displacement-based assessment framework.

2. Analytical curve generation methodology

The proposed methodology for vulnerability curve derivation prescribes the analysis of a population of RC buildings subjected to a number of earthquake records with distinct characteristics. It is thus able to account for the effect of variability in seismic input and structural characteristics on the damage statistics simulated for the building class (“system”), and evaluate the associated uncertainty in the vulnerability prediction. The procedure is summarised in Fig. 1 and may be regarded as consisting of four main steps. Step 1, “system definition”, consists in the selection and design of a single structure with material, configuration and seismic resistance characteristics that are representative of the building class being assessed. Deviation in seismic resistance of buildings within the class is considered through the analysis of a population of building models, generated from the general system design by varying its structural properties. Step 2, “definition of ground motion input”, involves the selection of suites of earthquake records for the analysis. Within the proposed methodology different suites of accelerograms are adopted in the derivation of each limit

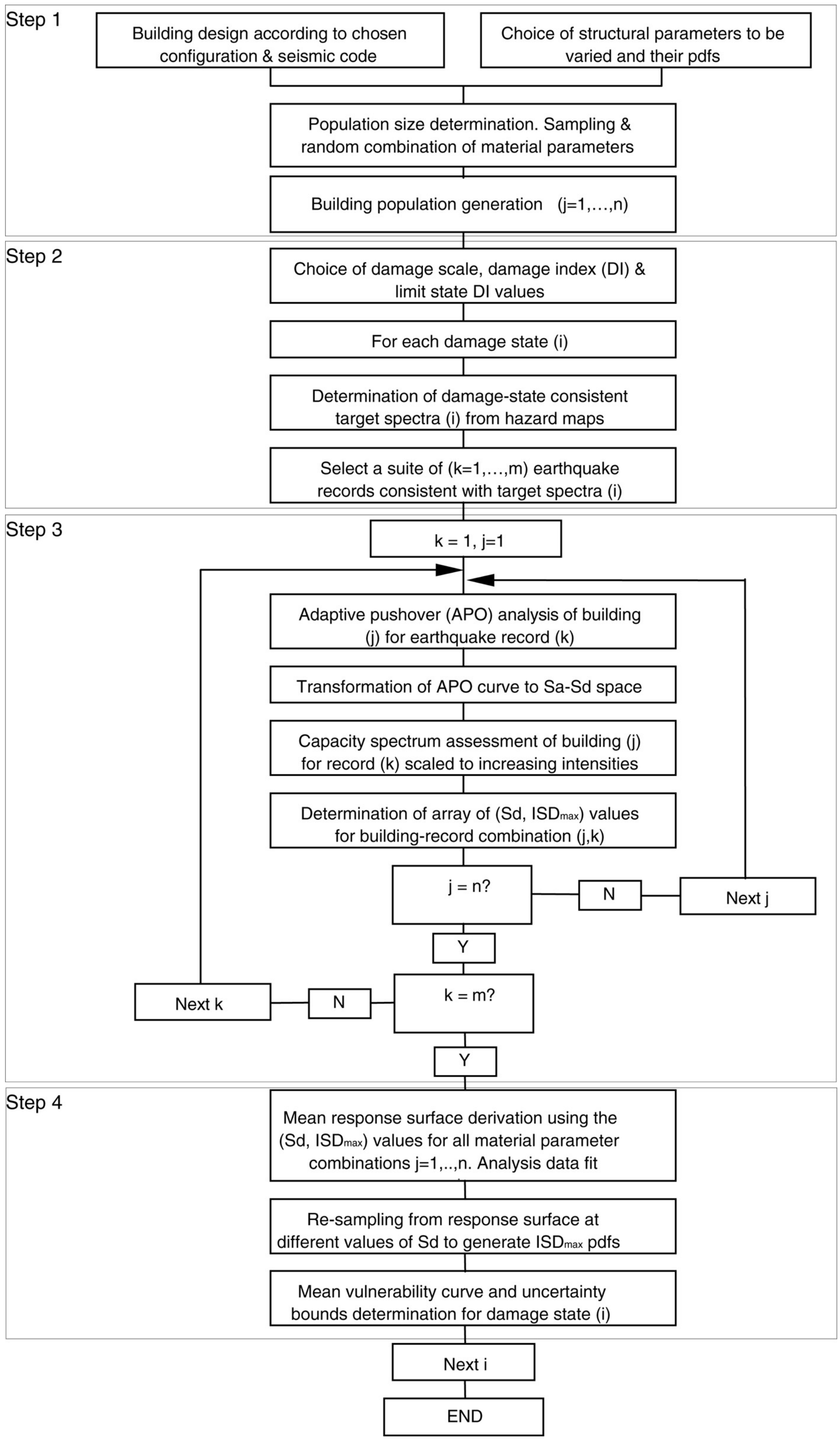


Fig. 1. The flow chart of the proposed analytical vulnerability curve derivation method.

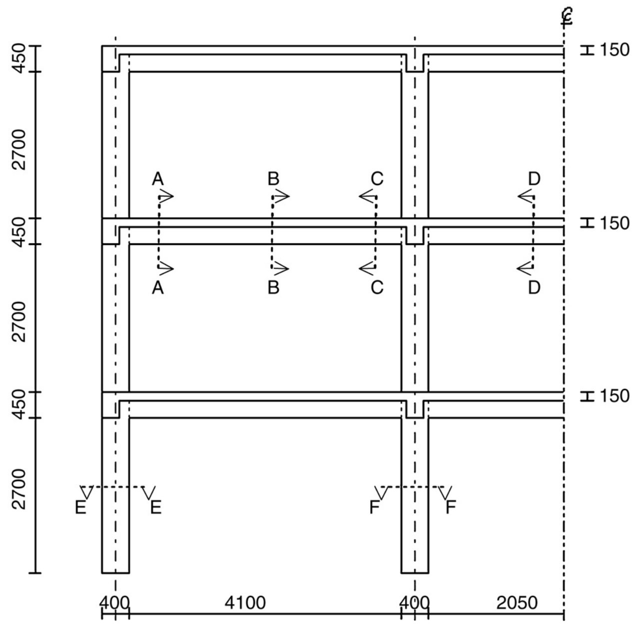
state curve. These "performance-consistent" record suites are selected in accordance with spectra that are characteristic of a seismic event with a return period, for which the structural damage state defining the curve is acceptable or is expected. Designs of experiment procedures are considered in the selection of the population and earthquake record suite sizes for the analyses. This is done in order to optimise computational effort and to guarantee convergence of results. Step 3, the "model evaluation", is carried out using an innovative adaptive pushover analysis technique within a capacity spectrum framework of assessment. Adaptive pushover analyses can account for the effect of ground motion characteristics on structural response and require reduced computational effort compared to time-history analyses. The maximum inter-storey drift response of each structure within the population is assessed, for increasing intensities of ground motion, using a modified capacity spectrum method. This assessment method avoids the repetition of analyses for increasing ground motions, and further reduces the analysis number from several thousands to a few hundred. Step 4, the "statistical processing of analysis results", involves the definition of response surfaces, relating the observed inter-storey drift response to the structural property and ground motion parameter values. Response surface equations are defined for each hazard scenario, through separate consideration of the analysis statistics resulting from each suite of performance-consistent records. A re-sampling technique is adopted to generate building damage statistics from the response surfaces, for a range of ground motion severities. Hence, vulnerability curves are plotted. The response surface equation used to develop each damage state curve is selected according to the hazard level associated with the satisfaction of a desired performance objective. The generated vulnerability curves may therefore be defined as being "performance consistent". Uncertainty in the damage state prediction, and its variation with increasing ground motion intensity, is accounted for through vulnerability curve confidence bounds. These are determined from consideration of the fit of the analytical observations of maximum inter-storey drift to the response surfaces, within the damage histogram generation process.

The choice of building design and material properties assumed for the model are dependent on the composition of the building stock in the assessed region. The buildings must be grouped into categories with similar lateral load resistance and different vulnerability curves must be derived for each building class. The overall risk to the population for an earthquake of given size may then be obtained by combining the damage state exceedence probabilities for each structure class according to their relative proportions in the assessed region. It is emphasised that any structural typology may be assessed within the framework of the proposed methodology through an appropriate selection of structural model, structural properties for variation and their corresponding probability distributions, and the ground motion input. However, in order to better illustrate the proposed analytical vulnerability curve derivation methodology, in the following text each step of the procedure is explained within an example application to a population of low-rise infilled RC frames of typical European construction, which are designed to old seismic codes (not including capacity design concepts). Conclusions are then drawn as regards the ability of the resulting curves to reproduce observational damage data.

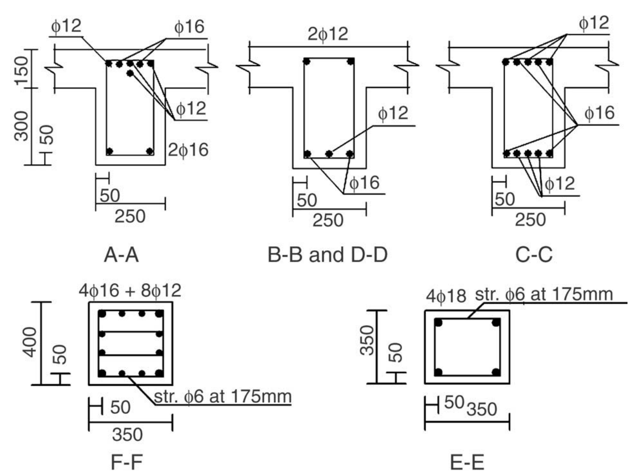
2.1. System definition

A regular, three-storey infilled frame configuration is chosen to represent the system (i.e. the low-rise infilled RC frame structural class used as an example here). The frame is designed according to the prescriptions for loading, material, member dimensioning and detailing of the seismic and gravity load design codes in place in Italy in 1982. This code is chosen as being representative of the seismic design of the existing European building stock. The full design of the infilled frame is presented in [6] and the elevation, beam and column details are illustrated in Fig. 2. The structure consists of four frames with constant inter-storey heights and bay widths of 3 m and 4.5 m, respectively. It is symmetrical in plan and elevation, with the external and internal frame designs differing due to changes in the design loads. An intermediate seismic zone (Zone 2, $S = 9$), approximately corresponding to a *pga* of 0.07 g (10% exceedence in 50 years), is adopted in the design, and results in a design base shear of 8.4% of the structure weight. Concrete with a characteristic compressive strength (f_{ck}) of 30 MPa and FeB38 grade ribbed steel bars with characteristic yield stress (f_{yk}) of 380 MPa are used for the structure. It is observed that despite the inclusion of seismic actions in the design, the shear reinforcement in all columns and members is insufficient for significant levels of section confinement.

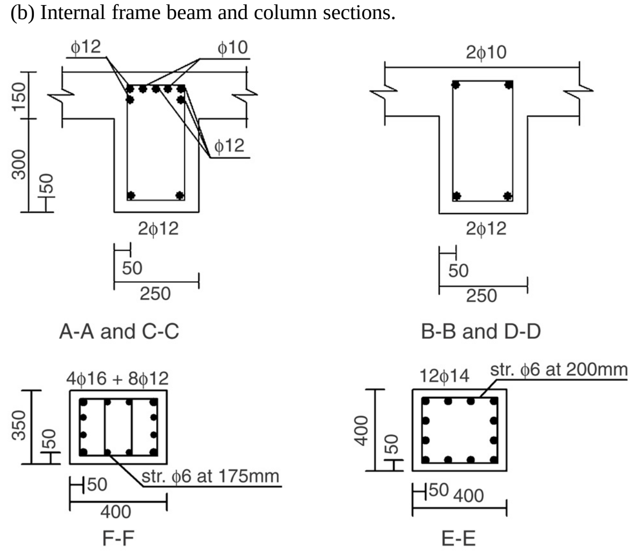
The system is modelled using the Inelastic Dynamic Analysis of Structure finite-element package, INDYAS [7]. The program allows a 3D finite-element model of the building to be constructed, wherein the distribution of reinforcement within sections is modelled bar by bar and the non-linear behaviour of materials is taken into account. The seismic behaviour prediction accuracy and stability of the INDYAS program has been proven by Pinho [8] and Rossetto [6], amongst others. Due to symmetry, only half the structure is modelled for the analysis. The sub-division of the frames into finite elements is carried out considering the spread of plasticity in members. The Mander et al. [9] model is used to represent the effects of concrete confinement. However, in the present case where members have negligible confinement, the choice of confinement model may be regarded as an insignificant source of response uncertainty. A bi-linear elasto-plastic model with kinematic strain hardening is used to represent the reinforcing steel behaviour. The infill panel response is modelled via inclined rectangular struts that act in compression only. The stress–strain curve proposed by Panagiotakos and Fardis [10] is modified to account for the



(a) Elevation of internal and external frames.



(b) Internal frame beam and column sections.



(c) External frame beam and column sections.

Fig. 2. Drawings of the low-rise infilled RC frame designed to the 1982 Italian seismic code.

effect of openings using the equations of Mosalam [11], and used to represent the non-linear behaviour of the infills. Values of 1240, 2515, 0.26 and 2.4 MPa are used for the infill shear modulus, Young's modulus, mean diagonal cracking strength and mean horizontal compressive stress, respectively, according to wallette tests carried out at the University of Pavia, Italy [12].

A population of frames with different dynamic response characteristics can be generated through the treatment of selected structural parameters as random variables. Once the variables are selected, they are assigned probability distribution functions based on experimental observations. Values of the parameters are then extracted from the distributions using appropriate sampling techniques. Finally, the sampled values are combined to define a series of frames with different characteristics, all nominally representing the same structure. Clearly, consideration of all possible uncertainties in the global and local structural characteristics yields an extremely large number of permutations for the analysis. It is recommended that the general plan and elevation configuration of the system are treated deterministically, as changes in these are expected to alter the design forces, code prescriptions, member design and detailing, and hence warrant a separate vulnerability curve derivation. If the typical practice of providing minimum section sizes and reinforcement for the code defined loads is considered, the rebar configuration, section and member geometry can also be regarded as constant. Variation in material properties is large within a population, and derives from differences in manufacturing processes and local construction practices. Therefore, it is proposed that the concrete unconfined compressive strength (f_c), the infill compressive strength (f_{cw}) and the beam and column reinforcing bar steel yield strength (f_y) are chosen to be randomly varied between frames. In the latter case, a correlation coefficient of 0.7 is assumed to exist between the sampled parameter values within the elements of single frames. It is highlighted that the probability distributions and degree of inter-correlation for the material parameters should be selected in accordance with the typical construction practice and reliability of the building material manufacturing processes used in the assessed area. Table 2 summarises the probability distribution functions assigned to each material property for the assessment of the example infilled frame population. These were determined through consideration of numerous tests on European construction materials dating from the assumed time of construction (e.g. Pipa and Carvalho [13], Petersons [14], amongst others).

Designs of experiment techniques offer an increase in the efficiency of simulation-based assessment of structural reliability, and are used in the proposed method to optimise the population size and expedite convergence of the results. The population size is determined via the 2^n factorial composite method [15]. This method prescribes $(2^n + 2n + 1)$ parameter combinations for the generation of

Table 2
Material parameter probability distribution functions used in the population generation

Material	Concrete	Steel	Masonry infill
Random variable	Mean f_c	COV f_y[b]	Mean f_{cw}
Distribution	Normal	Log-normal	Log-normal
Mean	Nominal f_c	0.06	Nominal f_{cw}
COV[a]	0.12	0.25	0.20
Design variable	Mean f_c	Mean f_y[c]	Mean f_{cw}

[a] Coefficient of variation.
[b] Use of the COV of f_y as the random variable allows a constant characteristic yield strength to be maintained and a realistic variation in mean yield strength to be obtained.
[c] Calculated for f_{yk} = nominal f_y.

a second-order response surface that fully represents the uncertainty associated with *n* independent random variables. A smaller number of permutations might be used in the present case study as correlation exists between two of the parameters. However, a sample size of 25 is seen to ensure convergence of the fragility curves for the four parameters used. Therefore, 25 values are sampled from each of the three material parameter probability distributions using the Latin hypercube method. During the sampling, a correlation coefficient of 0.7 is maintained between the column and beam steel yield strength values. The parameter values are shuffled and combined according to the algorithm adopted by Law and Kelton [16]. Pushover analysis of the thus derived population of 25 infilled frames shows an average coefficient of variation (COV) of 14%, 37% and 10% for the initial stiffness, yield displacement and yield period of the system, respectively. The latter results in an average COV of 21% for the ground motion response ($S_{d5\%}(T)$). A comparable average COV of 22% in the predicted $\mathrm{ISD}_{\max\%}$ response of the population is also observed to result from the material uncertainty.

2.2. Generation of ground motion input

In the generation of vulnerability curves, random variability in the ground motion parameter (i.e. $S_{d5\%}(T)$) is not considered, as this should be included in the determination of the hazard within the total risk assessment. Within an analytical curve derivation procedure, the ground motion parameter is deterministic, and is evaluated from the earthquake records used as input to the analyses. However, the characteristics of records producing any single value of the ground motion parameter can vary significantly and introduce uncertainty in the structural response and consequent damage predictions. Suites of accelerograms with different peak amplitude, frequency content, cycle number and duration characteristics must therefore be adopted in the population analysis. However, no formal guidance exists for the selection of accelerogram suites for use in vulnerability curve generation, where structural response must be evaluated over a very wide range of ground motion severities and different performance objectives checked.

Seismic performance of buildings is assessed conditionally to the probability of occurrence of an earthquake event, (e.g. “Serviceability” for frequent earthquakes and “Collapse prevention” for very rare events). Hence risk assessment tools should assess the attainment of different damage limit states using records that are representative of events with return periods that are consistent with the performance objective. In most existing vulnerability studies (e.g. [2]), sets of natural records are selected (or records are artificially generated) to be consistent with a response spectrum that represents the seismic hazard in the assessed region. These records are then scaled by various means, to represent the entire range of ground motion severities used for the curve generation. However, record scaling does not account for the change in the constitutive characteristics of records deriving from earthquake events of very different magnitudes and return periods. It is therefore proposed that three “Target” spectra representative of scenarios corresponding to the FEMA 273 [17] performance states of “Serviceability”, “Damage control” and “Collapse prevention” are defined for the selection of the accelerograms used in the derivation of the “Slight” and “Light”, “Moderate” and “Extensive”, “Partial Collapse” and “Collapse” fragility curves, respectively. Following a review of existing literature (Bentler [18] and FEMA 273 [17] amongst others), the three return periods of 95, 475 and 2475 years are found to be representative of the events associated with the three performance objectives. A back analysis of the Italian hazard maps of Albarello et al. [19], associated data and attenuation rules results in the determination of the earthquake characteristics shown in Table 3 for the three soil classes of Eurocode 8 [20] and three hazard scenarios. These values are evaluated over the land classified as Zone 2 by the Italian code (moderate–high seismic hazard), in order to be consistent with the seismic loads assumed in the design of the infilled building. The procedure followed is explained in greater detail in [6]. The derived mean *pga* corresponds well with the values of proposed by Campos-Costa and Pinto [21], for the same hazard scenario in Portugal. The attenuation relationships of Ambraseys et al. [22] and Sabetta and Pugliese [23] are used with equal weighting (as per the derivation of the hazard maps), to establish the “target” acceleration

Table 3
Characteristics of the earthquake events defining the “Target” spectra

Soil category	Rock			Firm			Soft		
Return period (years)	95	475	2475	95	475	2475	95	475	2475
Target *pga* (g cm/s^2)	6.70	12.20	21.70	6.10	11.90	20.70	6.10	11.80	22.30
Lower bound *pga* (g cm/s^2)	4.10	7.60	13.80	3.50	7.30	12.80	3.50	7.20	14.40
Upper bound *pga* (g cm/s^2)	9.30	18.50	34.90	8.70	18.20	33.90	8.70	18.10	35.50
Surface magnitude (M_s)	5.75	6.05	6.25	5.55	5.85	6.15	4.95	5.35	5.45
Fault distance (r, km)	22	14	8	30	18	12	16	10	4

spectra from the mean values of *pga*, surface magnitude (M_s) and fault distance (r) for firm soil conditions presented in Table 3. Equivalent displacement “target” spectra are also derived using the attenuation relationship of Bommer et al. [5] and are shown in Fig. 3. It can be inferred from the work of Wen and Wu [24] that ten accelerograms are sufficient for the representation of the ground motion variability associated with a given spectral shape. Three suites of ten natural records corresponding to the three damage state spectra are, therefore, selected from the European Strong-Motion Database [25]. The records are chosen for different site soil conditions, considering earthquakes with values of M_s and r within $\pm 0.5 M_s$ and ± 10 km of those defining the target curves, such that the average spectral shape of the selected suite of records closely approximates the target spectrum. Ranges of *pga* are also used in the record selection; these are defined by considering the deviation in *pga* values characterising the hazard in the Italian seismic Zone 2 (Table 3). The records thus selected are presented in full in [6]. Notwithstanding the close fit of the mean spectrum of the records with the “target” shape (Fig. 3), a large variation in the time histories and spectra used to represent each of the earthquake return period scenarios is seen. Uncertainty in ground motion introduces an average coefficient of variation of 27.0% in the seismic spectral displacement demand imposed on the infilled structure population, for all scenarios. This leads to an average variation in the $\mathrm{ISD}_{\max\%}$ predicted for each structure, at a given level of *pga*, of up to 37.2% (COV). The latter value is greater than that resulting from material property variation within the structural population, and emphasises the importance of including ground motion uncertainty in analytical vulnerability evaluations.

Although the adopted record selection technique is time-consuming, the resulting record suites may be used to carry out vulnerability analyses in all areas with similar seismic activity and fault mechanisms to those considered (i.e. medium European seismicity, shallow crustal faulting). For areas with very different seismicity or characterised by subduction earthquakes, ranges of peak ground accelerations, earthquake surface magnitudes and fault distances can be found from historical earthquake catalogues or from local hazard maps using a method similar to that shown above. Target spectra can then be developed using appropriate attenuation relationships and suites of

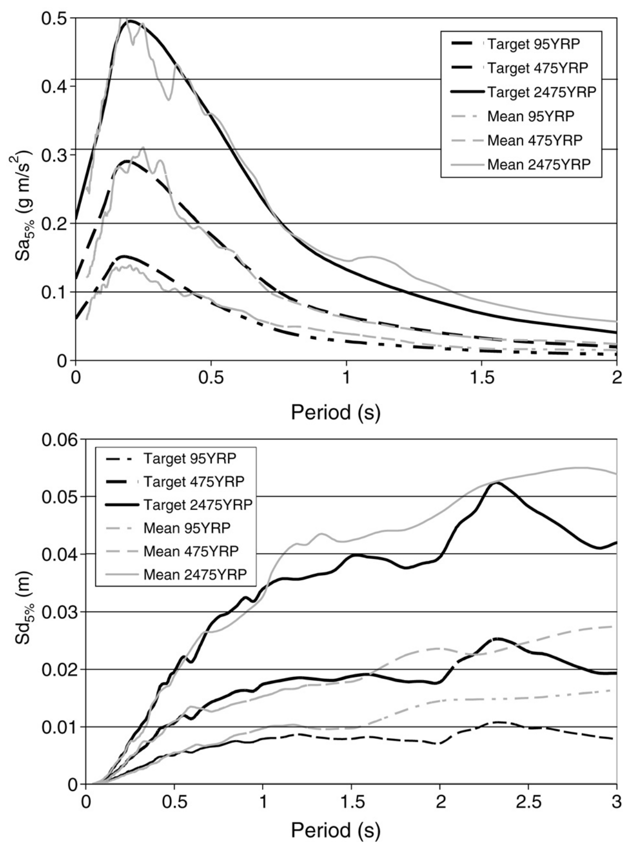


Fig. 3. The “Target” and ensemble record suite mean acceleration and displacement spectra for the 95, 475 and 2475 year return period earthquake scenarios and firm soil conditions.

records for analysis chosen from catalogues of earthquakes with similar fault mechanisms.

2.3. Model evaluation

In order to generate vulnerability curves, the building population determined above must be assessed for damage for increasing levels of seismic load. Despite the use of experiment design techniques to limit the number of accelerograms and building models required for the full representation of structural and ground motion uncertainties, a large number of analyses are still required to assess the population over a wide range of ground motions. Within the vulnerability curve derivation methodology proposed here, the number of analyses is reduced through the use of an adaptive pushover analysis procedure within a capacity spectrum framework of assessment.

Pushover analyses are associated with analysis times that are a fraction of those required for full non-linear dynamic time-history analyses and are thus amenable for use in cases where numerous analyses are required. In conventional pushover procedures, a constant distribution of forces is applied to a structural model incorporating inelastic material properties, and is incremented to structural failure. The analysis is unable to account for the characteristics of earthquake records and the variation in applied seismic demand with increasing structural degradation. Furthermore, a poor representation of the deformed shape of structures is often seen if these do not respond predominantly in the first mode. Conventional pushover analyses can therefore not be used in the present assessment, where ground motion uncertainty is considered, buildings with inadequate seismic design are addressed and $\mathrm{ISD}_{\mathrm{max}\%}$ is used as the damage measure. An adaptive pushover (APO) analysis method is proposed in [26], which follows the same procedure as conventional pushover analysis, but at each load increment updates the lateral applied load distribution to take into account the instantaneous structural stiffness, modal properties and consequent ground motion demand. The validity of the adaptive pushover technique is verified by Rossetto [6] with respect to the dynamic time-history analysis of seismically designed eight-storey structures and an irregular, non-seismically designed RC frame. A reasonable correlation between the structural top-drift, base-shear and inter-storey drift responses predicted by the two methods is observed and the sequence of formation of local and global collapse mechanisms is satisfactorily predicted. The APO method of Antoniou [26] implemented within the framework of the INDYAS finite-element package [7] is adopted in the analysis of the structure population here. The result of the APO analysis is a set of 750 base-shear versus top-displacement curves, which describe the capacity of each of the 25 buildings for the 30 seismic events considered. A single adaptive pushover curve is sufficient for the evaluation of the structure vulnerability, over all ground motion severities, for any earthquake of a given spectral shape. The repetition of structural analyses for each ground motion scale factor increment is eliminated and the number of analyses required for the determination of the vulnerability curves is considerably reduced (750 analyses, compared to 11 250 for time-history analyses). The consequent reduction in analysis time and cost render the adaptive pushover technique a highly desirable tool for application in vulnerability studies.

Capacity Spectrum Methods (CSM) are proposed in the literature for the seismic assessment of building performance using the results of static pushover analyses (Fajfar [27] amongst others). They are incorporated in codes of practice such as FEMA 273 [18], and are seen by many to represent the future of seismic assessment of buildings, as they have simple visual interpretations and involve elements of both acceleration and displacement responses. A direct comparison is made between earthquake spectra representing the seismic demand and a transformed force–displacement curve representing the seismic capacity of the structure. Both demand and capacity curves are plotted in spectral displacement–spectral acceleration (SASD) space, with the “performance point” (PP), defining the maximum response of the structure for the given earthquake, being determined from the location of their intersection. The implicit assumption of a fundamental mode of response for the building is the main impediment to the adopting of the existing capacity spectrum method in the assessment of structural damage using the results of APO analysis. In the case of APO analysis a single transformation cannot be applied to the pushover curve as the relative contribution of each mode changes with each applied load increment. Hence, an approximate method for the transformation is proposed here, where the instantaneous displaced shape and storey forces at each increment step of the APO method are used to transform the force–displacement curves into SASD space. The same expressions as for the single-degree-of-freedom transformation are adopted:

$$S_a = \frac{V_b}{M^*}; \qquad S_d = \frac{u_N}{\Gamma \phi_N};$$
$$\Gamma = \frac{\sum_{i=1}^{N} m_i \phi_i}{\sum_{i=1}^{N} m_i \phi_i^2}; \qquad M^* = \frac{\left(\sum_{i=1}^{N} m_i \phi_i\right)^2}{\sum_{i=1}^{N} m_i \phi_i^2}. \tag{1}$$

However, the current displaced shape of the structure normalised to the top displacement (Φ_n) replaces the fundamental mode shape. Consequently, ϕ_i is the component of Φ_n corresponding to the ith storey, m_i is the lumped mass at the ith floor and u_N and V_b are the top displacement and base shear at the current load increment, respectively. The reasoning behind this transformation method is that the force distribution and resulting displacement distribution implicitly incorporate the modal combinations. This assumption may not be theoretically justified, but is observed to yield reasonable assessment results [6]. Another drawback of existing capacity spectrum procedures is that they are iterative, graphical methods of assessment. Due to the large number of assessments necessary for the simulation of damage statistics in vulnerability curve generation, the adopting of such a procedure is impractical. A modified approach to the visualisation of CSM using inelastic earthquake spectra is therefore proposed, which both aids the automation and increases the efficiency of the assessment procedure. Following transformation of the APO base-shear–top-drift response curves to SASD space, according to the procedure outlined above, the transformed pushover curve is idealised as a multi-linear curve. This curve is then discretised into a series of points, denominated by the capacity–demand checking points (CDCP). The idealised curve shape up to

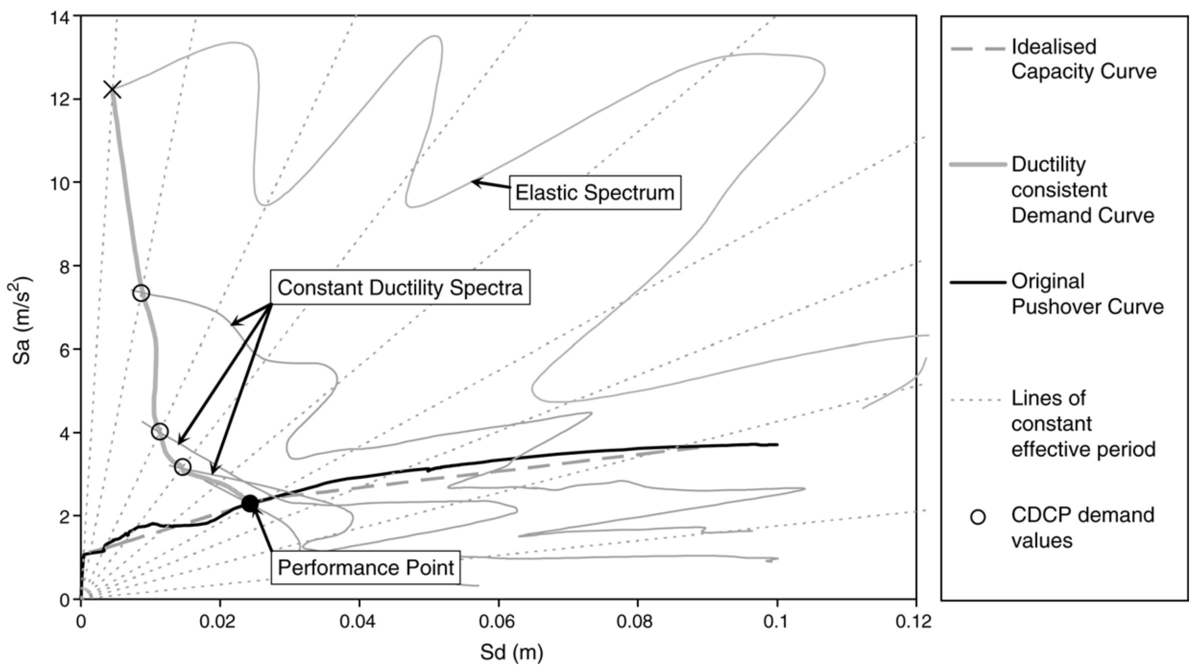


Fig. 4. Illustration of the proposed method for performance point solution.

each CDCP location defines the elastic period, ductility and non-linear response curve characteristics of a corresponding single-degree-of-freedom system (SDOF). The resulting series of SDOFs are analysed dynamically for the applied scaled earthquake records, to obtain a set of inelastic spectral acceleration and displacement values. These demand values may be visualised as corresponding to the structure capacity along the radii of the SASD plot that intersect the CDCPs. Using these results, a single inelastic demand curve may be drawn and the performance point defined directly from the crossing of this curve with the capacity curve (Fig. 4). The maximum inter-storey drift ratio ($ISD_{max\%}$) response is determined at the PP from the results of the adaptive pushover analysis and used to determine the damage state of the building. The same capacity curve is used to assess a structure, over the full range of ground motions required for the vulnerability curve generation, through record scaling.

A capacity assessment program (CAsP) is created for the implementation of the proposed procedure in the assessment of the infilled frame population. CAsP is coded in Visual Basic 5.0 [28] and is presented fully in [6]. In the evaluation of populations of building models, a large number of capacity curves of differing shape must be dealt with (750 in the case of the infilled frame population assessed in this study). CAsP is designed to automatically detect the location of yield and the ultimate point locations that best describe the curve shape, according to the behavioural model and yield criteria specified by the user. Three curve idealisation models can be used to model the pushover response of the structures: the elastic–perfectly plastic (EPP) model, the linear strain hardening (LSH) model and tri-linear model (TLM). Several options exist for definition of the global yield point and strain hardening slope in each case. An adaptive strain hardening option is also included, where the strain hardening varies according to the point on the curve being assessed. The choice of curve idealisation model strongly affects the performance point predictions for infilled frames, where plateaus in the pushover response are seen to occur due to the sudden failure of infill panels. Neither the LSH model nor the EPP model can model this behaviour, the latter model resulting in errors of up to 60% in the performance point spectral displacement location. All current capacity spectrum assessment procedures adopt EPP- or LSH-type curves in the determination of the structural capacity and seismic demand. The above observations therefore raise concern over the adequacy of these methods for the assessment of infilled frames, and promote the use of programs such as CAsP that allow more complicated capacity curve modelling. Furthermore, in the proposed assessment method, consistent curve shapes are used to determine both the demand and capacity, unlike in previous studies (e.g. [29] and [27]). It is concluded that the TLM with variable strain hardening gives the best representation of the infilled structure non-linear pushover response and it is therefore adopted in the example population assessment here. A simpler bi-linear curve idealisation may be adequate for bare frame response idealisation.

Through use of the proposed assessment approach, only a single inelastic dynamic analysis of the equivalent SDOF system is required at each equivalent period considered, compared to the series of analyses for different ductility values implied by a graphical solution. Consequently, a significantly lower computation time is achieved. Within

CAsP, a full Newton–Raphson iterative scheme is used to solve the dynamic non-linear equilibrium equation for the response evaluation of each SDOF. In the case study presented, each of the 25 APO curves defining the population is assessed approximately twenty times for the increasingly scaled accelerogram used in their analysis, until an inter-storey drift value is obtained which exceeds that of the most severe damage state of interest. The program is then run for a new earthquake and a corresponding set of APO analysis results.

2.4. Statistical processing

The capacity spectrum assessment for each frame of defined properties (f_c, f_{cw}, f_y, f_{yb}) yields values of the maximum inter-storey drift response ($\mathrm{ISD}_{\max}$) for increasing values of ground motion intensity. The results of the population assessment are used to construct second-order response surfaces of the form of the following equation:

$$\frac{\mathrm{ISD}_{\max}}{S_{d5\%}(T)} = a_1 f_c^2 + a_2 f_{cw}^2 + a_3 f_{yc}^2 + a_4 f_{yb}^2 + a_5 f_c + a_6 f_{cw} + a_7 f_{yc} + a_8 f_{yb} + a_9 f_c f_{cw} + a_{10} f_c f_{yc} + a_{11} f_c f_{yb} + a_{12} f_{cw} f_{yc} + a_{13} f_{cw} f_{yb} + a_{14} f_{yc} f_{yb} + C. \quad (2)$$

A different response surface is generated for each damage state scenario, using the results of the population assessment for the corresponding “performance-compatible” record suite. These are each regressed from an average of 3750 points using the non-linear regression module of STATISTICA [30]. An example of the derived scenario response curves, and their fit to the regression data, is shown in Fig. 5. The coefficients of correlation (R^2) of the curves with the data range between 0.59 and 0.71. This scatter in the data is later accounted for in the confidence bound derivation of the vulnerability curves. The quantity of regression data exceeds the minimum prescribed by the 2^n factorial composite method [15] for the derivation of a second-order response surface from the five random variables involved (i.e. the four material parameters and $\mathrm{ISD}_{\max}$). This quantity of data is deemed sufficient for the detection of severe discontinuities or singularities. Values of the elastic spectral displacement (for 5% critical damping) corresponding to the mean of the yield periods for the population (defined on the basis of the point of first deviation from elastic behaviour of each model and evaluated as $T = 0.125$ s) are used to characterise the seismic demand in the response surfaces and resulting vulnerability curves. The effects of material variability and structure inelastic behaviour on both the building capacity and seismic demand are included in the proposed method for structure performance assessment. Hence, the influence of these factors on the population vulnerability will implicitly be included in the y-axis of the fragility curves developed. Use of an effective period for the calculation of the demand parameter characterising the x-axis of the vulnerability curves is therefore superfluous.

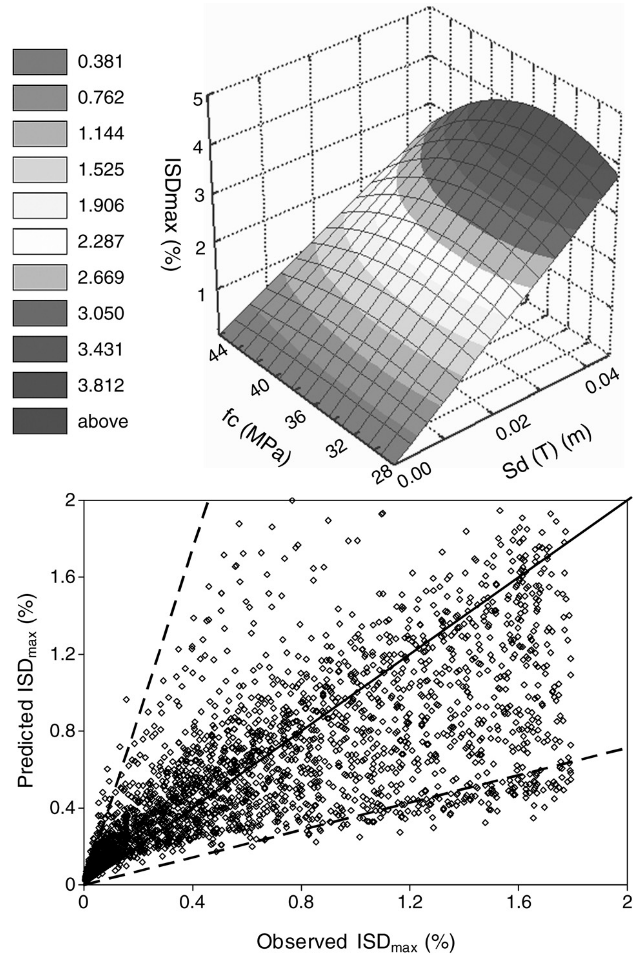


Fig. 5. Illustrations of the 2475YRP response surface (top) and its fit to the regression data (bottom; the dashed lines indicate the 5th and 95th percentile observed/predicted $\mathrm{ISD}_{\max\%}$ bounds).

Damage statistics are generated for a system through the multiple selection of material parameter value combinations for input into the response surfaces. Values of $\mathrm{ISD}_{\max}$ response are evaluated at a series of spectral displacements, for each material parameter combination. Frequency plots are obtained for each spectral displacement value and can be used to define the shape parameters of probability distribution functions for $\mathrm{ISD}_{\max}$. Damage state exceedence probabilities are calculated from the latter functions considering the threshold $\mathrm{ISD}_{\max}$ values for the HRC damage states presented in Table 1. In the case of the infilled frame population, 650 material parameter combinations are used to generate the $\mathrm{ISD}_{\max}$ histograms from each scenario response surface, at 100 spectral displacement values. The proportion of buildings exceeding each damage state is calculated at each ground motion level and plotted against the corresponding spectral displacement value for the vulnerability curve shape regression. The non-linear regression tool of STATISTICA [30] is used to regress for the parameters of the log-normal cumulative probability functions characterising the vulnerability curve shapes.

Table 4
Summary of the infilled frame population vulnerability curve equation parameters

HRC damage state	Mean[c]		95% upper bound[d]		5% lower bound[e]	
	μ[a]	σ[b]	μ	σ	μ	σ
Slight damage	−7.80	0.60	−8.25	0.60	−6.30	0.42
Light damage	−7.15	0.40	−7.82	0.60	−5.76	0.34
Moderate damage	−5.78	0.21	−6.32	0.20	−4.51	0.22
Extensive damage	−4.44	0.21	−4.92	0.21	−3.12	0.22
Partial collapse	−3.49	0.22	−3.98	0.22	−2.12	0.27
Collapse	−2.99	0.22	−3.50	0.22	−1.67	0.24

[a] Mean values defining the cumulative log-normal vulnerability relationships.
[b] Standard deviation values defining the cumulative log-normal vulnerability relationships.
[c] Mean confidence bound curves.
[d] Upper 95th percentile confidence bound curves.
[e] Lower 5th percentile confidence bound curves.

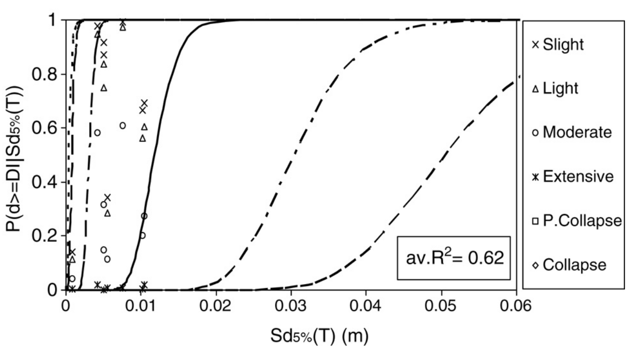


Fig. 6. Comparison of the infilled frame population fragility curves with eight observed post-earthquake damage distributions for like populations of structures.

A very close fit of the regressed curves to the analytical data is achieved in all cases ($R^2 > 0.98$). The vulnerability relationships for the "Slight" and "Light", "Moderate" and "Extensive", and "Partial Collapse" and "Collapse" damage states are selected from the curve sets derived from the 95YRP, 475YRP and 2475YRP response surfaces, respectively. These relationships are combined to form a "performance-consistent" set of vulnerability curves. The curves are illustrated in Fig. 6 and their equations are summarised in Table 4.

The fit of the analysis data to the response surfaces is a direct indication of the uncertainty in response prediction due to input parameter variability. The 5th and 95th percentile values of the ratio of analytically observed to predicted ISD_{max} response are therefore applied to the mean ISD_{max} predicted by the response surface for each system variation. New exceedence probability statistics are generated using the extreme ISD_{max} estimates. These are plotted against the corresponding ground motion values and used in the regression of 90% confidence bounds for the vulnerability curves (Table 4). The confidence bounds are wide and increase in width for higher damage states. It is observed that similar sized confidence bounds are generated where the response surfaces are derived from data relating to a single scenario and when they are derived from the combined data from all scenarios. This indicates that the variation in material and ground motion is fully represented in the analyses for each earthquake scenario.

3. Comparison with observational damage

In Fig. 6, the analytical performance-consistent vulnerability curves for the three-storey infilled RC frame designed to the Italian standards of 1982 are compared to damage statistics deriving from eight post-earthquake surveys carried out on populations of infilled RC structures (1154 buildings in total). These data consist of a subset of the observational damage distribution database adopted in [4] for the generation of empirical vulnerability curves. Although the quantity of data for comparison is limited, the analytical curves are seen to give a reasonable, slightly conservative, fit to the empirical damage data, with an average correlation coefficient (R^2) of 0.62. The analytical curves show improved prediction of observed damage compared to other existing RC structure curves [4] and compared to the infilled frame fragility curves of Mosalam et al. [3] (with $R^2 = 0.39$). Comparison of the empirical vulnerability curves derived by Rossetto and Elnashai [4] for low-rise, old seismic code, infilled RC frames with those derived analytically shows the latter to give a better correlation to the observed damage data ($R^2 = 0.62$ compared to 0.42). This is caused by to the scarcity and highly scattered nature of the post-earthquake damage surveys available for infilled RC frames, which result in unreliable curve shapes being derived for the latter empirical relationships. The width of the confidence bounds associated with the analytical relationships, in terms of maximum exceedence probability interval (at constant spectral displacement), is observed to be comparable to that observed for the empirical curves. The width in terms of maximum spectral displacement interval (for constant damage state exceedence probability) is instead a fraction of that observed for the empirical curves. The difference is mainly due to the uncertainty introduced in the derivation of the latter curves due to scarcity of observational data for high levels of ground motion. These observations give rise to substantial doubt as regards the reliability of observation-based vulnerability functions and confirm the importance of analytical methods for the generation of fragility curves.

4. Conclusions

The main accomplishment of the study described above is a clear methodology for the derivation of vulnerability curves using analytical damage statistics. The procedure addresses problems of uncertainty in the input ground motion and damage state identification, and yields curves that are appropriate for use within a displacement-based assessment framework. The proposed methodology represents the best possible inelastic static analysis solution given current knowledge and capabilities and has the added benefit that, through appropriate consideration in modelling, it can be applied to any structural type and seismo-tectonic environment. Comparison of the curves derived for a population of low-rise infilled RC frames of inadequate seismic design with observational data and empirical curves for these structures shows that the proposed methodology is capable of producing analytical vulnerability curves that give reasonable predictions of observed post-earthquake building damage. Further verification of the vulnerability curve derivation procedure for different structural systems and greater quantities of observational data are needed.

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