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# Determination of effecting dimensional parameters on warpage of thin shell plastic parts using integrated response surface method and genetic algorithm<sup>☆</sup>

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## Abstract

Decreasing warpage is a very significant topic to improve the quality of injection-molded parts. Dimensional stability is an important factor for the minimum warpage of thin shell plastic part. In this study, efficient minimization of warpage on thin shell plastic parts by integrating finite element analysis, statistical design of experiment method, response surface methodology and genetic algorithm is investigated. A thin shell plastic part model is considered as an example. To achieve the minimum warpage, optimum process condition dimensional parameters are determined.  $X$  dimension,  $Y$  dimension, and  $Z$  dimension are considered as model variables. Another parameters of effecting minimum warpage are taken into consideration as constant, such as mold temperature, melt temperature, injection time, injection pressure, etc. Finite element analyses are conducted for combination of process parameters organized using statistical three-level full factorial experimental design. A predictive model for warpage is created using response surface methodology exploiting finite element analysis results. Response surface model is interfaced with an effective genetic algorithm to find the optimum process parameter values.

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*Keywords:* Warpage; Design of experiment; Plastic injection molding; Response surface model; Genetic algorithm

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## 1. Introduction

As the wall thickness of plastic parts become thinner, the injection molding operation becomes more difficult. However, the industry demand for techniques of plastic injection molding to produce plastic parts with thin wall features. The procedure of injection molding is described, such as plastication, injection, packing, cooling, ejection and process part/part quality control applications. When the interior of cavity has become stable, the product is ejected from the mold. Defects of the products, such as warpage, shrinkage, sink marks, and residual stress, are caused by many factors during the production process. These defects influence the quality and accuracy of the products. Dimensional stability is an important factor for the minimum warpage of thin shell plastic part. Reducing warpage is one of the top priorities to improve the quality of injection-molded parts [1–7]. During production of plastic parts, the quality problems arise from dimensional ratio of the parts designed. Designs of dimensional process parameters are investigated from several aspects in the literature. Several researches have been conducted on the warpage of thin shell plastic parts [8,9]. However, very few of them are devoted to the optimization of such parts [10,11]. In this study, an efficient optimization method by coupling finite element analysis, response surface methodology and genetic algorithm is introduced to minimize warpage of thin shell plastic parts. The developed optimization method is applied to a thin shell plastic part model. During the optimization process, finite element (FE) analyses of the part model base are conducted for combination of process parameters organized based on statistical full factorial experimental design.  $X$  dimension,  $Y$  dimension, and  $Z$  dimension are considered as process conditions dimensional parameters influencing warpage. Another parameters of effecting minimum warpage are taken into consideration as constant, such as mold temperature, melt temperature, injection time, injection pressure, etc. A predictive model for warpage in terms of the critical process parameters is then created using response surface methodology. Response surface model is coupled with an effective genetic algorithm to find the optimum process parameter values. The following sections explain in detail the generation of predictive models for minimum warpage.

## 2. Experimental study

Design of experiment (DOE) has been implemented to select manufacturing process parameters that could result in a better quality product. The DOE is an effective approach to optimize the

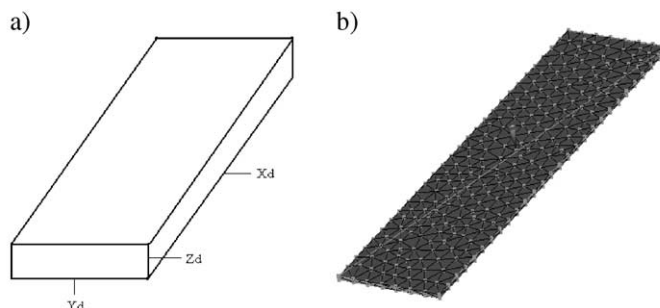


Fig. 1. (a) Base and (b) mesh geometry of thin shell plastic model.

Table 1  
Properties of the material used

Commercial product names	Bayer ABS Limited, Absolac 300, ABS
Density of solution (g/cm <sup>3</sup> )	1.0574
Viscosity (Pa s)	VI(240)91
Recommended die temperature (°C)	60
Recommended solution temperature (°C)	235
Material characteristics	Amorphous

various manufacturing process parameters [12]. Three independent variables consist of injection parameters, each with three levels, for warpage were applied total of  $3^3=27$  experimental runs [12]. In this study, three independent variables, such as  $X$  dimension ( $X_d$ ),  $Y$  dimension ( $Y_d$ ), and  $Z$  dimension ( $Z_d$ ) had total of  $3^3=27$  experimental runs. The simulation model of the thin shell plastic part was designed using Pro/Engineering 2001 CAD software. Base and mesh geometry are shown in Fig. 1a,b. To develop a simulation model, the geometry of the thin shell plastic part is executed using Fusion mesh with MoldFlow. It is created by MoldFlow Plastic Insight 3.0 which is a commercial software based on hybrid finite-element/finite-difference method for solving pressure, flow and temperature fields. The part is made of ABS (Absolac 300 Bayer Limited) [13]. Material details are shown in Table 1.

Impax supreme was used as the die material. Mold density of 7.8 g/cm<sup>3</sup>, mold specific heat of 460 J/kg °C, and mold thermal conductivity of 29 W/m °C were considered [13]. Sprue of conic, runner of circular, and gate of conic were used to inject the melted material into cavity (constant dimension taken into consideration). In this study, in order to keep the mold temperature constant, cooling water's inlet temperature of 20 °C was used. Simulation modeling's cooling channel diameter of  $d$  (6 mm), cooling channel's center distance of  $3d$  (18 mm) in horizontal direction, cooling channel center and parting surface distance of 10 mm were taken into consideration (as

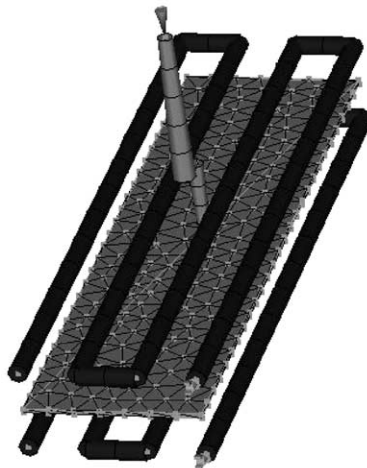


Fig. 2. Finite element modal with cooling channel for thin shell plastic part model.

Table 2  
Manufacturing parameters employed in MoldFlow analysis

Injection time (s)	1.45
Injection pressure (MPa)	85
Packing pressure (MPa)	90
Packing time (s)	10
Cooling time (s)	12
Cooling channel diameter (mm)	6
Between cooling channel's center distance (mm)	18
Number of gate	1
Between cooling channel's (circuit) center and parting surface distance (mm)	10
Cooling channel of only one section departure length (mm)	According to Xd values varies
Water inlet temperature (°C)	20
Water flow/rate (l/min)	2.29
Reynolds number	10,000

shown in Fig. 2). Manufacturing parameters employed in MoldFlow analysis were shown in Table 2.

### 3. Response surface model for warpage

Computationally cost FE model is not suitable for large number of repetitive analyses which are often required in an optimization process. Therefore, in this study, the FE model for warpage is replaced by a simpler and more efficient predictive model created by response surface methodology (RSM). RSM is a model building technique based on statistical design of experiment and least square error fitting. Steps taken in creating response surface (RS) models by RSM are illustrated in Fig. 3. RSM creates polynomial models for the available data set as follows:

$$f = a_0 + \sum_{i=1}^n a_i x_i + \sum_{i=1}^n \sum_{j=1}^n a_{ij} x_i x_j + \dots \quad (1)$$

where  $a_0$ ,  $a_i$  and  $a_{ij}$  are tuning parameters and  $n$  is the number of model parameters (i.e. process parameters). The polynomial models generated by RSM are often referred to as response surface (RS)

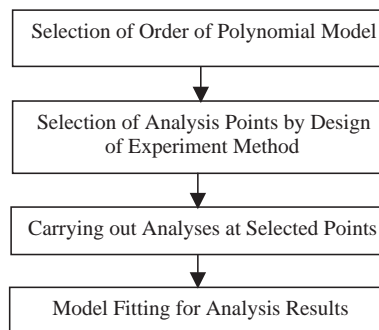


Fig. 3. Steps taken in creating a response surface model by RSM.

Table 3a  
Three factors and three levels

Factors	Level 1	Level 2	Level 3
Xd	30	85	160
Yd	5	45	85
Zd	0.8	1.2	1.6

models in the literature. RSM was originally developed for the model fitting of physical experiments by Box and Draper [14] and later adopted in other fields. To create RS models, a computer program has been written in MATLAB language. The program has the capability of creating RS polynomials up to 10th order if sufficient data exist. All cross terms in the models can be taken into account. RS models can also be generated in terms of inverse of parameters. That is,  $x_i$  can be replaced as  $1/x_i$  (i.e. inversely) in RS model if desired. RS models of varying orders from first order to third order are created and tested with the developed program. The data set consists of  $3^3=27$  analysis results and corresponds to the combination of three-dimensional parameters affecting the warpage. Therefore, RS models generated describe warpage in terms of the dimensional parameters ( $X$  dimension (Xd),  $Y$  dimension (Yd), and  $Z$  dimension (Zd)). The data set is divided into two parts; one part to create the model, other part to check the accuracy of the created model. These data sets are shown in Tables 3a–3c.

Table 3b  
Training data sets blocks

Experiment no.	Training data sets blocks and results			
	$X$ dimension, Xd (mm)	$Y$ dimension, Yd (mm)	$Z$ dimension, Zd (mm)	Warpage (mm)
1	30	5	0.8	0.0977
2	95	5	0.8	0.3017
3	160	5	0.8	0.4978
4	30	45	0.8	0.1737
5	95	45	0.8	0.3063
6	160	45	0.8	0.5101
7	95	85	0.8	0.3871
8	160	85	0.8	0.5411
9	30	5	1.2	0.0990
10	95	5	1.2	0.3106
11	160	5	1.2	0.5011
12	30	45	1.2	0.1758
13	95	45	1.2	0.3491
14	30	85	1.2	0.2905
15	95	85	1.2	0.4173
16	160	85	1.2	0.5578
17	30	5	1.6	0.0995
18	95	5	1.6	0.3160
19	160	5	1.6	0.5062
20	30	45	1.6	0.1766
21	160	45	1.6	0.5227
22	30	85	1.6	0.2917
23	95	85	1.6	0.4201
24	160	85	1.6	0.5699

Table 3c  
Check data sets blocks

Experiment no.	Check data sets blocks and results			
	X dimension, Xd (mm)	Y dimension, Yd (mm)	Z dimension, Zd (mm)	Warpage (mm)
1	30	85	0.8	0.2847
2	160	45	1.2	0.5149
3	95	45	1.6	0.3524

Afterwards, each factor was made of symbol digit 0 (directly) and 1 (inversely). Factor statement was changed directly and inversely on function base ( $2^3=8$ ). Linear, parabolic, and cubic polynomial functions were used to see check data sets result percentage error. Accuracy of the created RS models is shown in Table 4. As seen from Table 4, cubic polynomial, where all dimensional parameters are included directly, gives the least error of acceptable value (1.43%) and this model is therefore utilized in the warpage optimization with genetic algorithm in the following section. Approximately empirical Eq. (2) is shown as:

$$f = a_0 + (a_1Xd + a_2Xd^2 + a_3Xd^3) + (a_4Yd + a_5Yd^2 + a_6Yd^3) + (a_7Zd + a_8Zd^2 + a_9Zd^3) + a_{10}XdYd + a_{11}YdZd + a_{12}XdZd + a_{13}XdYdZd + \dots \quad (2)$$

where  $a_0$ ,  $a_i$  and  $a_{ij}$  are tuning parameters and X dimension (Xd), Y dimension (Yd), and Z dimension (Zd) are significant dimensional parameters of minimum warpage for a thin shell plastic parts. The minimum warpage was predicted by 1.43% error using cubic polynomial function (Tables 5a and 5b).

#### 4. Warpage optimization by genetic algorithm

##### 4.1. Optimization problem formulation and solution

In this research, the best (optimum) warpage condition within the range given in Tables 3a–3c is determined by using an optimization method. For this purpose, minimum warpage process is defined in

Table 4  
Comparison check data sets result percentage error

Reciprocal flag	Polynomial function types and percentage error		
	Linear (needed training data is 4)	Parabolic (needed training data is 10)	Cubic (needed training data is 20)
[0 0 0]	22	7.2	1.43
[0 1 0]	33.17	13.58	2.74
[0 0 1]	23.15	8.04	1.98
[1 0 0]	28.32	2.01	1.94
[1 1 0]	39.31	10.71	3.28
[0 1 1]	34.27	14.47	1.82
[1 0 1]	28.83	2.16	4.94
[1 1 1]	40.05	11.23	3.04

Table 5a  
 Comparison of training data sets results for cubic polynomial function and FE

Experiment no.	Comparison of training data sets results (mm)	
	FE results	Cubic polynomial function results
1	0.0977	0.0998
2	0.3017	0.2959
3	0.4978	0.5002
4	0.1737	0.1683
5	0.3063	0.3202
6	0.5101	0.5038
7	0.3871	0.3853
8	0.5411	0.5417
9	0.0990	0.1004
10	0.3106	0.3091
11	0.5011	0.5034
12	0.1758	0.1769
13	0.3491	0.3432
14	0.2905	0.2943
15	0.4173	0.4117
16	0.5578	0.5618
17	0.0995	0.0979
18	0.3160	0.3191
19	0.5062	0.5034
20	0.1766	0.1767
21	0.5227	0.5248
22	0.2917	0.2898
23	0.4201	0.4234
24	0.5699	0.5672

the standard optimization problem format that can be solved by a numerical optimization algorithm. Standard optimization problem definition requires an objective function to be minimized or maximized and constraint functions to be satisfied in terms of optimization parameters. For a thin shell plastic part model, optimization problem can be defined as follows:

$$\text{Find : } X_d, Y_d, Z_d \tag{3a}$$

$$\text{Minimize : } \text{Warpage } (X_d, Y_d, Z_d) \tag{3b}$$

$$\text{Subjected to constraints : } \text{Warpage} \leq 0.0977 \text{ mm} \tag{3c}$$

Table 5b  
 Comparison of check data sets results for cubic polynomial function and FE

Experiment no.	Comparison of check data sets results (mm)		
	FE results	Cubic polynomial function results	Error (%)
1	0.2847	0.2842	0.17
2	0.5149	0.5188	0.75
3	0.3524	0.3574	1.43

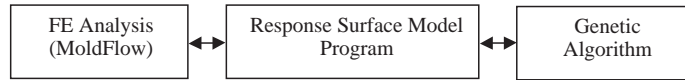


Fig. 4. Interaction of simulation software, RS model and GA during warpage optimization.

Within ranges:

$$30 \text{ mm} \leq X_d \leq 160 \text{ mm} \tag{3d}$$

$$5 \text{ mm} \leq Y_d \leq 85 \text{ mm} \tag{3e}$$

$$0.8 \text{ mm} \leq Z_d \leq 1.6 \text{ mm} \tag{3f}$$

Ranges of variables for  $X_d/Z_d$  ratio of ABS material based on the minimum and maximum values that should be used in MoldFlow User manual were selected [13]. Ranges of variable for  $Y_d$  were selected to take into consideration  $X_d/Z_d$  ratio, randomly. There are several numerical optimization algorithms available to solve the above optimization problem. In this study, a global optimization method, genetic algorithm (GA) [15], is used to converge a global optimum among several possible local optimums. To solve the above optimization problem, an effective GA is coupled with the response surface model for warpage to yield a global optimum as shown in Fig. 4.

Genetic algorithm (GA) simulates biological evolution process; Darwin’s theory of survival of the fittest. The solution of the optimization problem with genetic algorithm begins with a set of potential solutions or *chromosomes* (usually in the form of bit strings) that are randomly generated or selected. The entire set of these chromosomes comprises a *population*. The chromosomes evolve during several iterations or *generations*. New generations (*offspring*) are generated using the crossover and mutation technique. Crossover involves splitting two chromosomes and then combining one half of each chromosome with the other pair. Mutation involves flipping a single bit of a chromosome. The chromosomes are then evaluated using a certain fitness criteria and the best ones are kept while the others are discarded. This process repeats until one chromosome has the best fitness and thus is taken as the best solution of the problem.

The critical parameters of genetic algorithm in solving an optimization problem effectively are the size of the population, mutation rate, number of iterations (i.e. generations), etc. In this study, population size of 50, crossover rate of 1.0, mutation rate of 0.1, bit number for each variable of 16, and number of generations of 500 are employed.

#### 4.2. Optimization results

Optimization problem in Eqs. (3a)–(3f) is solved with and without constraints to search the effect of several what-if scenarios such as the existence/non-existence and the magnitude of constraint limits on

Table 6  
Dimension parameters of minimum warpage condition values and after optimization

	Injection molding process parameters		
	$X_d$ (mm)	$Y_d$ (mm)	$Z_d$ (mm)
After optimization	30	5	0.9



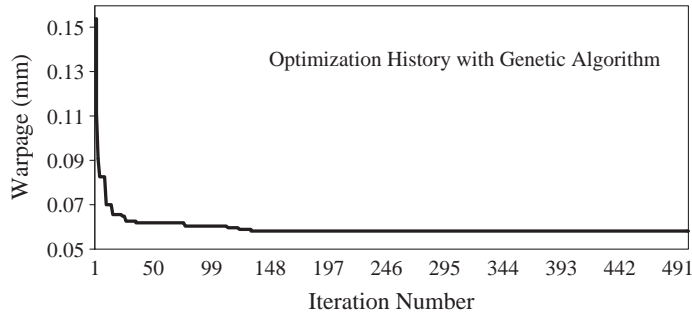


Fig. 5. Optimization history with iterations for warpage.

optimum values of parameters. In the with constraint option, solutions are conducted for variety of constraint limits. Constraint upper limits for maximum warpage are 0.0977 mm. With a solution, a set of optimum values is obtained to provide the user with a wide range of selection of warpage dimensional parameters. Validation of the optimum values obtained from the GA program with FE simulations is performed for some values. These values are shown in Table 6. Optimization history has 500 iterations and these are demonstrated in Fig. 5. Warpage distribution on the thin shell plastic part model based on optimum dimensional parameter set result according to Table 6 is shown in Fig. 6.

When maximum warpage is considered, it is seen that maximum warpage on the thin shell plastic parts model, according to Table 3b is 0.0977 mm before the optimization, is reduced to 0.0582 mm by 40.4% after optimization. The constraint of minimum warpage imposed is seen in Table 6.

### 5. Conclusions

In this study, an efficient optimization methodology using RSM and genetic algorithm was introduced in minimizing warpage of thin shell plastic parts manufactured by injection molding. A thin shell plastic

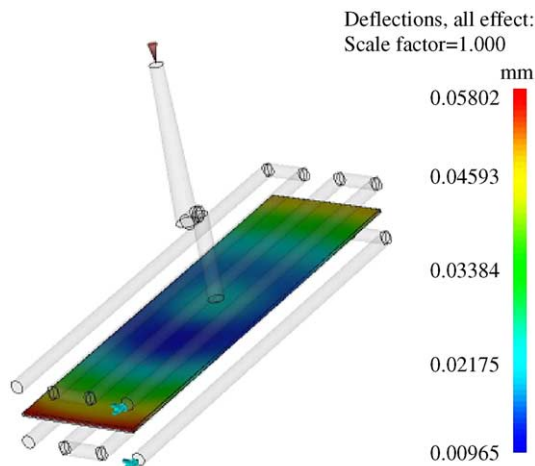


Fig. 6. Warpage distribution on the thin shell plastic part model based on optimum dimensional parameters.

part model was designed. To achieve the minimum warpage, the appropriate dimensional parameters were determined.  $X$  dimension ( $X_d$ ),  $Y$  dimension ( $Y_d$ ), and  $Z$  dimension ( $Z_d$ ) were considered as dimensional parameters. Finite element analyses were conducted for combination of dimensional parameters organized using statistical three-level full factorial experimental design method. A predictive model for warpage was created in terms of the dimensional parameters ( $X$  dimension ( $X_d$ ),  $Y$  dimension ( $Y_d$ ), and  $Z$  dimension ( $Z_d$ )) using response surface methodology to reduce the computational cost of the optimization process. Response surface model was interfaced with an effective genetic algorithm to find the optimum process parameter values. GA had reduced the warpage of the initial model significantly. Warpage was improved by 40.4%. This indicated that the optimization methodology in this study could also be employed to improve other thin shell plastic parts.

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