Available online at www.sciencedirect.com
SCIENCE DIRECT
OCEAN ENGINEERING
Ocean Engineering 32 (2005) 1332–1349
www.elsevier.com/locate/oceaneng

Neural network for design and reliability analysis of rubble mound breakwaters

Dong Hyawn Kim[a,*], Woo Sun Park[b]

[a]Department of Ocean System Engineering, Kunsan National University, Kunsan, Jeonbuk 573-701, South Korea
[b]Coastal Engineering and Harbor Research Division, KORDI, Ansan, Kyonggi 426-744, South Korea

Received 28 June 2004; accepted 24 November 2004
Available online 25 February 2005

Abstract

Artificial neural networks were applied to the design of rubble mound breakwater. Five neural networks with different network structures were trained with the same training data. Then they were compared with conventional empirical model and one another. It was found that the neural network technique gives more accurate results than conventional empirical model and the extent of accuracy can be affected by the structure of neural network. After that, how to integrate the trained neural network into reliability analysis technique is proposed. Since the neural network technique shows better performance than empirical model based approach in breakwater design, it is expected that the neural network integrated reliability analysis gives more improved results for probability of failure than it is done with empirical model.

Keywords: Breakwater; Stability; Neural network; Reliability based design; Monte Carlo simulation

1. Introduction

The stability of rubble mound breakwater is usually analyzed by the well-known empirical formulae by Hudson (1958) and van der Meer (1988a). Those formulae are used to determine the individual weight of armor blocks of a breakwater. Although those formulae were derived from a number of experimental data, they show too much

* Corresponding author. Tel.: +82 63 469 1862; fax: +82 63 463 9493.
E-mail address: eastlite@kunsan.ac.kr (D.H. Kim).

0029-8018/$ - see front matter
doi:10.1016/j.oceaneng.2004.11.008

disagreement between the measured stability numbers and the predicted ones. The uncertainties in the empirical formulae inevitably increase the factor of safety and, eventually, the construction cost. Therefore, a number of studies have been carried out to develop an advanced empirical formula for breakwater stability.

Kaku (1990) and Kaku et al. (1991) proposed an empirical formula for damage level prediction based on the van der Meer's experimental data. Smith et al. (1992) compared their own test results with the prediction by Kaku et al. (1991). Hanzawa et al. (1996) proposed an empirical formula for stability number based on their own test data. Although, several empirical formulae have been proposed for decades, remarkable improvement cannot be seen. Recently, Mase et al. (1995) proposed a new stability evaluation method by using neural network. The neural network technique seems to make a breakthrough in the design of rubble mound breakwater. Actually, the damage levels predicted by the neural network agree better than those by the van der Meer's. The stability number, however, still need to be improved.

In this study, several neural network models are proposed to predict the stability number of armor blocks of breakwater. The same training data set is used for the neural networks but the structures of neural network, the number of nodes at input and hidden layer, differ from one another. Based on the numerical examples, it was shown that the neural network technique gives more accurate results than conventional empirical model and the extent of accuracy can be affected by the structure of neural network.

After that, a new reliability analysis technique is proposed by combining the trained neural network with Monte Carlo simulation. Since the neural network technique shows better performance than empirical model based approach in breakwater design, it is expected that the neural network integrated reliability analysis gives more improved results for probability of failure than it is done with empirical model.

2. Empirical formula for stability

van der Meer proposed a stability model by analysing a large number of irregular wave tests on rock stability in his Thesis van der Meer (1988b). He first surveyed the influential design parameters which should be included in the empirical model such as the significant wave height (H_s), the mean wave period (T_m), the relative density of stone ($\Delta = \rho_s/\rho_w - 1$), the nominal diameter of stone (D_{n50}), the permeability of breakwater (P), the number of wave attack (N_w), the slope angle (α), etc. The stability formula using these parameters was given by

$$N_s = \begin{cases} 6.2P^{0.18}\left(\dfrac{S_d}{\sqrt{N_w}}\right)^{0.2}\dfrac{1}{\sqrt{\xi_m}} & \xi_m < \xi_c \\ 1.0P^{-0.13}\left(\dfrac{S_d}{\sqrt{N_w}}\right)^{0.2}\sqrt{\cot\alpha}\,\xi_m^P & \xi_m \ge \xi_c \end{cases} \tag{1}$$

where N_s is the stability number defined as

$$N_s = H_s/\Delta D_{n50} \tag{2}$$

and S_d is the damage level defined by using the eroded area (A) of the breakwater cross-section as

$$S_d = \frac{A}{D_{n50}^2} \tag{3}$$

and ξ_m is the surf similarity parameter of the following form

$$\xi_m = \frac{\tan\alpha}{\sqrt{2\pi H_s/gT_m^2}} \tag{4}$$

The transition condition of surf similarity is expressed as

$$\xi_c = (6.2P^{0.31}\sqrt{\tan\alpha})^{(1/(P+0.5))} \tag{5}$$

Fig. 1 shows the predicted stability numbers together with the measured ones for van der Meer's 641 data. As can be seen in the figure, the degree of agreement between the measured stability numbers and the predicted ones is not so good. The scattering of the predicted stability numbers from the measured ones cannot be negligible. Due to the uncertainty of the prediction results, unnecessary increases in the factor of safety, and simultaneously, the construction cost is inevitable. Hence, a new prediction model with higher accuracy is certainly required.

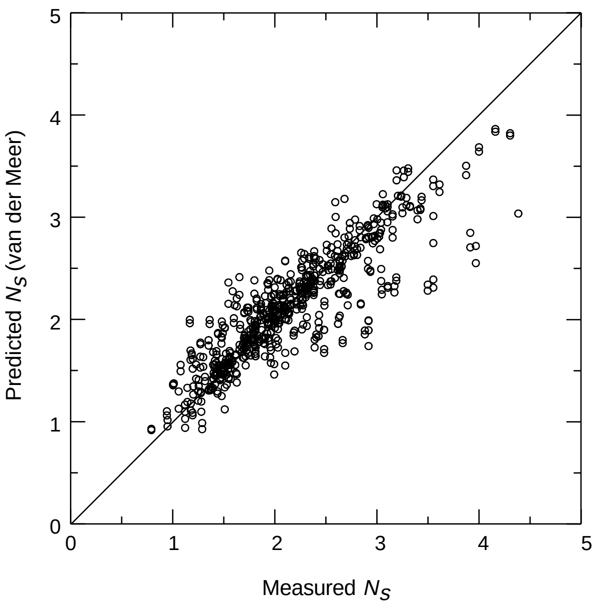

Fig. 1. Stability numbers predicted by van der Meer's formula.

3. Stability model using neural network

3.1. Neural network

Neural network is an information processing unit that was invented by modeling the human brain. It has ability to reproduce any kind of input–output relationship after it is properly trained. Due to the learning capability, the neural network has been widely used in many engineering problems, which cannot be easily solved by conventional mathematical approaches. Stability assessment of rubble mound breakwater is one of the suitable examples to which the learning capability of neural network is well applied.

Fig. 2 shows the typical layout of neural network. It consists of input layer, hidden layer, and output layer. And each layer has some nodes, which is the basis of information process. If there are $n_{\mathbf{x}}$, $n_{\mathbf{y}}$, $n_{\mathbf{z}}$ nodes at each layers, and the neural network receives $\mathbf{x}(n_{\mathbf{x}}\times 1)$ as input, the output of hidden layer can be obtained as

$$\mathbf{y} = f(\mathbf{W}_1\mathbf{x} + \mathbf{b}_1) \tag{6}$$

where $\boldsymbol{W}_1(n_{\mathbf{y}}\times n_{\mathbf{x}})$ is the weighting matrix between input and hidden layer and $\boldsymbol{b}_1(n_{\mathrm{y}}\times 1)$ is the bias vector at hidden layer, and f denotes an activation function, with constant u_0, which is given as

$$f(u) = \frac{1}{2}\left\{1 + \tanh\left(\frac{u}{u_0}\right)\right\} \tag{7}$$

Likewise, the out of neural network can be obtained simply as

$$\mathbf{z} = f(\mathbf{W}_2\mathbf{y} + \mathbf{b}_2) \tag{8}$$

where $\boldsymbol{W}_2(n_{\mathbf{z}}\times n_{\mathbf{y}})$ is also the weighting matrix between hidden and output layer, $\boldsymbol{b}_2(n_{\mathrm{z}}\times 1)$ is the bias vector at output layer. When the neural network is applied to breakwater design, $\mathbf{x}$ includes the design parameter such as wave height, period, etc. and $\mathbf{z}$ is the stability number (N_{s}) to be estimated.

With initially chosen weights, the neural network cannot estimate the required output. Therefore, the weights should be first found in a reasonable way that is the so-called training. To train a neural network, error function is defined as

$$E = \|\boldsymbol{\tau} - \mathbf{z}\|^2 \tag{9}$$

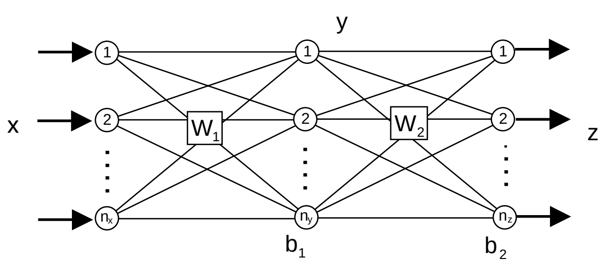

Fig. 2. A typical layout of neural network.

where $\boldsymbol{\tau}$ is the target value to be sought. By steepest gradient approach, it was already shown that if the weight matrix is updated according to the following rule, the error function can be minimized.

$$\Delta \mathbf{W}_2 = \alpha \mathbf{d}_2 \mathbf{y}^{\mathbf{T}} \tag{10}$$

$$\Delta \mathbf{W}_1 = \alpha \mathbf{d}_1 \mathbf{x}^{\mathbf{T}} \tag{11}$$

where α denotes the learning rate, $\mathbf{T}$ means the transpose of a vector, and $\mathbf{d}_2$ and $\mathbf{d}_1$ can be written as

$$\mathbf{d}_2 = (\boldsymbol{\tau} - \mathbf{z}) \otimes \mathbf{z}' \tag{12}$$

$$\mathbf{d}_1 = [\mathbf{W}_2^{\mathbf{T}} \mathbf{d}_2] \otimes \mathbf{y}' \tag{13}$$

In the above equation, prime denotes the derivative of a function and $\otimes$ is the element-by-element operator that can be expressed as

$$\begin{Bmatrix} q_1 \\ q_2 \\ \vdots \\ q_n \end{Bmatrix} \otimes \begin{Bmatrix} r_1 \\ r_2 \\ \vdots \\ r_n \end{Bmatrix} = \begin{Bmatrix} q_1 r_1 \\ q_2 r_2 \\ \vdots \\ q_n r_n \end{Bmatrix} \tag{14}$$

3.2. *Mase et al.'s approach (1995)*

Using randomly chosen 100 data from the experimental data of van der Meer (1988a), Mase et al. trained two neural network models to predict the stability number and damage level, respectively. The neural network consisted of six input nodes such that P, N_{w}, S_{d}, ξ_{m}, dimensionless water depth (h/H_{s}), and spectral shape (SS). They newly added h/H_{s} and SS to the neural network input to improve the prediction accuracy. The predicted damage levels actually agree better to the calculated ones than van der Meer's. The correlation coefficient for damage levels was increased from 0.7 (van der Meer's formula) to 0.81 (neural network). However, the stability numbers still need to be improved. Actually the correlation coefficient was decreased from 0.92 (van der Meer's formula) to 0.91 (neural network). For design purpose, the stability number is much more useful. Therefore, an advanced prediction model for the stability number is required.

More valuable finding in the study is that there exist some optimal numbers of training epoch. In common sense, it can be thought that the more training iteration gives the better prediction results. However, it is not true in case of the breakwater design. The numerical examples showed that too many training iteration could lead the network to be over-trained. Therefore, the training iteration was restricted to 5000 times in the study.

3.3. *Neural network models for benchmarking*

To compare the performance of neural network for stability prediction, five neural network models were employed. Each model has its own input parameters as in Table 1.

Table 1
Input parameters of neural network models

Model	Input parameters
ANN I	P, N, S_d, ξ_m, h/H_s, SS
ANN II	P, N, S_d, ξ_m, SS
ANN III	P, N, S_d, ξ_m, h/L_s, SS
ANN IV	P, N, S_d, $\cos\theta$, H_s/L_s, h/H_s, SS
ANN V	P, N, S_d, $\cos\theta$, H_s, T_s, h/H_s, SS

ANN I was first proposed by Mase et al. and it is used in this study for comparison with other models. ANN II is presented to identify the effect of water depth parameter, h/H_s, on the prediction performance. In ANN III, the water depth parameter is replaced by h/L_s where L_s is the period of significant wave. ANN IV is to show the effect of expansion of input dimension of neural network. Usually, the predictability of neural network increases to some extent when the input dimension increases. Therefore, the surf similarity is replaced by $\cot\theta$ and H_s/L_s. ANN V is to identify whether the performance can be improved or not when the neural network input is further expanded to include the wave height and the wave period.

To evaluate the stability models in more reasonable way, the so-called index of agreement of the following equation is used (Willmott, 1981).

$$I_a = 1 - \frac{\sum_{i=1}^{n}(e_i - m_i)^2}{\sum_{i=1}^{n}[|e_i - \bar{m}| + |m_i - \bar{m}|]^2} \tag{15}$$

where e_i and m_i denotes the estimated and the measured stability number; $\bar{m}$ is the average of measured stability. As I_a closes to one, the predicted set agree well to the measured set. The correlation coefficient has been widely used in many engineering data analysis. But it is not thought to be the best criterion for the problems whose purpose is to check how well

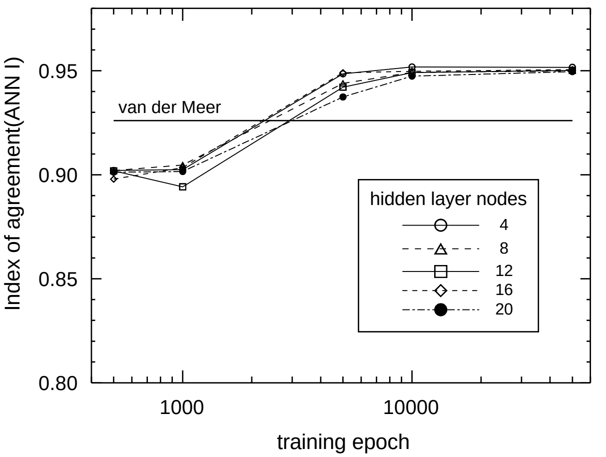

Fig. 3. Index of agreement during training (ANN I).

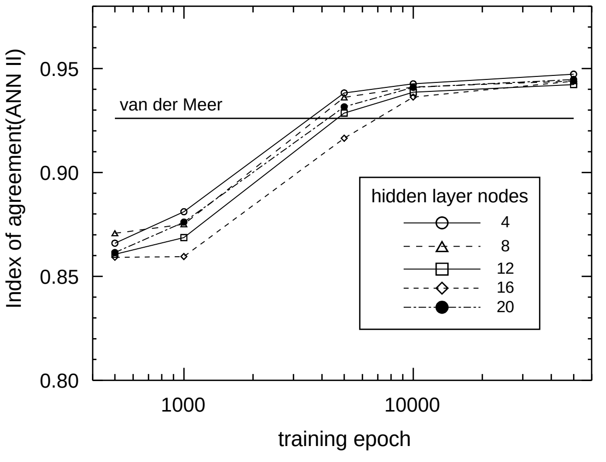

Fig. 4. Index of agreement during training (ANN II).

the predicted values agree to the measured ones. The correlation coefficient is basically defined to evaluate not the coincidence but the linearity between any two data sets. Therefore, the index of agreement is more suitable for the evaluation of estimation accuracy.

The same 100 data randomly chosen from the van der Meer's 641 data are used to train the five neural network models. Then, the accuracy of the trained neural network is evaluated with 641 data. Figs. 3–7 show the index of agreements during training. The index of agreements of all the models are larger than that of van der Meer's at above 1×10^4 training epochs and they increase with further training. ANN I is, however, optimal at 1×10^4 training epochs and get worse with further training. The optimal condition for five

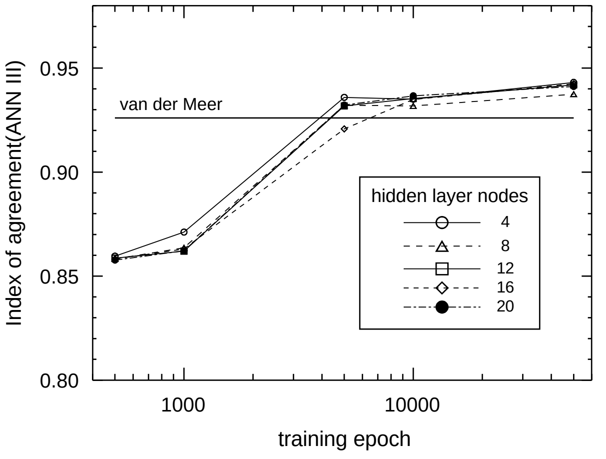

Fig. 5. Index of agreement during training (ANN III).

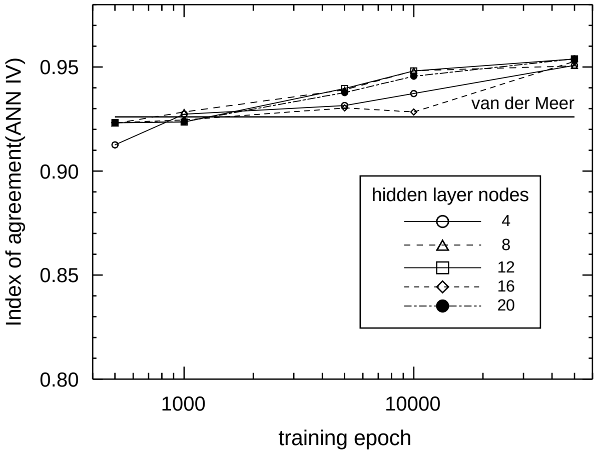

Fig. 6. Index of agreement during training (ANN IV).

models are summarized in Table 2. The index of agreement of ANN I is more closer to one than that of ANN II. It implies that the water depth parameter, h/H_s, is effective in estimating the stability number. In addition, ANN IV and ANN V are better than ANN I–ANN III implying that any neural network becomes effective when using the design parameters as independent network input rather than using artificial parameters such as the surf similarity as one input. It is well-known characteristics of neural network. The stability numbers predicted by the optimal neural network models are shown in Figs. 8–12, respectively.

However, one should note that the stability models are trained by using the data obtained from the scale model test. Actually, the training wave height data ranges from 0.0461 to 1.18 m and the mean wave period from 1.24 to 4.4 s which are quite small compared with practical design ranges. Usually, when a neural network receives input

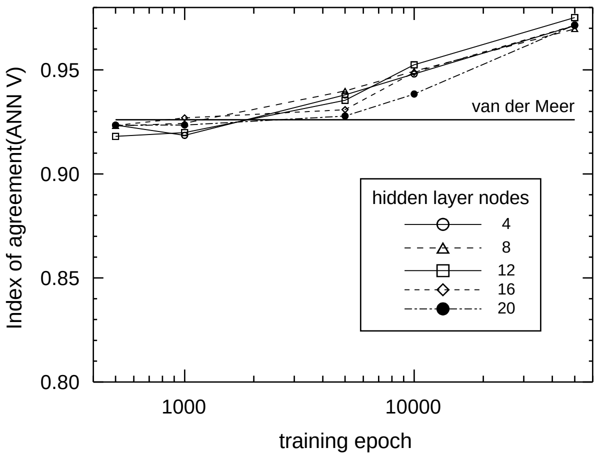

Fig. 7. Index of agreement during training (ANN V).

Table 2
Performance of stability models

Criteria	Model					
	VM	ANN I	ANN II	ANN III	ANN IV	ANN V
Index of agreement	0.926	0.951	0.947	0.944	0.954	0.975
(Epoch)	–	(1×10^4)	(5×10^4)	(5×10^4)	(5×10^4)	(5×10^4)
(Hidden layer node)	–	(4)	(4)	(4)	(20)	(12)
Correlation coefficient	0.876	0.914	0.906	0.902	0.915	0.952
(Epoch)	–	(1×10^4)	(5×10^4)	(5×10^4)	(5×10^4)	(5×10^4)
(Hidden layer node)	–	(4)	(20)	(4)	(20)	(12)

data, which out-range the data used in the training, it cannot predict proper output. So, the ANN V cannot be considered to be useful for design purposes. This can be seen from Fig. 12. The design weights of armor units are calculated according to the damage level. The other design parameters are fixed as (case I): $h=6$ m; $H_s=3.5$ m; $T_m=10.0$ s; $T_p=11$ s; $T_s=10.406$ s; cot $\theta=3.0$; $P=0.5$; $N=1000$; $\Delta=1.63$; SS=Pierson Moskowitz Spectrum. As can be seen from the figure, the weights of amour layer cannot be accepted as physically meaningful ones (Fig. 13).

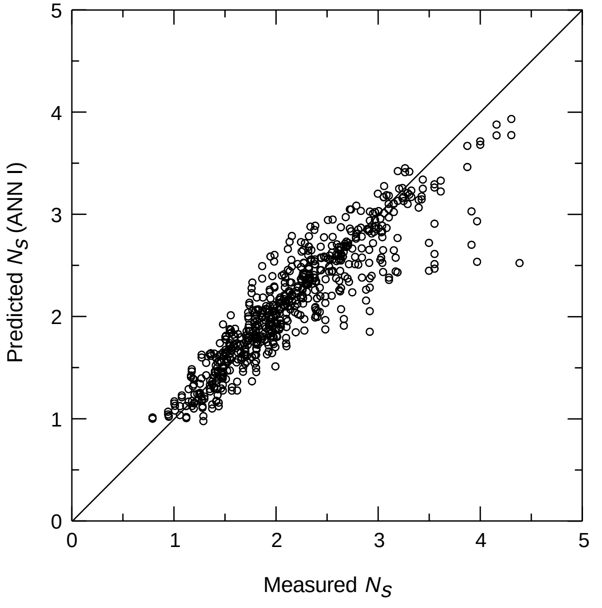

Fig. 8. Stability numbers predicted by ANN I.

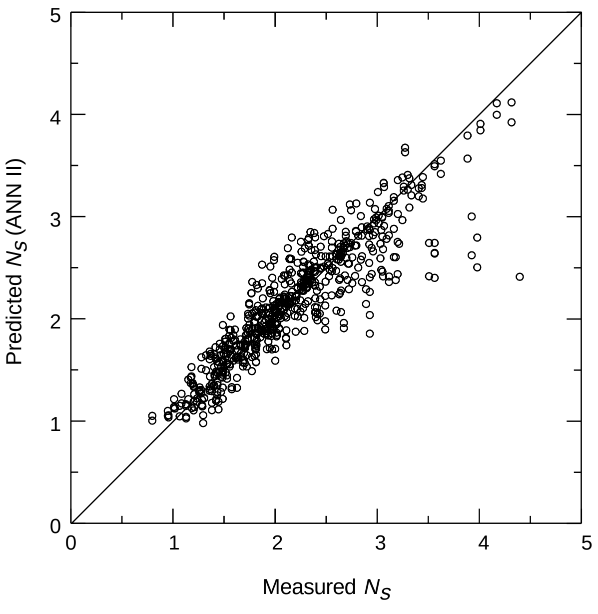

Fig. 9. Stability numbers predicted by ANN II.

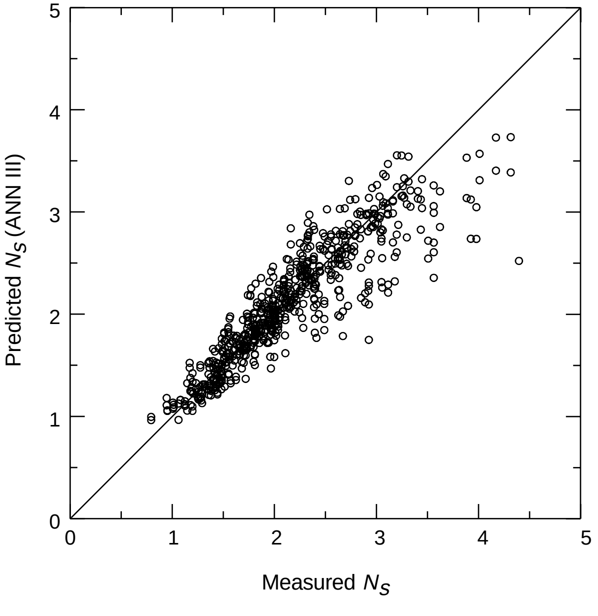

Fig. 10. Stability numbers predicted by ANN III.

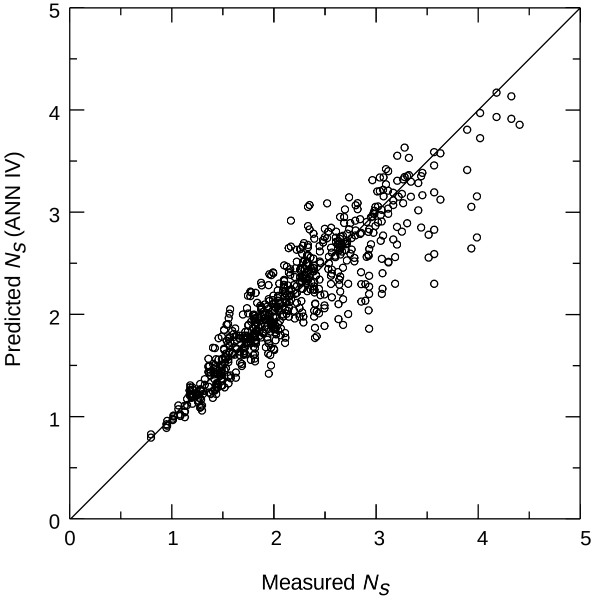

Fig. 11. Stability numbers predicted by ANN IV.

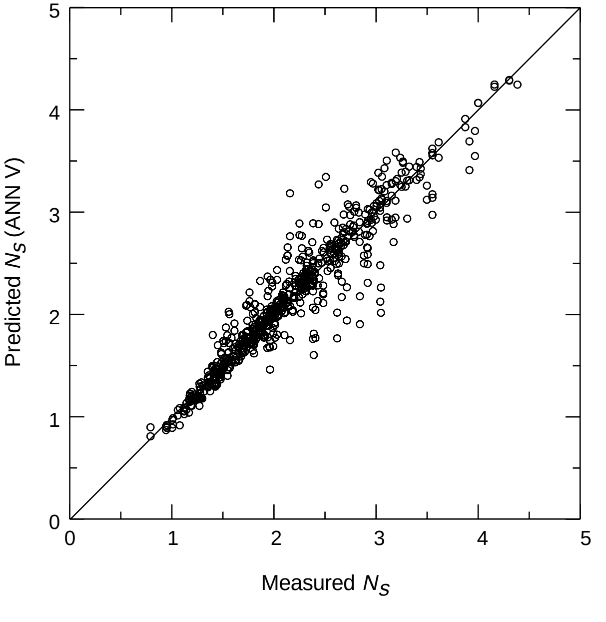

Fig. 12. Stability numbers predicted by ANN V.

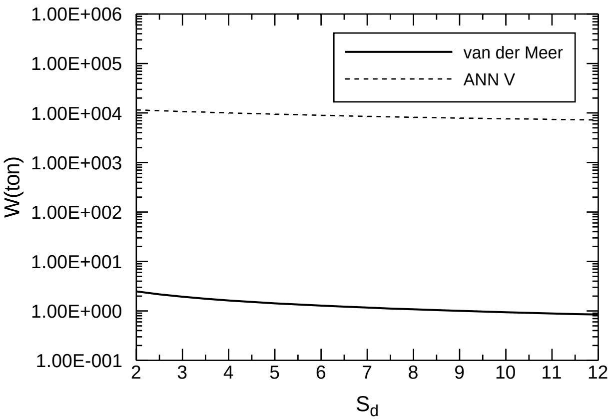

Fig. 13. Weight of amour layer by ANN V (case I).

On the other hand, ANN I–ANN IV give physically acceptable results as shown in Fig. 14. It can be concluded, therefore, that ANN IV is the most effective and practically useful model among the five models.

In case I, the van der Meer's formula gives much conservative results than neural network models. But, for a second case (case II) where the parameters are selected as: $h = 3$ m; $H_s = 2.0$ m; $T_m = 5.0$ s; $T_p = 5.5$ s; $T_s = 5.2$ s; cot $\theta = 3.0$; $P = 0.5$; $N = 1000$; $\Delta = 1.63$; SS = Pierson Moskowitz Spectrum; the weights of amour blocks by ANN III and ANN IV give much conservative design results than van der Meer's as shown in Fig. 15. Likewise, it is case by case whether van der Meer's formula and neural network model give relatively lighter or heavier design weights. Therefore, it may be dangerous to design a breakwater based only on an empirical formula such as van der Meer's. It is recommended that advanced stability model such as neural network models should be used to design a armor units in more economic and safer way.

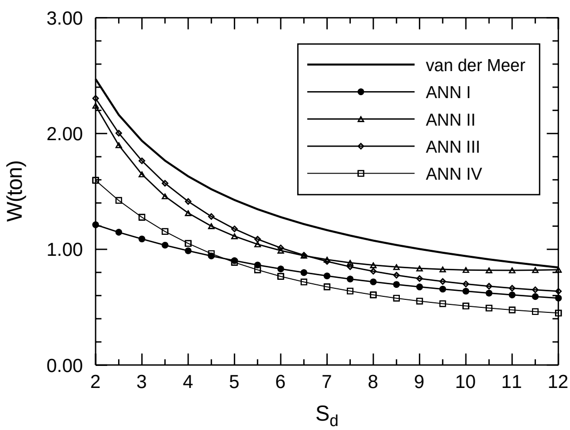

Fig. 14. Weight of amour layer by ANN I–ANN IV (case I).

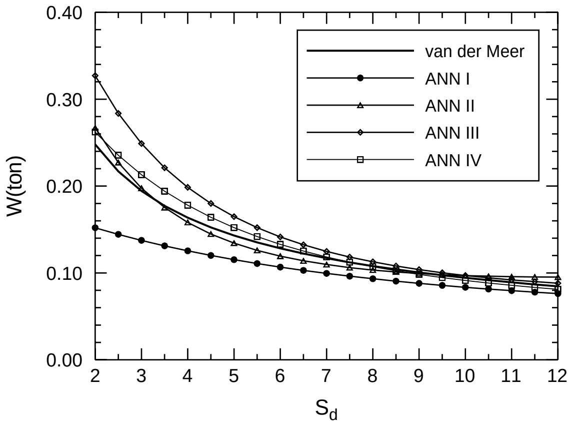

Fig. 15. Weight of amour layer by ANN I–ANN IV (case II).

4. Reliability analysis combined with neural network

Reliability based design approach using the trained neural network model is proposed. The so-called Monte Carlo simulation, one of the level III approach, is used to analyze the probability of failure for a given design point. The flow chart of Monte Carlo simulation is described in Fig. 16.

The n set of design parameters can be generated by using the distribution parameters such as means and standard deviations. Then, the stability numbers can be predicted by the trained neural network, or by the empirical formula for each n set. Using the predicted stability number, the reliability function value, Z, is calculated. If Z is positive, then the design set is considered to be safe. But if Z is negative, then it is expected to be in failure. Repeat this process until i equals n. The failure probability is finally obtained by dividing the failure cases (f) by the total populations (n).

In the case of neural network approach, the reliability function is defined by

$$Z^{\mathrm{NN}} = N_{\mathrm{s}}^{\mathrm{NN}} \Delta D_{\mathrm{n50}} - H_{\mathrm{s}} \tag{16}$$

where $N_{\mathrm{s}}^{\mathrm{NN}}$ denotes the stability number predicted by neural network. For empirical stability model, the following reliability function is used

$$Z = \begin{cases} 6.2P^{0.18}\left(\frac{S}{\sqrt{N}}\right)^{0.2} \dfrac{1}{\sqrt{\xi_{\mathrm{m}}}}\,\Delta D_{\mathrm{n50}} - H_{\mathrm{s}} & \xi_{\mathrm{m}} < \xi_{\mathrm{c}} \\ 1.0P^{-0.13}\left(\frac{S}{\sqrt{N}}\right)^{0.2} \sqrt{\cot\alpha}\,\xi_{\mathrm{m}}^{P}\,\Delta D_{\mathrm{n50}} - H_{\mathrm{s}} & \xi_{\mathrm{m}} \geq \xi_{\mathrm{c}} \end{cases} \tag{17}$$

For random simulation, 2×10^4 design parameter sets are generated. Some of them are constants and others show normal distribution with means and standard deviations listed in

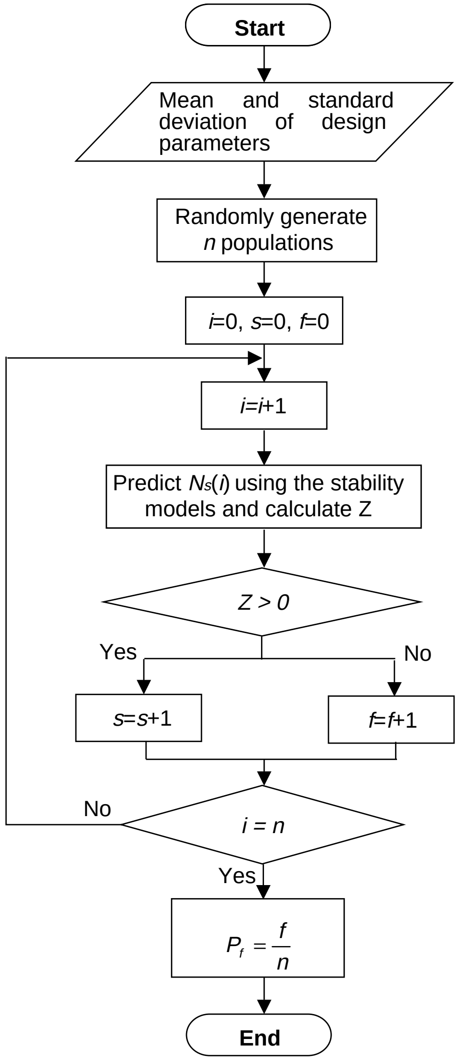

Fig. 16. Flow chart of Monte Carlo simulation.

Table 3 (case I). For simplicity, the peak wave period and significant wave period are assumed to be $T_p = 1.3T_m$; $T_s = 0.946T_p$.

While, the wave height shows Weibull distribution of the following form

$$\mathrm{Prob}[H \geq H_s] = \exp\left[-\left(\frac{H_s - C}{B}\right)^{\gamma}\right] \tag{18}$$

in which B is the scale parameter; C is the background noise level; γ is the shape parameter.

Table 3
Distribution of design parameters used in reliability analysis (case I)

Parameter	Distribution	Average	Standard deviation
S_d	Normal	6.0	1.0
D_{n50}	Normal	1.0	0.03
H_s	Weibull	($B=0.3$, $C=2.53$, $\gamma=1.0$)	
T_m	Normal	7.0	0.5
cot α	Normal	3.0	0.15
Δ	Normal	1.63	0.05
h	Normal	6	0.2
N_w	Const.	1000	
P	Const.	0.1	
SS	Const.	Pierson Moscowitz spectrum	

Fig. 17 shows the probability density functions constructed with 2×10^4 data sets. In the graph, the failure probability equals the integrated area below the curve from $Z=0$ to negative infinite. The failure probabilities and reliability indices are summarized in Table 4. The failure probability by van der Meer's formula is the largest among the models. In this case, van der Meer's formula gives conservative failure probability. On the other hand, for the design case (case II) with parameter distributions of Table 5, probability density function and failure probabilities are shown in Fig. 18 and Table 6. On the contrary to case I, failure probability with ANN IV is the largest among them. Therefore, it can be said that the failure probability based on only one empirical formula may lead absolutely conservative (case I) or dangerous (case II) design results.

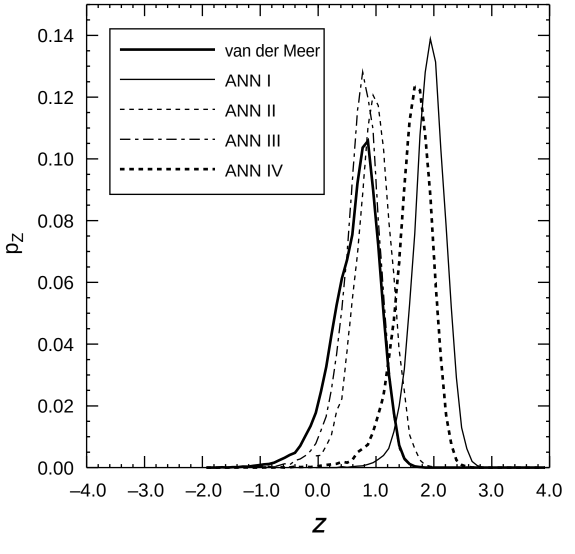

Fig. 17. Probability density functions of Z (case I).

Table 4
Failure probabilities and reliability indices (case I)

Stability model	Failure probability (%)	Reliability index (β)
van der Meer	6.84	1.488
ANN I	0.005	3.891
ANN II	1.05	2.308
ANN III	2.64	1.937
ANN IV	0.17	2.929

Table 5
Distribution of design parameters used in reliability analysis (case II)

Parameter	Distribution	Average	Standard deviation
S_d	Normal	6.0	1.0
D_{n50}	Normal	0.8	0.03
H_s	Weibull	($B=0.2$, $C=2.2$, $\gamma=1.0$)	
T_m	Normal	4.0	0.5
$\cot\alpha$	Normal	3.0	0.15
Δ	Normal	1.63	0.05
h	Normal	3.0	0.1
N_w	Const.	1000	
P	Const.	0.1	
SS	Const.	Pierson Moscowitz spectrum	

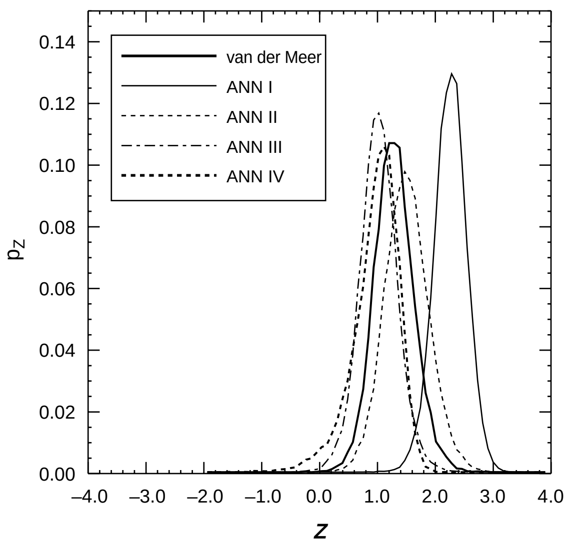

Fig. 18. Probability density functions of Z (case II).

Table 6
Failure probabilities and reliability indices (case II)

Stability model	Failure probability (%)	Reliability index (β)
van der Meer	0.04	3.353
ANN I	0.00	∞
ANN II	0.005	3.891
ANN III	0.245	2.814
ANN IV	2.36	1.985

To avoid this situation, one should adopt more advanced stability models such as neural network models in designing a armor units rather than using only one empirical formula.

5. Conclusions

In numerical examples, it was shown that the neural network technique gives more advanced results than the empirical model in estimating the stability number of breakwater and the estimation performance can be affected by the neural network structures employed. Especially, the neural network using wave steepness and slope angle as separate inputs shows better performance than those with surf similarity. The neural network with inputs such as wave height and mean wave period turned out to be useless in practical design because the parameters are usually beyond the range of training data set.

It was shown that the trained neural network model can be embedded into Monte Carlo simulation technique which estimates the failure probability of breakwater.

Since the neural network technique shows better performance than empirical model based approach in breakwater design, it is expected that the neural network integrated reliability analysis gives more advanced results for probability of failure than it is done with empirical model.

Acknowledgements

This research was supported by the Korea Ocean Research and Development Institute (KORDI) under grant number PE89900. The financial support is gratefully acknowledged.

References

Hanzawa, M., Sato, H., Takahashi, S., Shimosako, K., Takayama, T., Tanimoto, K., 1996. New stability formula for wave-dissipating concrete blocks covering horizontally composite breakwaters, Proceedings of 25th Coastal Engineering Conference, ASCE, Orlando 1996 pp. 1665–1678.

Hudson, R.Y., 1958. Design of Quarry Stone Cover Layer For Rubble Mound Breakwaters, Research Report No. 2-2. Waterways Experiment Station, Coastal Engineering Research Centre, Vicksburg, Miss.

Kaku, S., 1990. Hydraulic stability of rock slopes under irregular wave attack. Master Thesis, University of Delaware, Newark, Del.

Kaku, S., Kobayashi, N., Ryu, C.R., 1991. Design formulas for hydraulic stability of rock slopes under irregular wave attack, Proceedings of 38th Japanese Conference Coastal Engineering, Tokyo, Japan 1991 pp. 661–665, (in Japanese).

Mase, H., Sakamoto, M., Sakai, T., 1995. Neural network for stability analysis of rubble-mound breakwaters. J. Wtrwy. Port. Coast. Ocean Eng. 121 (6), 294–299.

Smith, W.G., Kobayashi, N., Kaku, S., 1992. Profile changes of rock slopes by irregular waves, Proceedings of 23th International Conference Coast Engineering ASCE, New York, NY 1992 pp. 1559–1572.

van der Meer, J.W., 1988a. Deterministic and probabilistic design of breakwater armor layers. J. Wtrwy. Port Coast., Ocean Eng. 114 (1), 66–80.

van der Meer, J.W., 1988b. Rock slopes and gravel beaches under wave attack. PhD Thesis, Delft University of Technology, Delft, The Netherlands.

Willmott, C.J., 1981. On the validation of models. Phys. Geogr. 2 (2), 184–194.