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Adaptive neural control of MIMO nonlinear state time-varying delay systems with unknown dead-zones and gain signs ☆

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Abstract

In this paper, adaptive neural control is proposed for a class of uncertain multi-input multi-output (MIMO) nonlinear state time-varying delay systems in a triangular control structure with unknown nonlinear dead-zones and gain signs. The design is based on the principle of sliding mode control and the use of Nussbaum-type functions in solving the problem of the completely unknown control directions. The unknown time-varying delays are compensated for using appropriate Lyapunov–Krasovskii functionals in the design. The approach removes the assumption of linear functions outside the deadband as an added contribution. By utilizing the integral Lyapunov function and introducing an adaptive compensation term for the upper bound of the residual and optimal approximation error as well as the dead-zone disturbance, the closed-loop control system is proved to be semi-globally uniformly ultimately bounded. Simulation results demonstrate the effectiveness of the approach.

Keywords: Adaptive control; Neural networks; Sliding mode control; Dead-zone; Nonlinear time-varying delay systems

1. Introduction

In the past decade, adaptive control system design of nonlinear systems using universal function approximators has received a great deal of attention (Ge, Hang, Lee, & Zhang, 2001; Lee & Tomizuka, 2000; Sanner & Slotine, 1992; Su & Stepanenko, 1994; Yesildirek & Lewis, 1995). Typically, these methods use either neural networks (NNs) or fuzzy logic systems to parametrize the unknown nonlinearities (Sanner & Slotine, 1992; Su & Stepanenko, 1994; Yesildirek & Lewis, 1995). Direct adaptive tracking control was proposed for a class of continuous-time nonlinear systems using radial basis function NNs (Sanner & Slotine, 1992). Using a families of novel integral Lyapunov functions for avoiding the possible controller singularity problem without using projection, adaptive neural controls have been investigated for a class of nonlinear systems in nonlinear parametrization (Ge, Hang, & Zhang, 1999b) and in a Brunovsky form (Zhang, Ge, & Hang, 2000), and for a class of MIMO nonlinear systems with a triangular structure in the control inputs (Ge, Zhang, & Hang, 2000). Decentralized indirect adaptive fuzzy control was proposed for a class of nonlinear systems with unknown constant control gains and unknown function control gains (Zhang, 2001).

When there is no a priori knowledge about the signs of control gains, adaptive control of such systems becomes much more difficult. The first solution was given for a class of first-order linear systems (Nussbaum, 1983), where the Nussbaum-type gain was originally proposed. When the high-frequency control gains and their signs are unknown, gains of Nussbaum-type (Nussbaum, 1983) have been effectively used in control design in solving the difficulty of unknown control directions for higher order systems (Ye & Jiang, 1998), and nonlinear systems with unknown time-delays (Ge, Hong, & Lee, 2004), among others.

Dead-zone is one of the most important non-smooth nonlinearities in many industrial processes, which can severely limit

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system performance, and its study has been drawing much interest in the control community for a long time (Cho & Bai, 1998; Selmic & Lewis, 2000; Tao & Kokotovic, 1994, 1995; Taware & Tao, 2003; Wang, Hong, & Su, 2003, 2004). To handle systems with unknown dead-zones, adaptive dead-zone inverses were proposed (Cho & Bai, 1998; Tao & Kokotovic, 1994, 1995). Continuous and discrete adaptive dead-zone inverses were built for linear systems with unmeasurable dead-zone outputs (Tao & Kokotovic, 1994, 1995). Asymptotical adaptive cancelation of unknown dead-zone is achieved analytically (Cho & Bai, 1998) under the condition that the output of the dead-zone is measurable. A compensation scheme was presented for general nonlinear actuator dead-zones of unknown width (Selmic & Lewis, 2000). Given a matching condition to the reference model, adaptive control with adaptive dead-zone inverse has been investigated (Wang et al., 2003). For a dead-zone with equal slopes, robust adaptive control was developed for a class of nonlinear systems (Wang et al., 2004) without constructing the inverse of the dead-zone. In the work (Shyu, Liu, & Hsu, 2005), decentralized variable structure control was proposed for a class of uncertain large-scale systems with state time-delay and dead-zone input. However, the parameters u_{i-}, u_{i+} of the dead-zones (Shyu et al., 2005) and gain signs need to be known, and the disturbances satisfy the matching condition. Adaptive output feedback control using backstepping and smooth inverse function of the dead-zone was proposed for a class of SISO nonlinear systems with unknown dead-zone (Zhou, Wen, & Zhang, 2006). However, the problem of over-parametrization still exists.

Time-delay is often encountered in various systems, for example, in the turbojet engines, aircraft systems, microwave oscillators, nuclear reactors, rolling mills, chemical processes, and hydraulic systems, etc. (Liu & Su, 1998). The existence of time-delays in a system frequently becomes a source of instability, and may degrade the control performance. Therefore, a number of different approaches have been proposed in order to stabilize such systems with time-delays (Nguang, 2000; Niculescu, 2001; Richard, 2003). Using appropriate *Lyapunov–Krasovskii functionals* to compensate for the uncertainties from unknown time-delays (Hale, 1977), thorough adaptive neural controls were presented for classes of nonlinear systems with unknown time delays and virtual control coefficients as either unknown constants or unknown functions with known or unknown sign (Ge, Hong, & Lee, 2003, 2005; Ge et al., 2004).

In this paper, we consider a class of uncertain MIMO nonlinear state time-varying delay systems with both unknown nonlinear dead-zones and unknown gain signs. To the best of our knowledge, there are few works dealing with such kinds of systems in the literature at present stage. The main contributions of the paper include:

(i) the novel description of a general nonlinear dead-zone model which makes the control system design possible without necessarily constructing a dead-zone inverse using the mean value theorem;
(ii) the removal of the need for known parameter bounds of dead-zones and the linear functions outside the deadband;
(iii) the use of integral Lyapunov function in avoiding the controller singularity problem which may be caused by time-varying gain functions;
(iv) the use of the Nussbaum-type functions and multilayer NNs in solving the problem of both unknown control directions and unknown control gain functions; and
(v) the combination of Lyapunov–Krasovskii functional and Young's inequality in eliminating the unknown time-varying delay $\tau_i(t)$ in the upper bounding function of the Lyapunov functional derivative, which makes NN parametrization with known inputs possible.

This paper is organized as follows. The problem formulation and preliminaries are given in Section 2. In Section 3, adaptive NN control is firstly developed for SISO time-varying delay systems with nonlinear dead-zones by using integral Lyapunov functions, then, it is extended to MIMO systems. Simulation results are performed to demonstrate the effectiveness of the approach in Section 4, followed by conclusion in Section 5.

2. Problem formulation and preliminaries

2.1. Problem formulation

Consider a class of uncertain MIMO nonlinear time-varying delay systems with dead-zones in the following form

Plant:

$$\begin{cases} \dot{x}_{1j} = x_{1,j+1}, \quad j = 1, \ldots, n_1 - 1, \\ \dot{x}_{1n_1} = f_1(x) + f_{1,\tau}(x_1(t-\tau_1(t)), \ldots, \\ \qquad x_m(t-\tau_m(t))) + b_1(x_1)u_1, \\ \dot{x}_{ij} = x_{i,j+1}, \quad j = 1, \ldots, n_i - 1, \\ \dot{x}_{in_i} = f_i(x, u_1, \ldots, u_{i-1}) + f_{i,\tau}(x_1(t-\tau_1(t)), \ldots, \\ \qquad x_m(t-\tau_m(t))) + b_i(x_1, \ldots, x_i)u_i, \\ \qquad i = 2, \ldots, m, \\ x_i(t) = \phi_i(t), \quad t \in [-\tau_{\max}, 0], \; i = 1, \ldots, m, \\ y_1 = x_{11}, \ldots, y_m = x_{m1}. \end{cases} \tag{1}$$

Dead-zone:

$$u_i = D_i(v_i) = \begin{cases} g_{ir}(v_i) & \text{if } v_i \geqslant b_{ir}, \\ 0 & \text{if } b_{il} < v_i < b_{ir}, \\ g_{il}(v_i) & \text{if } v_i \leqslant b_{il}. \end{cases} \tag{2}$$

where $x = [x_1^{\mathrm{T}}, x_2^{\mathrm{T}}, \ldots, x_m^{\mathrm{T}}]^{\mathrm{T}} \in R^n$ is the state vector, $x_i = [x_{i1}, \ldots, x_{in_i}]^{\mathrm{T}}$, $i = 1, \ldots, m$, $n = \sum_{i=1}^{m} n_i$; functions $g_{ir}(v_i), g_{il}(v_i)$ are unknown smooth nonlinear functions; $y_i \in R$ denotes the ith subsystem output; $f_1(x), f_2(x, u_1), \ldots, f_m(x, u_1, \ldots, u_{m-1}), f_{i,\tau}(x_1(t-\tau_1(t)), \ldots, x_m(t-\tau_m(t)))$ are the unknown continuous functions; $b_1(x_1), b_2(\bar{x}_2), \ldots, b_m(\bar{x}_m)$ are the unknown differentiable control gains, $\bar{x}_i = [x_1^{\mathrm{T}}, x_2^{\mathrm{T}}, \ldots, x_i^{\mathrm{T}}]^{\mathrm{T}}$; $\tau_1(t), \ldots, \tau_m(t)$ are unknown time-varying delays, $\phi_1(t), \ldots, \phi_m(t)$ are known continuous initial state vector functions, $\tau_{\max}$ as will be defined later is a known positive constant; $u_i \in R$ is the output of

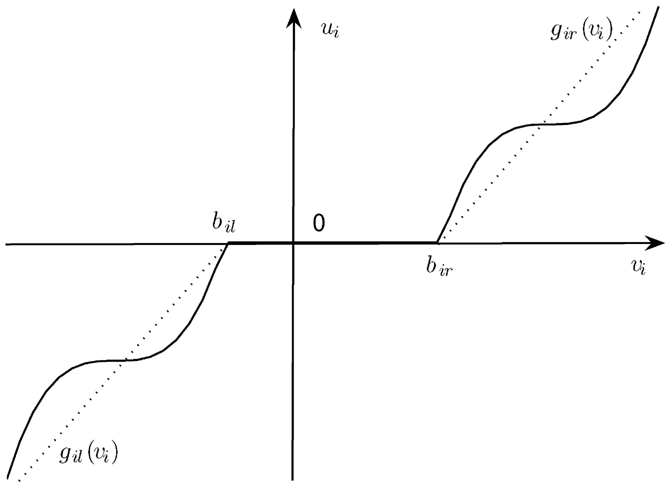

Fig. 1. Nonsymmetric nonlinear dead-zone model.

the ith dead-zone (and the input to the ith subsystem), $v_i(t) \in R$ is the input to the ith dead-zone, b_{il} and b_{ir} are the unknown parameters of the ith dead-zone; and the nonsymmetric nonlinear dead-zone with the input v_i is shown in Fig. 1.

Assumption 1. The dead-zone outputs $u_1, \ldots, u_m$ are not available.

Assumption 2. The dead-zone parameters b_{ir} and b_{il} are unknown constants, but their signs are known, i.e., $b_{ir} > 0$ and $b_{il} < 0$, $i = 1, \ldots, m$.

Assumption 3. The growth of the ith dead-zone's left and right functions, $g_{il}(v_i)$ and $g_{ir}(v_i)$, are smooth, and there exist unknown positive constants $k_{il0}, k_{il1}, k_{ir0}$, and k_{ir1} such that

$$0 < k_{il0} \leqslant g'_{il}(v_i) \leqslant k_{il1}, \quad \forall v_i \in (-\infty, b_{il}], \tag{3}$$

$$0 < k_{ir0} \leqslant g'_{ir}(v_i) \leqslant k_{ir1}, \quad \forall v_i \in [b_{ir}, +\infty), \tag{4}$$

where $g'_{il}(v_i) = \mathrm{d}g_{il}(z)/\mathrm{d}z|_{z=v_i}$ and $g'_{ir}(v_i) = \mathrm{d}g_{ir}(z)/\mathrm{d}z|_{z=v_i}$.

For convenience, $g_{il}(v_i)$ and $g_{ir}(v_i)$ in (3), (4) are assumed to be true for $v_i \in (-\infty, b_{ir}]$, and for $v_i \in [b_{il}, +\infty)$, respectively.

According to the differential mean value theorem, we know that there exist $\xi_{il}(v_i) \in (-\infty, b_{ir})$ and $\xi_{ir}(v_i) \in (b_{il}, +\infty)$ such that

$$\begin{aligned} g_{il}(v_i) &= g_{il}(v_i) - g_{il}(b_{il}) = g'_{il}(\xi_{il}(v_i))(v_i - b_{il}), \\ &\text{for } \xi_{il}(v_i) \in (v_i, b_{il}) \text{ or } (b_{il}, v_i), \end{aligned} \tag{5}$$

$$\begin{aligned} g_{ir}(v_i) &= g_{ir}(v_i) - g_{ir}(b_{ir}) = g'_{ir}(\xi_{ir}(v_i))(v_i - b_{ir}), \\ &\text{for } \xi_{ir}(v_i) \in (v_i, b_{ir}) \text{ or } (b_{ir}, v_i). \end{aligned} \tag{6}$$

Define vectors $\Phi_i(t)$ and $K_i(t)$ as follows:

$$\Phi_i(t) = [\varphi_{ir}(t), \varphi_{il}(t)]^{\mathrm{T}},$$

$$K_i(t) = [g'_{ir}(\xi_{ir}(v_i(t))), g'_{il}(\xi_{il}(v_i(t)))]^{\mathrm{T}}$$

with

$$\varphi_{ir}(t) = \begin{cases} 1 & \text{if } v_i(t) > b_{il}, \\ 0 & \text{if } v_i(t) \leqslant b_{il} \end{cases} \tag{7}$$

$$\varphi_{il}(t) = \begin{cases} 1 & \text{if } v_i(t) < b_{ir}, \\ 0 & \text{if } v_i(t) \geqslant b_{ir}. \end{cases} \tag{8}$$

Based on Assumption 3, the dead-zone (2) can be rewritten as follows:

$$u_i = D_i(v_i) = K_i^{\mathrm{T}}(t)\Phi_i(t)v_i + d_i(v_i), \tag{9}$$

where

$$d_i(v_i) = \begin{cases} -g'_{ir}(\xi_{ir}(v_i))b_{ir} & \text{if } v_i \geqslant b_{ir}, \\ -[g'_{il}(\xi_{il}(v_i)) & \\ \quad +g'_{ir}(\xi_{ir}(v_i))]v_i & \text{if } b_{il} < v_i < b_{ir}, \\ -g'_{il}(\xi_{il}(v_i))b_{il} & \text{if } v_i \leqslant b_{il} \end{cases} \tag{10}$$

and $|d_i(v_i)| \leqslant p_i^*$, p_i^* is an unknown positive constant with $p_i^* = (k_{ir1} + k_{kl1}) \max\{b_{ir}, -b_{il}\}$.

The control objective is to design adaptive control $v_i(t)$ for system (1) such that the output y_i follows the specified desired trajectory y_{id}, $i = 1, \ldots, m$.

Define x_{id} and e_i as

$$x_{id} = [y_{id}, \dot{y}_{id}, \ldots, y_{id}^{(n_i-1)}]^{\mathrm{T}},$$

$$e_i = x_i - x_{id} = [e_{i1}, e_{i2}, \ldots, e_{in_i}]^{\mathrm{T}}$$

and the filtered tracking error s_i as

$$s_i = \left(\frac{\mathrm{d}}{\mathrm{d}t} + \lambda_i\right)^{n_i-1} e_{i1} = \sum_{j=1}^{n_i-1} \lambda_{ij} e_{ij} + e_{in_i}, \tag{11}$$

where $\lambda_{ij} = C_{n_i-1}^{j-1} \lambda_i^{n_i-j}$, $j = 1, \ldots, n_i - 1$, $\lambda_i > 0$, $i = 1, \ldots, m$ are positive constants, specified by the designer.

Assumption 4. Smooth functions $b_i(\bar{x}_i)$ and their signs are unknown, and there exist constants b_{i0} and b_{i1} such that $0 < b_{i0} \leqslant |b_i(\bar{x}_i)| \leqslant b_{i1}, \forall \bar{x}_i \in R^{\bar{n}_j}$ with $\bar{n}_j = \sum_{j=1}^{i} n_j$, $i = 1, \ldots, m$.

Assumption 5. The desired trajectory vectors are continuous and available, and $\bar{x}_{id} = [x_{id}^{\mathrm{T}}, y_{id}^{(n_i)}]^{\mathrm{T}} \in \Omega_{id} \subset R^{n_i+1}$ with known compact set Ω_{id}, $i = 1, \ldots, m$.

Assumption 6. The unknown continuous functions $f_{i,\tau}(x_1(t - \tau_1(t)), \ldots, x_m(t - \tau_m(t)))$ satisfy the inequality $|f_{i,\tau}(x_1(t - \tau_1(t)), \ldots, x_m(t - \tau_m(t)))| \leqslant \sum_{k=1}^{m} \varrho_{ik}(x_k(t - \tau_k(t)))$ with $\varrho_{ik}(x_k(t))$ being known positive continuous functions, $i = 1, \ldots, m$.

Assumption 7. The unknown state time-varying delays $\tau_i(t)$ satisfy:

$$0 \leqslant \tau_i(t) \leqslant \tau_{\max}, \quad \dot{\tau}_i(t) \leqslant \bar{\tau}_{\max} < 1, \quad i = 1, \ldots, m \tag{12}$$

with the known constants $\tau_{\max}$ and $\bar{\tau}_{\max}$.

2.2. Nussbaum function properties

In order to deal with the unknown control gain sign, the Nussbaum gain technique is employed in this paper. A function $N(\zeta)$ is called a Nussbaum-type function if it has the following properties:

$$\text{(i)} \lim_{s\to+\infty} \sup \frac{1}{s}\int_0^s N(\zeta)\,\mathrm{d}\zeta = +\infty, \tag{13}$$

$$\text{(ii)} \lim_{s\to+\infty} \inf \frac{1}{s}\int_0^s N(\zeta)\,\mathrm{d}\zeta = -\infty. \tag{14}$$

Commonly used Nussbaum functions include: $\zeta^2\cos(\zeta)$, $\zeta^2\sin(\zeta)$, and $\exp(\zeta^2)\cos((\pi/2)\zeta)$ (Ge et al., 2004; Nussbaum, 1983; Ryan, 1991). For clarity, the even Nussbaum function, $N(\zeta) = \mathrm{e}^{\zeta^2}\cos((\pi/2)\zeta)$ is used throughout this paper.

Lemma 1 (*Ge et al., 2004*). *Let $V(\cdot), \zeta(\cdot)$ be smooth functions defined on $[0, t_f)$ with $V(t) \geqslant 0, \forall t \in [0, t_f)$, and $N(\cdot)$ be an even smooth Nussbaum-type function. If the following inequality holds*:

$$V(t) \leqslant c_0 + \mathrm{e}^{-c_1 t}\int_0^t g(x(\tau))N(\zeta)\dot{\zeta}\mathrm{e}^{c_1\tau}\,\mathrm{d}\tau + \mathrm{e}^{-c_1 t}\int_0^t \dot{\zeta}\mathrm{e}^{c_1\tau}\,\mathrm{d}\tau, \quad \forall t \in [0, t_f), \tag{15}$$

where c_0 represents some suitable constant, c_1 is a positive constant, and $g(x(\tau))$ is a time-varying parameter which takes values in the unknown closed intervals $I = [l^-, l^+]$, with $0 \notin I$, then $V(t)$, $\zeta(t)$, $\int_0^t g(x(\tau))N(\zeta)\dot{\zeta}\,\mathrm{d}\tau$ must be bounded on $[0, t_f)$.

According to Proposition 2 (Ryan, 1991), if the solution of the resulting closed-loop system is bounded, then $t_f = \infty$.

2.3. Multilayer neural networks (MNNs)

NNs have been widely used in modeling and control of nonlinear systems because of their good capabilities of nonlinear function approximation, learning, and fault tolerance. In this paper, three-layer NNs will be used to approximate a continuous function $h(z) : R^p \to R$ as described by (Ge et al., 2001; Lewis, Yesildirek, & Liu, 1996)

$$h_{\mathrm{nn}}(z, W, V) = W^{\mathrm{T}}S(V^{\mathrm{T}}\bar{z}), \tag{16}$$

where $z = [z_1, \ldots, z_p]^{\mathrm{T}}$, $\bar{z} = [z^{\mathrm{T}}, 1]^{\mathrm{T}}$; $V = [v_1, \ldots, v_l] \in R^{(p+1)\times l}$, $W = [w_1, \ldots, w_l]^{\mathrm{T}} \in R^l$ are the first-to-second layer and the second-to-third layer weights, respectively; $S(V^{\mathrm{T}}\bar{z}) = [s(v_1^{\mathrm{T}}\bar{z}), \ldots, s(v_{l-1}^{\mathrm{T}}\bar{z}), 1]^{\mathrm{T}}$ with $s(z_\alpha) = 1/(1+\mathrm{e}^{-\gamma z_\alpha})$ and constant $\gamma > 0$; and the NN node number $l > 1$.

In this paper, $\|\cdot\|$ denotes the 2-norm, $\|\cdot\|_{\mathrm{F}}$ denotes the Frobenius norm, $\|A\|_1 = \sum_{k=1}^{l}|a_k|$ with $A = [a_1, \ldots, a_l] \in R^l$, $\lambda_{\min}(B)$ and $\lambda_{\max}(B)$ denote the smallest and largest eigenvalues of a square matrix B, respectively.

Let

$$h(z) = h_{\mathrm{nn}}(z, W^*, V^*) + \varepsilon(z), \quad \forall z \in \Omega_z \subset R^p, \tag{17}$$

where W^*, V^* are ideal NN weights, $\Omega_z \subset R^p$ is a compact set, and $\varepsilon(z)$ is the NN approximation error.

The ideal weights W^* and V^* are "artificial" required for analytical purposes. According to the discussion (Ge et al., 2001; Polycarpou & Mears, 1998), W^* and V^* are defined as follows:

$$(W^*, V^*) = \arg\min_{(W,V)}\left[\sup_{z\in\Omega_z}|h_{\mathrm{nn}}(z, W, V) - h(z)|\right], \tag{18}$$

which are unknown and need to be estimated in control design. Let $\hat{W}$ and $\hat{V}$ be the estimates of W^* and V^*, respectively, and $\tilde{(\cdot)} = \hat{(\cdot)} - (\cdot)$.

Lemma 2 (*Ge et al., 2001*). *For NN* (16), *the NN estimation error can be expressed as*

$$\hat{W}^{\mathrm{T}}S(\hat{V}^{\mathrm{T}}\bar{z}) - W^{*\mathrm{T}}S(V^{*\mathrm{T}}\bar{z}) = \tilde{W}^{\mathrm{T}}(\hat{S} - \hat{S}'\hat{V}^{\mathrm{T}}\bar{z}) + \hat{W}^{\mathrm{T}}\hat{S}'\tilde{V}^{\mathrm{T}}\bar{z} + d_u, \tag{19}$$

where $\hat{S} = S(\hat{V}^{\mathrm{T}}\bar{z})$, $\hat{S}' = \mathrm{diag}\{\hat{s}'_1, \ldots, \hat{s}'_{l-1}, 0\}$ with $s'(\hat{v}_k^{\mathrm{T}}\bar{z}) = d[s(z_\alpha)]/dz_\alpha|_{z_\alpha=\hat{v}_k^{\mathrm{T}}\bar{z}}$, $k = 1, \ldots, l-1$, and the residual term d_u is bounded by

$$|d_u| \leqslant \|V^*\|_{\mathrm{F}}\|\bar{z}\hat{W}^{\mathrm{T}}\hat{S}'\|_{\mathrm{F}} + \|W^*\|\|\hat{S}'\hat{V}^{\mathrm{T}}\bar{z}\| + \|W^*\|_1. \tag{20}$$

From Eqs. (17) and (19), we obtain

$$h(z) = \hat{W}^{\mathrm{T}}S(\hat{V}^{\mathrm{T}}\bar{z}) - \tilde{W}^{\mathrm{T}}(\hat{S} - \hat{S}'\hat{V}^{\mathrm{T}}\bar{z}) - \hat{W}^{\mathrm{T}}\hat{S}'\tilde{V}^{\mathrm{T}}\bar{z} - d_u + \varepsilon(z). \tag{21}$$

The following even function $q(x|c)$ is introduced for the purpose of the control design in Section 3.1:

$$q(x|c) = \begin{cases} 1 & \text{for } |x| \geqslant c, \\ 0 & \text{for } |x| < c, \end{cases} \quad \forall x \in R \tag{22}$$

with any given positive constant $c > 0$.

3. Control system design and stability analysis

3.1. Adaptive NN control for SISO system ($m = 1$)

To illustrate the design methodology clearly, we first consider the SISO system ($m = 1$).

From Eqs. (1), (9) and (10), we obtain

$$\dot{s}_1 = f_1(x_1) + \gamma_1 + b_1(x_1)K_1^{\mathrm{T}}(t)\Phi_1(t)v_1(t) + f_{1,\tau}(x_1(t-\tau_1(t))) + b_1(x_1)d_1(v_1(t)), \tag{23}$$

where $\gamma_1 = \sum_{j=1}^{n_1-1}\lambda_{1j}e_{1,j+1} - y_{1d}^{(n_1)}$.

For (23), motivated by the definition of integral Lyapunov functions (Ge et al., 1999b, 2001), define a smooth scalar function as follows:

$$V_{s1} = \int_0^{s_1}\frac{\sigma}{|b_1(\bar{x}_1^+, \sigma + \beta_1)|}\,\mathrm{d}\sigma, \tag{24}$$

where $\beta_1 = y_{1d}^{(n_1-1)} - \sum_{j=1}^{n_1-1}\lambda_{1j}e_{1j}$, $\bar{x}_1^+ = [x_{11}, \ldots, x_{1,n_1-1}]^{\mathrm{T}}$.

By Second Mean Value Theorem for Integrals, V_{s1} can be rewritten as $V_{s1}=s_1^2/2|b_1(\bar{x}_1^+,\lambda_{s1}s_1+\beta_1)|$ with $\lambda_{s1}\in(0,1)$. Because $0<b_{10}\leqslant|b_1(x_1)|$, it is shown that V_{s1} is positive definitive with respect to s_1.

Differentiating V_{s1} with respect to time t, we obtain

$$\dot{V}_{s1}=\frac{s_1}{|b_1(x_1)|}\dot{s}_1+\int_0^{s_1}\sigma\left[\sum_{k=1}^{n_1-1}\frac{\partial|b_1^{-1}(\bar{x}_1^+,\sigma+\beta_1)|}{\partial x_{1k}}x_{1,k+1}\right.$$
$$\left.+\frac{\partial|b_1^{-1}(\bar{x}_1^+,\sigma+\beta_1)|}{\partial\beta_1}\dot{\beta}_1\right]\mathrm{d}\sigma. \tag{25}$$

Because $\dot{\beta}_1=-\gamma_1$ and $\partial|b_1^{-1}(\bar{x}_1^+,\sigma+\beta_1)|/\partial\sigma=\partial|b_1^{-1}(\bar{x}_1^+,\sigma+\beta_1)|/\partial\beta_1$, it is shown that

$$\int_0^{s_1}\sigma\frac{\partial|b_1^{-1}(\bar{x}_1^+,\sigma+\beta_1)|}{\partial\beta_1}\dot{\beta}_1\,\mathrm{d}\sigma$$
$$=-\frac{\gamma_1 s_1}{|b_1(x_1)|}+\int_0^{s_1}\frac{\gamma_1}{|b_1(\bar{x}_1^+,\sigma+\beta_1)|}\,\mathrm{d}\sigma. \tag{26}$$

Substituting (23), and (26) into (25), and applying Eq. (9), we obtain

$$\dot{V}_{s1}\leqslant s_1g_1(t)v_1+s_1Q_1(z_1)+\frac{s_1^2}{2b_1^2(x_1)}$$
$$+\frac{1}{2}\varrho_{11}^2(x_1(t-\tau_1(t)))+|s_1|p_1^*, \tag{27}$$

where

$$g_1(t)=\frac{b_1(x_1)}{|b_1(x_1)|}K_1^{\mathrm{T}}(t)\Phi_1(t), \tag{28}$$

$$Q_1(z_1)=\frac{f_1(x_1)}{|b_1(x_1)|}+s_1\int_0^1\left[\theta\sum_{k=1}^{n_1-1}\frac{\partial|b_1^{-1}(\bar{x}_1^+,\theta s_1+\beta_1)|}{\partial x_{1k}}\right.$$
$$\left.\times x_{1,k+1}+\frac{\gamma_1}{|b_1(\bar{x}_1^+,\theta s_1+\beta_1)|}\right]\mathrm{d}\theta \tag{29}$$

with $z_1=[x_1^{\mathrm{T}},s_1,\gamma_1,\beta_1]^{\mathrm{T}}\in R^{p_1}$, $p_1=n_1+3$.

To overcome the design difficulties from the unknown time-delay $\tau(t)$, the following Lyapunov–Krasovskii functional can be considered

$$V_{U_1}(t)=\frac{1}{2(1-\bar{\tau}_{\max})}\int_{t-\tau_1(t)}^{t}U_1(x_1(\tau))\,\mathrm{d}\tau \tag{30}$$

with $U_1(x_1(t))=\varrho_{11}^2(x_1(t))$.

The time derivative of $V_{U_1}(t)$ is

$$\dot{V}_{U_1}(t)=\frac{1}{2(1-\bar{\tau}_{\max})}[\varrho_{11}^2(x_1(t))$$
$$-\varrho_{11}^2(x_1(t-\tau_1(t)))(1-\dot{\tau}_1(t))], \tag{31}$$

which can be used to cancel the time-delay term on the right-hand side of (27), and thus eliminate the design difficulty from the unknown time-varying delay $\tau_1(t)$ without introducing any uncertainties to the system. For notation conciseness, the time variables t and $t-\tau_1(t)$ will be omitted, after the time-varying delay term are eliminated, from here onward. Accordingly, we obtain

$$\dot{V}_{s1}+\dot{V}_{U_1}\leqslant s_1g_1(t)v_1+s_1h_1(z_1)+|s_1|p_1^*, \tag{32}$$

where

$$h_1(z_1)=Q_1(z_1)+\frac{0.5s_1}{b_1^2(x_1)}+\frac{0.5}{(1-\bar{\tau}_{\max})s_1}\varrho_{11}^2(x_1). \tag{33}$$

Define a compact set

$$\Omega_{z_1}=\{[x_1^{\mathrm{T}},s_1,\gamma_1,\beta_1]^{\mathrm{T}}|x_1\in\Omega_1,\bar{x}_{1d}\in\Omega_{1d}\}, \tag{34}$$

where $\Omega_1\subset R^{n_1}$ is a sufficiently large compact set satisfying $\Omega_1\supset\Omega_{10}$ as defined later in Theorem 1.

Note that if $h_1(z_1)$ is utilized to construct the control law, controller singularity may occur since $(1/2(1-\bar{\tau}_{\max})s_1)\times\varrho_{11}^2(x_1)$ is not well-defined at $s_1=0$. Therefore, care must be taken to guarantee the boundedness of the control as discussed (Ge et al., 2004).

For similarily, let us define sets $\Omega_{c_{s_1}}\subset\Omega_{z_1}$ and $\Omega_{z_1}^0$ as follows:

$$\Omega_{c_{s_1}}=\{z_1||s_1|<c_{s_1},x_{1d}\in\Omega_{1d}\}, \tag{35}$$

$$\Omega_{z_1}^0=\Omega_{z_1}-\Omega_{c_{s_1}}, \tag{36}$$

where c_{s_1} is a positive design constant that can be chosen arbitrarily small and "−" in (36) is used to denote the complement of set $\Omega_{c_{s_1}}$ in set Ω_{z_1}. In addition, it has been shown that $\Omega_{z_1}^0$ is a compact set in Ge et al. (2004).

Let $\hat{W}_1^{\mathrm{T}}S(\hat{V}_1^{\mathrm{T}}\bar{z}_1)$ be the approximation of the three-layer NNs, which are discussed in Section 2.3, on the compact $\Omega_{z_1}^0$ to $h_1(z_1)$, then we have

$$h_1(z_1)=\hat{W}_1^{\mathrm{T}}S(\hat{V}_1^{\mathrm{T}}\bar{z}_1)-\tilde{W}_1^{\mathrm{T}}(\hat{S}_1-\hat{S}_1'\hat{V}_1^{\mathrm{T}}\bar{z}_1)$$
$$-\hat{W}_1^{\mathrm{T}}\hat{S}_1'\tilde{V}_1^{\mathrm{T}}\bar{z}_1-d_{u1}+\varepsilon_1(z_1), \tag{37}$$

where $z_1=[z_{11},\ldots,z_{1p_1}]^{\mathrm{T}}$, $\bar{z}_1=[z_1^{\mathrm{T}},1]^{\mathrm{T}}$; $\hat{V}_1=[\hat{v}_{11},\ldots,\hat{v}_{1l_1}]\in R^{(p_1+1)\times l_1}$ and $\hat{W}_1=[\hat{w}_{11},\ldots,\hat{w}_{1l_1}]^{\mathrm{T}}\in R^{l_1}$ denote the estimates of ideal constant weights W_1^* and V_1^*, respectively, $\hat{S}_1=S_1(\hat{V}_1^{\mathrm{T}}\bar{z}_1)=[s(\hat{v}_{11}^{\mathrm{T}}\bar{z}_1),\ldots,s(v_{1,l_1-1}^{\mathrm{T}}\bar{z}_1),1]^{\mathrm{T}}$ with $s(z_\alpha)=1/(1+\mathrm{e}^{-\gamma_{10}z_\alpha})$,and constant $\gamma_{10}>0$, $\hat{S}_1'=\mathrm{diag}\{\hat{s}_{11}',\ldots,\hat{s}_{1,l_1-1}',0\}$ with $\hat{s}_{1k}'=s'(\hat{v}_{1k}^{\mathrm{T}}\bar{z}_1)=\mathrm{d}[s(z_\alpha)]/\mathrm{d}z_\alpha|_{z_\alpha=\hat{v}_{1k}^{\mathrm{T}}\bar{z}_1}$, $k=1,\ldots,l_1-1$; and the NN node number $l_1>1$; and the residual term d_{u1} is bounded by

$$|d_{u1}|\leqslant\|V_1^*\|_{\mathrm{F}}\|\bar{z}_1\hat{W}_1^{\mathrm{T}}\hat{S}_1'\|_{\mathrm{F}}+\|W_1^*\|\|\hat{S}_1'\hat{V}_1^{\mathrm{T}}\bar{z}_1\|+\|W_1^*\|_1, \tag{38}$$

the approximation error $\varepsilon_1(z_1)$ satisfies $|\varepsilon_1(z_1)|\leqslant\varepsilon_1^*$, $\forall z_1\in\Omega_{z_1}^0$ with constant $\varepsilon_1^*>0$.

Consider the following control law:

$$v_1(t) = q(s_1|c_{s_1})N(\zeta_1)[k_{10}(t)s_1 + \hat{W}_1^{\mathrm{T}} S_1(\hat{V}_1^{\mathrm{T}}\bar{z}_1) + \hat{\delta}_1 \xi_1(z_1)\tanh(s_1\xi_1(z_1)/\rho_1))], \quad (39)$$

$$\dot{\zeta}_1 = q(s_1|c_{s_1})[k_{10}(t)s_1^2 + \hat{W}_1^{\mathrm{T}} S_1(\hat{V}_1^{\mathrm{T}}\bar{z}_1)s_1 + \hat{\delta}_1\xi_1(z_1)s_1\tanh(s_1\xi_1(z_1)/\rho_1)], \quad (40)$$

where $q(\cdot|\cdot)$ is defined by Eq. (22), $N(\zeta_1) = \mathrm{e}^{\zeta_1^2}\cos((\pi/2)\zeta_1)$, ρ_1 is a positive constant, $\hat{\delta}_1$ is the estimate of δ_1 with $\delta_1 = \max\{\|V_1^*\|_{\mathrm{F}}, \|W_1^*\|, \|W_1^*\|_1 + \varepsilon_1^* + p_1^*\}$ at time t,

$$\xi_1(z_1) = \|\bar{z}_1\hat{W}_1^{\mathrm{T}}\hat{S}_1'\|_{\mathrm{F}} + \|\hat{S}_1'\hat{V}_1^{\mathrm{T}}\bar{z}_1\| + 1, \quad (41)$$

$k_{10}(t) = k_{11} + k_{12}(t)$ with k_{11} being any positive constant and $k_{12}(t)$ chosen as

$$k_{12}(t) = \frac{k_{13}q(s_1|c_{s_1})}{2(1-\bar{\tau}_{\max})s_1^2}\int_{t-\tau_{\max}}^{t} \varrho_{11}^2(x_1(\tau))\,\mathrm{d}\tau \quad (42)$$

with k_{13} a positive constant specified by the designer.

The adaptive tuning laws are defined as

$$\dot{\hat{W}}_1 = q(s_1|c_{s_1})\Gamma_{w1}[(\hat{S}_1 - \hat{S}_1'\hat{V}_1^{\mathrm{T}}\bar{z}_1)s_1 - \sigma_{w1}\hat{W}_1], \quad (43)$$

$$\dot{\hat{V}}_1 = q(s_1|c_{s_1})\Gamma_{v1}[\bar{z}_1\hat{W}_1^{\mathrm{T}}\hat{S}_1' s_1 - \sigma_{v1}\hat{V}_1], \quad (44)$$

$$\dot{\hat{\delta}}_1 = q(s_1|c_{s_1})\eta_1\left[s_1\xi_1(z_1)\tanh\left(\frac{s_1\xi_1(z_1)}{\rho_1}\right) - \sigma_1\hat{\delta}_1\right], \quad (45)$$

where $\Gamma_{w1} > 0$, $\Gamma_{v1} > 0$, $\sigma_{w1}, \sigma_{v1}, \eta_1, \rho_1$ and σ_1 are strictly positive constants.

Theorem 1. *Consider the closed-loop system consisting of the plant* (1), *the adaptive control given by* (39), (40), (43)–(45). *Under Assumptions* 1–7, *for bounded initial conditions*, *the overall closed-loop neural control system is semi-globally stable in the sense that all of the signals in the closed-loop system are bounded*, *the parameter estimates*

$$(\hat{W}_1, \hat{V}_1, \hat{\delta}_1) \in \Omega_{w1} = \left\{(\hat{W}_1, \hat{V}_1, \hat{\delta}_1) \,\middle|\, \|\tilde{W}_1\|^2 \leqslant \frac{2\mu_1}{\lambda_{\min}(\Gamma_{w1}^{-1})}, \|\tilde{V}_1\|_{\mathrm{F}}^2 \leqslant \frac{2\mu_1}{\lambda_{\min}(\Gamma_{v1}^{-1})}, |\tilde{\delta}_1|^2 \leqslant 2\eta_1\mu_1\right\}, \quad (46)$$

and $\forall x_1(0) \in \Omega_{10}$ (*as will be defined later in the proof*), *the state vector*

$$x_1 \in \Omega_{1c} = \left\{x_1 \,\middle|\, \|x_1 - x_{1d}\| \leqslant c_{10}(1 + \|\Lambda_1\|)\|\omega_1(0)\| + \left[1 + \frac{(1+\|\Lambda_1\|)c_{10}}{\lambda_1}\right]\max\{\sqrt{2b_{11}\mu_1}, c_{s_1}\}, \bar{x}_{1d} \in \Omega_{1d}\right\} \subset \Omega_1, \quad (47)$$

whose size can be adjusted by appropriately choosing the design parameters.

Proof. The proof includes two steps as discussed in (Ge, Hang, & Zhang, 1999a). We shall first assume that $x_1 \in \Omega_1, \forall t \geqslant 0$, on which NN approximation (37) is valid, and construct stable adaptive NN control over Ω_1. Then, we shall show that there exists nonempty initial set Ω_{10} such that the state x_1 indeed remains in the compact set Ω_1 for all $t \geqslant 0$, if initial state $x_1(0)$ initiates from Ω_{10}.

The proof is indeed a bit more complex than the model based adaptive control design where the model is valid over the entire space. Through the process of the proof, it is clear that there is a nonempty initial compact set, as long as initial state $x_1(0)$ starts from Ω_{10}, the state x_1 will never escape out of the conservative compact set, Ω_{1c}, belonging to the chosen compact set Ω_1, as will be shown later in the proof and in Fig. 2. Because NN approximation is only valid on a compact set, we have to present the idea in the above manner, and at the same time avoid the so-called circular argument as commonly understood in the classical model based control as detailed in Ge et al. (1999a).

Step 1: Suppose that $x_1 \in \Omega_1, \forall t \geqslant 0$, then NN approximation (37) is valid. Consider the Lyapunov function candidate

$$V_1(t) = V_{s1}(t) + V_{U_1}(t) + \frac{1}{2}\tilde{W}_1^{\mathrm{T}}\Gamma_{w1}^{-1}\tilde{W}_1 + \frac{1}{2}\mathrm{tr}\{\tilde{V}_1^{\mathrm{T}}\Gamma_{v1}^{-1}\tilde{V}_1\} + \frac{1}{2\eta_1}\tilde{\delta}_1^2. \quad (48)$$

Differentiating $V_1(t)$ with respect to time t leads to

$$\dot{V}_1(t) = \dot{V}_{s1}(t) + \dot{V}_{U_1}(t) + \tilde{W}_1^{\mathrm{T}}\Gamma_{w1}^{-1}\dot{\hat{W}}_1 + \mathrm{tr}\{\tilde{V}_1^{\mathrm{T}}\Gamma_{v1}^{-1}\dot{\hat{V}}_1\} + \frac{1}{\eta_1}\tilde{\delta}_1\dot{\hat{\delta}}_1. \quad (49)$$

Case i: If $|s_1| \geqslant c_{s_1}$, then $q_1(s_1|c_{s_1}) = 1$. In this case, substituting Eq. (27) into Eq. (49), and noting Eqs. (37) and (38), and using control law (39) and (40), it follows that

$$\begin{aligned}\dot{V}_1(t) &\leqslant s_1 g_1(t)v_1 + s_1 h_1(z_1) + |s_1|p_1^* + \tilde{W}_1^{\mathrm{T}}\Gamma_{w1}^{-1}\dot{\hat{W}}_1 + \mathrm{tr}\{\tilde{V}_1^{\mathrm{T}}\Gamma_{v1}^{-1}\dot{\hat{V}}_1\} + \frac{1}{\eta_1}\tilde{\delta}_1\dot{\hat{\delta}}_1 \\ &= g_1(t)N(\zeta_1)\dot{\zeta}_1 + \dot{\zeta}_1 - k_{10}(t)s_1^2 - \hat{W}_1^{\mathrm{T}}S_1(\hat{V}_1^{\mathrm{T}}\bar{z}_1)s_1 \\ &\quad - \hat{\delta}_1\xi_1(z_1)s_1\tanh(s_1\xi_1(z_1)/\rho_1) + [\hat{W}_1^{\mathrm{T}}S_1(\hat{V}_1^{\mathrm{T}}\bar{z}_1) \\ &\quad - \tilde{W}_1^{\mathrm{T}}(\hat{S}_1 - \hat{S}_1'\hat{V}_1\bar{z}_1) - \hat{W}_1^{\mathrm{T}}\hat{S}_1'\tilde{V}_1^{\mathrm{T}}\bar{z}_1 \\ &\quad - d_{u1} + \varepsilon_1(z_1)]s_1 + |s_1|p_1^* \\ &\quad + \tilde{W}_1^{\mathrm{T}}\Gamma_{w1}^{-1}\dot{\hat{W}}_1 + \mathrm{tr}\{\tilde{V}_1^{\mathrm{T}}\Gamma_{v1}^{-1}\dot{\hat{V}}_1\} + \frac{1}{\eta_1}\tilde{\delta}_1\dot{\hat{\delta}}_1. \end{aligned} \quad (50)$$

Using adaptive tuning laws (43)–(45), and the inequality: $0 \leqslant |x| - x\tanh(x/\varepsilon) \leqslant 0.2785\varepsilon$, for $\varepsilon > 0$, $x \in R$, and the fact that $\hat{W}_1^{\mathrm{T}}\hat{S}_1'\tilde{V}_1^{\mathrm{T}}\bar{z}_1 = \mathrm{tr}\{\tilde{V}_1^{\mathrm{T}}\bar{z}_1\hat{W}_1^{\mathrm{T}}\hat{S}_1'\}$, we obtain

$$\begin{aligned}\dot{V}_1(t) &\leqslant g_1(t)N(\zeta_1)\dot{\zeta}_1 + \dot{\zeta}_1 - k_{11}s_1^2 - \frac{k_{13}}{2(1-\bar{\tau}_{\max})} \\ &\quad \times \int_{t-\tau_{\max}}^{t} U_1(x_1(\tau))\,\mathrm{d}\tau + 0.2785\delta_1\rho_1 \\ &\quad - \sigma_{w1}\tilde{W}_1^{\mathrm{T}}\hat{W}_1 - \sigma_{v1}\mathrm{tr}\{\tilde{V}_1^{\mathrm{T}}\hat{V}_1\} - \sigma_1\tilde{\delta}_1\hat{\delta}_1. \end{aligned} \quad (51)$$

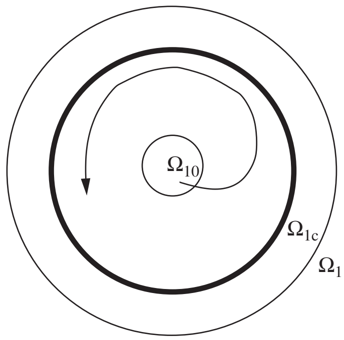

Fig. 2. Compact sets in Theorem 1.

By completion of squares, the following inequalities hold:

$$-\sigma_{w1}\tilde{W}_1^{\mathrm{T}}\hat{W}_1 \leqslant -\frac{\sigma_{w1}\|\tilde{W}_1\|^2}{2}+\frac{\sigma_{w1}\|W_1^*\|^2}{2}, \tag{52}$$

$$-\sigma_{v1}\mathrm{tr}\{\tilde{V}_1^{\mathrm{T}}\hat{V}_1\} \leqslant -\frac{\sigma_{v1}\|\tilde{V}_1\|_{\mathrm{F}}^2}{2}+\frac{\sigma_{v1}\|V_1^*\|_{\mathrm{F}}^2}{2}, \tag{53}$$

$$-\sigma_1\tilde{\delta}_1\hat{\delta}_1 \leqslant -\frac{\sigma_1\tilde{\delta}_1^2}{2}+\frac{\sigma_1\delta_1^2}{2}. \tag{54}$$

Therefore, we obtain

$$\begin{aligned}\dot{V}_1(t) \leqslant & -k_{11}s_1^2-\frac{k_{13}}{2(1-\bar{\tau}_{\max})}\int_{t-\tau_{\max}}^{t}U_1(x_1(\tau))\,\mathrm{d}\tau \\ & + g_1(t)N(\zeta_1)\dot{\zeta}_1+\dot{\zeta}_1-\frac{\sigma_{w1}\|\tilde{W}_1\|^2}{2} \\ & -\frac{\sigma_{v1}\|\tilde{V}_1\|_{\mathrm{F}}^2}{2}-\frac{\sigma_1\tilde{\delta}_1^2}{2}+0.2785\delta_1\rho_1 \\ & +\frac{\sigma_{w1}\|W_1^*\|^2}{2}+\frac{\sigma_{v1}\|V_1^*\|_{\mathrm{F}}^2}{2}+\frac{\sigma_1\delta_1^2}{2}. \end{aligned} \tag{55}$$

Define the following constants:

$$\begin{aligned}\lambda_{10} = & \min\{2k_{11}b_{10}, k_{13}, \sigma_{w1}/\lambda_{\max}(\Gamma_{w1}^{-1}), \\ & \sigma_{v1}/\lambda_{\max}(\Gamma_{v1}^{-1}), \sigma_1\eta_1\}, \end{aligned} \tag{56}$$

$$\mu_{10} = 0.2785\delta_1\rho_1+\frac{\sigma_{w1}\|W_1^*\|^2}{2}+\frac{\sigma_{v1}\|V_1^*\|_{\mathrm{F}}^2}{2}+\frac{\sigma_1\delta_1^2}{2}. \tag{57}$$

Thus, we have

$$\dot{V}_1(t) \leqslant -\lambda_{10}V_1(t)+\mu_{10}+(g_1(t)N(\zeta_1)+1)\dot{\zeta}_1. \tag{58}$$

Multiplying Eq. (58) by $\mathrm{e}^{\lambda_{10}t}$ yields

$$\frac{\mathrm{d}}{\mathrm{d}t}(V_1(t)\mathrm{e}^{\lambda_{10}t}) \leqslant \mathrm{e}^{\lambda_{10}t}[\mu_{10}+(g_1(t)N(\zeta_1)\dot{\zeta}_1+\dot{\zeta}_1)]. \tag{59}$$

Integrating Eq. (59) over $[0,t]$, we have

$$V_1(t) \leqslant C_{10}+\mathrm{e}^{-\lambda_{10}t}\int_0^t (g_1(\tau)N(\zeta_1)+1)\dot{\zeta}_1\mathrm{e}^{\lambda_{10}\tau}\,\mathrm{d}\tau \tag{60}$$

with $C_{10}=\mu_{10}/\lambda_{10}+V_1(0)$.

From Eqs. (7), (8), (28), and Assumption 3, we know that $g_1(t)\in[\min\{k_{1l},k_{1r}\}, 2\max\{k_{1l},k_{1r}\}]\subset(0,+\infty)$, $\forall t\geqslant 0$ or $g_1(t)\in[-2\max\{k_{1l},k_{1r}\}, -\min\{k_{1l},k_{1r}\}]\subset(-\infty,0)$, $\forall t\geqslant 0$. According to Lemma 1, we know that $V_1(t)$, $\zeta_1(t)$, $\int_0^t g_1(\tau) N(\zeta_1)\dot{\zeta}_1\,\mathrm{d}\tau$ are bounded on $[0,t_f)$. Therefore, $\tilde{\delta}_1$, $\|\tilde{W}_1\|$, $\|\tilde{V}_1\|_{\mathrm{F}}$ and $|s_i|$ are bounded on $[0,t_f)$ for all $t_f>0$, i.e., all signals in the closed-loop system are bounded on $[0,t_f)$ for all $t_f>0$. According to the discussion (Ryan, 1991), we know that the above conclusion is true for $t_f=+\infty$. Let $C_{\zeta 1}$ be the upper bound of $\mathrm{e}^{-\lambda_{10}t}\int_0^t(g_1(\tau)N(\zeta_1)+1)\dot{\zeta}_1\mathrm{e}^{\lambda_{10}\tau}\,\mathrm{d}\tau$ on $[0,\infty)$,

$$\mu_1=\frac{\mu_{10}}{\lambda_{10}}+V_1(0)+C_{\zeta 1}, \tag{61}$$

then $s_1^2\leqslant 2b_{11}V_1(t)\leqslant 2b_{11}\mu_1$. Similarly, $\|\tilde{W}_1\|^2\leqslant 2\mu_1/\lambda_{\min}(\Gamma_{w1}^{-1})$, $\|\tilde{V}_1\|_{\mathrm{F}}^2\leqslant 2\mu_1/\lambda_{\min}(\Gamma_{v1}^{-1})$, and $|\tilde{\delta}_1|^2\leqslant 2\mu_1\eta_1$.

Case ii: If $|s_1|<c_{s_1}$, then $q_1(s_1|c_{s_1})=0$. In this case, the control signal $v_1=0$, $\dot{\zeta}_1=0$, $\dot{\hat{W}}_1=0$, $\dot{\hat{V}}_1=0$ and $\dot{\hat{\delta}}_1=0$, i.e., ζ_1, $\hat{W}_1$, $\hat{V}_1$ and $\hat{\delta}_1$ are kept unchanged in bounded values.

Define $\omega_1=[e_{11},\ldots,e_{1,n_1-1}]^{\mathrm{T}}\in R^{n_1-1}$. From Eq. (11), we know that (i) there is a state space representation for mapping $s_1=[\Lambda^{\mathrm{T}}1]e_1$, i.e., $\dot{\omega}_1=A_{s_1}\omega_1+b_{s_1}s_1$ with $\Lambda_1=[\lambda_{11},\ldots,\lambda_{1,n_1-1}]^{\mathrm{T}}$, $b_{s_1}=[0,\ldots,0,1]^{\mathrm{T}}$, A_{s_1} being a stable matrix; (ii) there is a positive constant c_{10} such that $\|\mathrm{e}^{A_{s_1}t}\|\leqslant c_{10}\mathrm{e}^{-\lambda_1 t}$, and (iii) the solution for ω_1 is

$$\omega_1(t)=\mathrm{e}^{A_{s_1}t}\omega_1(0)+\int_0^t \mathrm{e}^{A_{s_1}(t-\tau)}b_{s_1}s_1(\tau)\,\mathrm{d}\tau.$$

Accordingly, it follows that

$$\|\omega_1(t)\|\leqslant c_{10}\|\omega_1(0)\|\mathrm{e}^{-\lambda_1 t}+c_{10}\int_0^t \mathrm{e}^{-\lambda_1(t-\tau)}|s_1(\tau)|\,\mathrm{d}\tau.$$

Let $\bar{\mu}_1=\max\{\sqrt{2b_{11}\mu_1}, c_{s_1}\}$. Therefore, we have

$$\|\omega_1(t)\|\leqslant c_{10}\|\omega_1(0)\|+\frac{c_{10}\bar{\mu}_1}{\lambda_1}. \tag{62}$$

Noting $s_1=\Lambda_1^{\mathrm{T}}\omega+e_{1n_1}$ and $e_1=[\omega_1^{\mathrm{T}}, e_{1n_1}]^{\mathrm{T}}$, we obtain

$$\|e_1\|\leqslant\|\omega_1\|+|e_{1n_1}|\leqslant(1+\|\Lambda_1\|)\|\omega_1\|+|s_1|.$$

Substituting Eq. (62) into the above inequality leads to

$$\|e_1\|\leqslant c_{10}(1+\|\Lambda_1\|)\|\omega_1(0)\|+\left[1+\frac{(1+\|\Lambda_1\|)c_{10}}{\lambda_1}\right]\bar{\mu}_1. \tag{63}$$

Since c_{10}, $\|\Lambda_1\|$ and λ_1 are positive constants, and $\omega_1(0)$ and $s_1(0)$ depend on $x_1(0)-x_{1d}(0)$, we conclude that there exists a positive constant $R_1(c_1, x_1(0), \tilde{W}_1(0), \tilde{V}_1(0), \tilde{\delta}_1(0))$ depending on c_1, $x_1(0)$, $\tilde{W}_1(0)$, $\tilde{V}_1(0)$ and $\tilde{\delta}_1(0)$ such that

$$\|e_1\|\leqslant R_1(c_1, x_1(0), \tilde{W}_1(0), \tilde{V}_1(0), \tilde{\delta}_1(0)),\quad \forall t\geqslant 0 \tag{64}$$

with $c_1=\mu_{10}/\lambda_{10}$.

Noting $x_1 = e_1 + x_{1d}$ and Assumption 5, we obtain

$$\|x_1\| \leqslant \|e_1\| + \|x_{1d}\| \leqslant c_{10}(1 + \|\Lambda_1\|)\|\omega_1(0)\| + \left[1 + \frac{(1+\|\Lambda_1\|)c_{10}}{\lambda_1}\right]\bar{\mu}_1 + \|x_{1d}\| \in L_\infty. \tag{65}$$

Therefore, we can conclude from Cases i and ii that all the closed-loop signals are semi-globally uniformly ultimately bounded for bounded initial conditions.

Step 2: In the following, we shall find the conditions such that $x_1 \in \Omega_1$, $\forall t \geqslant 0$. Firstly, define a set

$$\Omega_{10} = \{x_1(0) | \{x_1 | \|x_1 - x_{1d}\| < R_1(0, x_1(0), 0, 0, 0)\} \subset \Omega_1, \bar{x}_{1d} \in \Omega_{1d}\}, \tag{66}$$

which is not empty. It is easy to see that for all $x_1(0) \in \Omega_{10}$ and $\bar{x}_{1d} \in \Omega_{1d}$, we have $x_1 \in \Omega_1$, $\forall t \geqslant 0$. Then, for the system with $x_1(0) \in \Omega_{10}$, bounded $\hat{W}_1(0)$, $\hat{V}_1(0)$, $\hat{\delta}_1(0)$ and $\bar{x}_{1d} \in \Omega_{1d}$, the following constant c_1^* can be determined by

$$c_1^* = \sup_{c_1 \in R^+} \{c_1 | \{x_1 | \|x_1 - x_{1d}\| < R_1(c_1, x_1(0), \tilde{W}_1(0), \tilde{V}_1(0), \tilde{\delta}_1(0))\} \subset \Omega_1, \bar{x}_{1d} \in \Omega_{1d}\}. \tag{67}$$

From Eqs. (56) and (57), we know that if the adaptive control parameters $\sigma_{w1}, \sigma_{v1}, \sigma_1$, and ρ_1 are chosen to be sufficiently small, and $k_{11}, k_{13}, \eta_1, \lambda_{\min}(\Gamma_{w1})$ and $\lambda_{\min}(\Gamma_{v1})$ are taken to be sufficiently large, then the constant $c_1 = \mu_{10}/\lambda_{10}$ can be made arbitrary small. Therefore, for the initial condition $x_1(0) \in \Omega_{10}$, bounded $\hat{W}_1(0)$, $\hat{V}_1(0)$, $\hat{\delta}_1(0)$ and $\bar{x}_{1d} \in \Omega_{1d}$, if the adaptive control parameters are appropriately chosen such that $\mu_{10}/\lambda_{10} \leqslant c_1^*$, then system state x_1 indeed stays in Ω_1 for all time. This completes the proof. □

3.2. Adaptive NN control for MIMO system ($m \geqslant 2$)

In this section, the design in Section 3.1 is extended to MIMO system (1), which contains m interconnected subsystems. For the ith subsystem:

$$\begin{cases} \dot{x}_{ij} = x_{i,j+1}, \quad j = 1, \ldots, n_i - 1, \\ \dot{x}_{in_i} = f_i(x, u_1, \ldots, u_{i-1}) \\ \qquad + f_{i,\tau}(x_1(t-\tau_1(t)), \ldots, x_m(t-\tau_m(t))) \\ \qquad + b_i(x_1, \ldots, x_i)u_i, \quad i = 1, \ldots, m, \\ y_i = x_{i1}. \end{cases} \tag{68}$$

The filtering tracking error s_i is given by Eq. (11). From Eqs. (9), (10) and (68), we obtain

$$\dot{s}_i = f_i(x, u_1, \ldots, u_{i-1}) + \gamma_i + b_i(\bar{x}_i)K_i^{\mathrm{T}}(t)\Phi_i(t)v_i(t) + f_{i,\tau}(x_1(t-\tau_1(t)), \ldots, x_m(t-\tau_m(t))) + b_i(\bar{x}_i)d_i(v_i(t)), \tag{69}$$

where $\gamma_i = \sum_{j=1}^{n_i-1}\lambda_{ij}e_{i,j+1} - y_{id}^{(n_i)}$.

Define a smooth scalar function as follows:

$$V_{si} = \int_0^{s_i} \frac{\sigma}{|b_i(\bar{x}_i^+, \sigma + \beta_i)|}\,\mathrm{d}\sigma, \tag{70}$$

where $\beta_i = y_{id}^{(n_i-1)} - \sum_{j=1}^{n_i-1}\lambda_{ij}e_{ij}$, $\bar{x}_i^+ = [x_1^{\mathrm{T}}, \ldots, x_{i-1}^{\mathrm{T}}, x_{i1}, \ldots, x_{i,n_i-1}]^{\mathrm{T}}$.

By Second Mean Value Theorem for Integrals, V_{si} can be rewritten as $V_{si} = s_i^2/2|b_i(\bar{x}_i^+, \lambda_{si}s_i + \beta_i)|$ with $\lambda_{si} \in (0, 1)$. Because $0 < b_{i0} \leqslant |b_i(\bar{x}_i)|$, it is shown that V_{si} is positive definitive with respect to s_i.

Differentiating V_{si} with respect to time t, applying Eqs. (10) and (69), we obtain

$$\begin{aligned}\dot{V}_{si} = {} & \frac{s_i}{|b_i(\bar{x}_i)|}\dot{s}_i + \sum_{j=1}^{i-1} f_{j,\tau}(x_1(t-\tau_1(t)), \ldots, x_m(t-\tau_m(t)))\int_0^{s_i}\sigma\frac{\partial|b_i^{-1}(\bar{x}_i^+, \sigma+\beta_i)|}{\partial x_{jn_j}}\,\mathrm{d}\sigma \\ & + \int_0^{s_i}\sigma\left\{\sum_{j=1}^{i}\sum_{k=1}^{n_j-1}\frac{\partial|b_i^{-1}(\bar{x}_i^+\sigma+\beta_i)|}{\partial x_{jk}}x_{j,k+1} + \sum_{j=1}^{i-1}\frac{\partial|b_i^{-1}(\bar{x}_i^+, \sigma+\beta_i)|}{\partial x_{jn_j}}[f_j(x, u_1, \ldots, u_{j-1}) + b_j(\bar{x}_j)D_j(v_j)]\right\}\mathrm{d}\sigma \\ & - \frac{\gamma_i s_i}{|b_i(\bar{x}_i)|} + \gamma_i\int_0^{s_i}|b_i^{-1}(\bar{x}_i^+, \sigma+\beta_i)|\,\mathrm{d}\sigma \\ \leqslant {} & s_i g_i(t)v_i + s_i Q_i(z_i) + \frac{s_i f_{i,\tau}(x_1(t-\tau_1(t)), \ldots, x_m(t-\tau_m(t)))}{|b_i(\bar{x}_i)|} + |s_i|p_i^* \\ & + \sum_{j=1}^{i-1} f_{j,\tau}(x_1(t-\tau_1(t)), \ldots, x_m(t-\tau_m(t)))\int_0^{s_i}\sigma\frac{\partial|b_i^{-1}(\bar{x}_i^+, \sigma+\beta_i)|}{\partial x_{jn_j}}\,\mathrm{d}\sigma, \end{aligned} \tag{71}$$

where

$$g_i(t) = \frac{b_i(\bar{x}_i)}{|b_i(\bar{x}_i)|}K_i^{\mathrm{T}}(t)\Phi_i(t), \tag{72}$$

$$\begin{aligned} Q_i(z_i) = {} & \frac{f_i(x, u_1, \ldots, u_{i-1})}{|b_i(\bar{x}_i)|} + \int_0^1\left\{\theta\left[\sum_{j=1}^{i}\sum_{k=1}^{n_j-1}\frac{\partial|b_i^{-1}(\bar{x}_i^+, \theta s_i+\beta_i)|}{\partial x_{jk}}x_{j,k+1} + \sum_{j=1}^{i-1}\frac{\partial|b_i^{-1}(\bar{x}_i^+, \theta s_i+\beta_i)|}{\partial x_{jn_j}}(f_j(x, u_1, \ldots, u_{j-1}) + b_j(\bar{x}_j)D_j(v_j))\right] + \gamma_i|b_i^{-1}(\bar{x}_i^+, \theta s_i+\beta_i)|\right\}\mathrm{d}\theta, \end{aligned} \tag{73}$$

$$z_i = [x^{\mathrm{T}}, s_i, \gamma_i, \beta_i, v_1, \ldots, v_{i-1}]^{\mathrm{T}} = [z_{i1}, z_{i2}, \ldots, z_{ip_i}]^{\mathrm{T}}, \quad p_i = n + i + 2. \tag{74}$$

By utilizing Young's inequality and Assumption 6, we obtain

$$\sum_{j=1}^{i-1} f_{j,\tau}(x_1(t-\tau_1(t)),\ldots,x_m(t-\tau_m(t))) \times \int_0^{s_i} \sigma \frac{\partial |b_i^{-1}(\bar{x}_i^+, \sigma+\beta_i)|}{\partial x_{jn_j}} \mathrm{d}\sigma \leqslant \frac{m}{2}\sum_{j=1}^{i-1}\sum_{k=1}^{m} \varrho_{jk}^2(x_k(t-\tau_k(t))) + \frac{s_i^4}{2}\sum_{j=1}^{i-1}\left(\int_0^1 \theta \frac{\partial |b_i^{-1}(\bar{x}_i^+, \theta s_i+\beta_i)|}{\partial x_{jn_j}} \mathrm{d}\theta\right)^2 \tag{75}$$

and

$$\frac{s_i f_{i,\tau}(x_1(t-\tau_1(t)),\ldots,x_m(t-\tau_m(t)))}{|b_i(\bar{x}_i)|} \leqslant \frac{s_i^2}{2b_i^2(\bar{x}_i)} + \frac{m}{2}\sum_{k=1}^{m} \varrho_{ik}^2(x_k(t-\tau_k(t))). \tag{76}$$

Substituting Eqs. (75) and (76) into Eq. (71), we have

$$\dot{V}_{si} \leqslant s_i g_i(t) v_i + s_i Q_i(z_i) + \frac{s_i^2}{2b_i^2(\bar{x}_i)} + \frac{m}{2}\sum_{j=1}^{i}\sum_{k=1}^{m} \varrho_{jk}^2(x_k(t-\tau_k(t))) + |s_i| p_i^* + \frac{s_i^4}{2}\sum_{j=1}^{i-1}\left(\int_0^1 \theta \frac{\partial |b_i^{-1}(\bar{x}_i^+, \theta s_i+\beta_i)|}{\partial x_{jn_j}} \mathrm{d}\theta\right)^2. \tag{77}$$

To overcome the design difficulties from the unknown time-varying delays, $\tau_1(t),\ldots,\tau_m(t)$, the following Lyapunov–Krasovskii functional can be considered

$$V_{U_i}(t) = \frac{m}{2(1-\bar{\tau}_{\max})}\sum_{j=1}^{i}\sum_{k=1}^{m}\int_{t-\tau_k(t)}^{t} \varrho_{jk}^2(x_k(\tau))\,\mathrm{d}\tau. \tag{78}$$

The time derivative of $V_{U_i}(t)$ is

$$\dot{V}_{U_i}(t) = \frac{m}{2(1-\bar{\tau}_{\max})}\left[\sum_{j=1}^{i}\sum_{k=1}^{m} \varrho_{jk}^2(x_k(t)) - \sum_{j=1}^{i}\sum_{k=1}^{m} \varrho_{jk}^2(x_k(t-\tau_k(t)))(1-\dot{\tau}_k(t))\right], \tag{79}$$

which can be used to cancel the time-varying delay terms on the right-hand side of (71) and thus eliminate the design difficulty from the unknown time-delays, $\tau_1(t),\ldots,\tau_m(t)$, without introducing any uncertainties to the system. For notation conciseness, the time variables t, $t-\tau_1(t),\ldots,t-\tau_m(t)$ will be omitted after time-delay terms have been eliminated. Accordingly, we obtain

$$\dot{V}_{si} + \dot{V}_{U_i} \leqslant s_i g_i(t) v_i + s_i h_i(z_1) + |s_i| p_i^*, \tag{80}$$

where

$$h_i(z_i) = Q_i(z_i) + \frac{s_i}{2b_i^2(\bar{x}_i)} + \frac{m}{2(1-\bar{\tau}_{\max})s_i} \times \sum_{j=1}^{i}\sum_{k=1}^{m} \varrho_{jk}^2(x_k) + \frac{s_i^3}{2}\sum_{j=1}^{i-1}\left(\int_0^1 \theta \frac{\partial |b_i^{-1}(\bar{x}_i^+, \theta s_i+\beta_i)|}{\partial x_{jn_j}} \mathrm{d}\theta\right)^2. \tag{81}$$

Define the compact sets Ω_{z_1} and Ω_{z_i} as follows:

$$\Omega_{z_1} = \{[x^{\mathrm{T}}, s_1, \gamma_1, \beta_1]^{\mathrm{T}} | x_j \in \Omega_j, j=1,\ldots,m, \bar{x}_{1d} \in \Omega_{1d}\}, \tag{82}$$

$$\Omega_{z_i} = \{[x^{\mathrm{T}}, s_i, \gamma_i, \beta_i, v_1,\ldots,v_{i-1}]^{\mathrm{T}} | x_j \in \Omega_j, j=1,\ldots,m, \bar{x}_{kd} \in \Omega_{kd}, k=1,\ldots,i, (\hat{W}_j, \hat{V}_j, \hat{\delta}_j) \in \Omega_{wj}, j=1,\ldots,i-1\}, \tag{83}$$

where $\Omega_j \subset R^{n_j}$ is a sufficiently large compact set satisfying $\Omega_j \supset \Omega_{j0}$ which is similar to the definition of Ω_{10} in Theorem 1, $j=1,\ldots,m$; Ω_{wj} will be defined later in Theorem 2, $j=1,\ldots,i-1, i=2,\ldots,m$.

For ease of discussion, let us define sets $\Omega_{c_{s_i}} \subset \Omega_{z_i}$ and $\Omega_{z_i}^0$ as follows:

$$\Omega_{c_{s_i}} = \{z_i | |s_i| < c_{s_i}, \bar{x}_{id} \in \Omega_{id}\}, \tag{84}$$

$$\Omega_{z_i}^0 = \Omega_{z_i} - \Omega_{c_{s_i}}, \tag{85}$$

where c_{s_i} is a positive design constant that can be chosen arbitrarily small. As shown in Ge et al. (2004), we know that $\Omega_{z_i}^0$ is a compact set.

Let $\hat{W}_i^{\mathrm{T}} S(\hat{V}_i^{\mathrm{T}} \bar{z}_i)$ be the approximation of the three-layer NNs, which are discussed in Section 2.3, on the compact $\Omega_{z_i}^0$ to $h_i(z_i)$, we have

$$h_i(z_i) = \hat{W}_i^{\mathrm{T}} S_i(\hat{V}_i^{\mathrm{T}}\bar{z}_i) - \tilde{W}_i^{\mathrm{T}}(\hat{S}_i - \hat{S}_i'\hat{V}_i^{\mathrm{T}}\bar{z}_i) - \hat{W}_i^{\mathrm{T}}\hat{S}_i'\tilde{V}_i^{\mathrm{T}}\bar{z}_i - d_{ui} + \varepsilon_i(z_i), \tag{86}$$

where $z_i = [z_{i1},\ldots,z_{ip_i}]^{\mathrm{T}}$, $\bar{z}_i = [z_i^{\mathrm{T}}, 1]^{\mathrm{T}}$; $\hat{V}_i = [\hat{v}_{i1},\ldots,\hat{v}_{il_i}] \in R^{(p_i+1)\times l_i}$ and $\hat{W}_i = [\hat{w}_{i1},\ldots,\hat{w}_{il_i}]^{\mathrm{T}} \in R^{l_i}$ denote the estimates of W_i^* and V_i^*, respectively, W_i^* and V_i^* are ideal constant weights; $\hat{S}_i = S_i(\hat{V}_i^{\mathrm{T}}\bar{z}_i) = [s(\hat{v}_{i1}^{\mathrm{T}}\bar{z}_i),\ldots,s(v_{i,l_i-1}^{\mathrm{T}}\bar{z}_i), 1]^{\mathrm{T}}$with $s(z_\alpha) = 1/(1+\mathrm{e}^{-\gamma_{i0}z_\alpha})$, and constant $\gamma_{i0} > 0$, $\hat{S}_i' = \mathrm{diag}\{\hat{s}_{i1}',\ldots,\hat{s}_{i,l_i-1}', 0\}$ with $\hat{s}_{ik}' = s'(\hat{v}_{ik}^{\mathrm{T}}\bar{z}_i) = \mathrm{d}[s(z_\alpha)]/\mathrm{d}z_\alpha|_{z_\alpha=\hat{v}_{ik}^{\mathrm{T}}\bar{z}_i}$, $k=1,\ldots,l_i-1$; and the NN node number $l_i > 1$; and the residual term d_{ui} is bounded by

$$|d_{ui}| \leqslant \|V_i^*\|_{\mathrm{F}} \|\bar{z}_i \hat{W}_i^{\mathrm{T}}\hat{S}_i'\|_{\mathrm{F}} + \|W_i^*\| \|\hat{S}_i'\hat{V}_i^{\mathrm{T}}\bar{z}_i\| + \|W_i^*\|_1, \tag{87}$$

the approximation error $\varepsilon_i(z_i)$ satisfies $|\varepsilon_i(z_i)| \leqslant \varepsilon_i^*, \forall z_i \in \Omega_{z_i}^0$ with constant $\varepsilon_i^* > 0$.

Consider the following control law:

$$v_i(t) = q(s_i|c_{s_i})N(\zeta_i)[k_{i0}(t)s_i + \hat{W}_i^{\mathrm{T}} S_i(\hat{V}_i^{\mathrm{T}}\bar{z}_i) + \hat{\delta}_i \xi_i(z_i)\tanh(s_i\xi_i(z_i)/\rho_i))], \quad (88)$$

$$\dot{\zeta}_i = q(s_i|c_{s_i})[k_{i0}(t)s_i^2 + \hat{W}_i^{\mathrm{T}} S_i(\hat{V}_i^{\mathrm{T}}\bar{z}_i)s_i + \hat{\delta}_i \xi_i(z_i)s_i\tanh(s_i\xi_i(z_i)/\rho_i)], \quad (89)$$

where $q(\cdot|\cdot)$ is defined by Eq. (22), $N(\zeta_i) = \mathrm{e}^{\zeta_i^2}\cos((\pi/2)\zeta_i)$, ρ_i is a positive constant, $\hat{\delta}_i$ is the estimate of δ_i with $\delta_i = \max\{\|V_i^*\|_{\mathrm{F}}, \|W_i^*\|, \|W_i^*\|_1 + \varepsilon_i^* + p_i^*\}$ at time t,

$$\xi_i(z_i) = \|\bar{z}_i\hat{W}_i^{\mathrm{T}}\hat{S}_i'\|_{\mathrm{F}} + \|\hat{S}_i'\hat{V}_i^{\mathrm{T}}\bar{z}_i\| + 1, \quad (90)$$

$k_{i0}(t) = k_{i1} + k_{i2}(t)$ with k_{i1} being any positive constant, $k_{i2}(t)$ chosen as

$$k_{i2}(t) = \frac{mk_{i3}q(s_i|c_{s_i})}{2(1-\bar{\tau}_{\max})s_i^2}\sum_{j=1}^{i}\sum_{k=1}^{m}\int_{t-\tau_{\max}}^{t}\varrho_{jk}^2(x_k(\tau))\,\mathrm{d}\tau \quad (91)$$

with k_{i3} being a positive constant, specified by the designer.

The following adaptive tuning laws are shown as

$$\dot{\hat{W}}_i = q(s_i|c_{s_i})\Gamma_{wi}[(\hat{S}_i - \hat{S}_i'\hat{V}_i^{\mathrm{T}}\bar{z}_i)s_i - \sigma_{wi}\hat{W}_i], \quad (92)$$

$$\dot{\hat{V}}_i = q(s_i|c_{s_i})\Gamma_{vi}[\bar{z}_i\hat{W}_i^{\mathrm{T}}\hat{S}_i's_i - \sigma_{vi}\hat{V}_i], \quad (93)$$

$$\dot{\hat{\delta}}_i = q(s_i|c_{s_i})\eta_i[s_i\xi_i(z_i)\tanh(s_i\xi_i(z_i)/\rho_i) - \sigma_i\hat{\delta}_i], \quad (94)$$

where $\Gamma_{wi} > 0$, $\Gamma_{vi} > 0$, σ_{wi}, σ_{vi}, η_i, ρ_i and σ_i are strictly positive constants.

Theorem 2. *Consider the closed-loop system consisting of the plant* (1), *the control law* (88), *and adaptation laws* (92)–(94). *If Assumptions* 1–7 *hold*, *then for bounded initial conditions*, *the overall closed-loop neural control system is semi-globally stable in the sense that all signals in the closed-loop system are bounded*, *the parameter estimates*

$$(\hat{W}_i, \hat{V}_i, \hat{\delta}_i) \in \Omega_{wi} = \left\{(\hat{W}_i, \hat{V}_i, \hat{\delta}_i) \,|\, \|\tilde{W}_i\|^2 \leqslant \frac{2\mu_i}{\lambda_{\min}(\Gamma_{wi}^{-1})}, \|\tilde{V}_i\|_{\mathrm{F}}^2 \leqslant \frac{2\mu_i}{\lambda_{\min}(\Gamma_{vi}^{-1})}, |\tilde{\delta}_i|^2 \leqslant 2\eta_i\mu_i\right\}, \quad (95)$$

and $\forall x_i(0) \in \Omega_{i0}$, *the state vector*

$$x_i \in \Omega_{ic} = \left\{x_i \,|\, \|x_i - x_{id}\| \leqslant c_{i0}(1 + \|\Lambda_i\|)\|\omega_i(0)\| + \left[1 + \frac{(1+\|\Lambda_i\|)c_{i0}}{\lambda_i}\right]\max\{\sqrt{2b_{i1}\mu_i}, c_{s_i}\}, \bar{x}_{id} \in \Omega_{id}\right\} \subset \Omega_i, \quad (96)$$

whose size can be adjusted by appropriately choosing the design parameters; $\omega_i = [e_{i1}, \ldots, e_{i,n_i-1}]^{\mathrm{T}} \in R^{n_i-1}$, $\dot{\omega}_i = A_{s_i}\omega_i + b_{s_i}s_i$ *is one state space representation for mapping* $s_i = [\Lambda_i^{\mathrm{T}} 1]e_i$ *with* $\Lambda_i = [\lambda_{i1}, \ldots, \lambda_{i,n_i-1}]^{\mathrm{T}}$, $b_{s_i} = [0, \ldots, 0, 1]^{\mathrm{T}} \in R^{n_i-1}$, A_{s_i} *being a stable matrix*, *and* c_{i0} *being a positive constant satisfying* $\|\mathrm{e}^{A_{s_i}t}\| \leqslant c_{i0}\mathrm{e}^{-\lambda_i t}$, $i = 1, \ldots, m$.

Proof. The proof includes two steps as in Ge et al. (1999a). We first suppose that $x_i \in \Omega_i$ holds for all time, and find the upper bounds of system states. Later, for the appropriate initial condition $x_i(0)$ and the adaptive controller parameters, we prove that the state x_i indeed remains in the compact set Ω_i for all $t \geqslant 0$.

Suppose that $x_i \in \Omega_i$, $\forall t \geqslant 0$, then NN approximation (86) is valid. Consider the Lyapunov function candidate

$$V_i(t) = V_{si}(t) + V_{U_i}(t) + \frac{1}{2}\tilde{W}_i^{\mathrm{T}}\Gamma_{wi}^{-1}\tilde{W}_i + \frac{1}{2}\mathrm{tr}\{\tilde{V}_i^{\mathrm{T}}\Gamma_{vi}^{-1}\tilde{V}_i\} + \frac{1}{2\eta_i}\tilde{\delta}_i^2. \quad (97)$$

Differentiating $V_i(t)$ with respect to time t leads to

$$\dot{V}_i(t) = \dot{V}_{si}(t) + \dot{V}_{U_i}(t) + \tilde{W}_i^{\mathrm{T}}\Gamma_{wi}^{-1}\dot{\hat{W}}_i + \mathrm{tr}\{\tilde{V}_i^{\mathrm{T}}\Gamma_{vi}^{-1}\dot{\hat{V}}_i\} + \frac{1}{\eta_i}\tilde{\delta}_i\dot{\hat{\delta}}_i. \quad (98)$$

Case i: If $|s_i| \geqslant c_{s_i}$, then $q_i(s_i|c_{s_i}) = 1$. In this case, substituting Eq. (80) into Eq. (98), and using control law (88), (89), and (86) and (87), it follows that

$$\dot{V}_i(t) \leqslant g_i(t)N(\zeta_i)\dot{\zeta}_i + \dot{\zeta}_i - k_{i0}(t)s_i^2 - \hat{W}_i^{\mathrm{T}}S_i(\hat{V}_i^{\mathrm{T}}\bar{z}_i)s_i - \hat{\delta}_i\xi_i(z_i)s_i\tanh(s_i\xi_i(z_i)/\rho_i) + [\hat{W}_i^{\mathrm{T}}S_i(\hat{V}_i^{\mathrm{T}}\bar{z}_i) - \tilde{W}_i^{\mathrm{T}}(\hat{S}_i - \hat{S}_i'\hat{V}_i\bar{z}_i) - \hat{W}_i^{\mathrm{T}}\hat{S}_i'\tilde{V}_i^{\mathrm{T}}\bar{z}_i - d_{ui} + \varepsilon_i(z_i)]s_i + |s_i|p_i^* + \tilde{W}_i^{\mathrm{T}}\Gamma_{wi}^{-1}\dot{\hat{W}}_i + \mathrm{tr}\{\tilde{V}_i^{\mathrm{T}}\Gamma_{vi}^{-1}\dot{\hat{V}}_i\} + \frac{1}{\eta_i}\tilde{\delta}_i\dot{\hat{\delta}}_i. \quad (99)$$

Using adaptive tuning laws (92)–(94), and the inequality: $0 \leqslant |x| - x\tanh(x/\varepsilon) \leqslant 0.2785\varepsilon$, for $\varepsilon > 0$, $x \in R$, and the fact that $\hat{W}_i^{\mathrm{T}}\hat{S}_i'\tilde{V}_i^{\mathrm{T}}\bar{z}_i = \mathrm{tr}\{\tilde{V}_i^{\mathrm{T}}\bar{z}_i\hat{W}_i^{\mathrm{T}}\hat{S}_i'\}$, we obtain

$$\dot{V}_i(t) \leqslant g_i(t)N(\zeta_i)\dot{\zeta}_i + \dot{\zeta}_i - k_{i1}s_i^2 - \frac{mk_{13}}{2(1-\bar{\tau}_{\max})} \times \sum_{j=1}^{i}\sum_{k=1}^{m}\int_{t-\tau_{\max}}^{t}\varrho_{jk}^2(x_k(\tau))\,\mathrm{d}\tau + 0.2785\delta_i\rho_i - \sigma_{wi}\tilde{W}_i^{\mathrm{T}}\hat{W}_i - \sigma_{vi}\mathrm{tr}\{\tilde{V}_i^{\mathrm{T}}\hat{V}_i\} - \sigma_i\tilde{\delta}_i\hat{\delta}_i. \quad (100)$$

By completion of squares, the following inequalities hold:

$$-\sigma_{wi}\tilde{W}_i^{\mathrm{T}}\hat{W}_i \leqslant -\frac{\sigma_{wi}\|\tilde{W}_i\|^2}{2} + \frac{\sigma_{wi}\|W_i^*\|^2}{2}, \quad (101)$$

$$-\sigma_{vi}\mathrm{tr}\{\tilde{V}_i^{\mathrm{T}}\hat{V}_i\} \leqslant -\frac{\sigma_{vi}\|\tilde{V}_i\|_{\mathrm{F}}^2}{2} + \frac{\sigma_{vi}\|V_i^*\|_{\mathrm{F}}^2}{2}, \quad (102)$$

$$-\sigma_i\tilde{\delta}_i\hat{\delta}_i \leqslant -\frac{\sigma_i\tilde{\delta}_i^2}{2} + \frac{\sigma_i\delta_i^2}{2}. \quad (103)$$

Substituting Eqs. (101)–(103) into Eq. (100), we obtain

$$\dot{V}_i(t) \leqslant -k_{i1}s_i^2 - k_{i3}V_{U_i}(t) + g_i(t)N(\zeta_i)\dot{\zeta}_i + \dot{\zeta}_i + 0.2785\delta_i\rho_i - \frac{\sigma_{wi}\|\tilde{W}_i\|^2}{2} - \frac{\sigma_{vi}\|\tilde{V}_i\|_F^2}{2} - \frac{\sigma_i\tilde{\delta}_i^2}{2} + \frac{\sigma_{wi}\|W_i^*\|^2}{2} + \frac{\sigma_{vi}\|V_i^*\|_F^2}{2} + \frac{\sigma_i\delta_i^2}{2}. \quad (104)$$

Define the following constants:

$$\lambda_{i0} = \min\{2k_{i1}b_{i0}, k_{i3}, \sigma_{wi}/\lambda_{\max}(\Gamma_{wi}^{-1}), \sigma_{vi}/\lambda_{\max}(\Gamma_{vi}^{-1}), \sigma_i\eta_i\}, \quad (105)$$

$$\mu_{i0} = 0.2785\delta_i\rho_i + \frac{\sigma_{wi}\|W_i^*\|^2}{2} + \frac{\sigma_{vi}\|V_i^*\|_F^2}{2} + \frac{\sigma_i\delta_i^2}{2}. \quad (106)$$

Thus, we have

$$\dot{V}_i(t) \leqslant -\lambda_{i0}V_i(t) + \mu_{i0} + g_i(t)N(\zeta_i)\dot{\zeta}_i + \dot{\zeta}_i. \quad (107)$$

Multiplying Eq. (107) by $e^{\lambda_{i0}t}$ yields

$$\frac{d}{dt}(V_i(t)e^{\lambda_{i0}t}) \leqslant e^{\lambda_{i0}t}[\mu_{i0} + g_i(t)N(\zeta_i)\dot{\zeta}_i + \dot{\zeta}_i]. \quad (108)$$

Integrating Eq. (108) over $[0, t]$, we have

$$V_i(t) \leqslant C_{i0} + e^{-\lambda_{i0}t}\int_0^t (g_i(\tau)N(\zeta_i) + 1)\dot{\zeta}_i e^{\lambda_{i0}\tau}\, d\tau \quad (109)$$

with $C_{i0} = \mu_{i0}/\lambda_{i0} + V_i(0)$.

From Eqs. (7), (8), (72), and Assumption 3, we know that $g_i(t) \in [\min\{k_{il}, k_{ir}\}, 2\max\{k_{il}, k_{ir}\}] \subset (0, +\infty), \forall t \geqslant 0$ or $g_i(t) \in [-2\max\{k_{il}, k_{ir}\}, -\min\{k_{il}, k_{ir}\}] \subset (-\infty, 0), \forall t \geqslant 0$. According to Lemma 1, we have $V_i(t), \zeta_i(t), \int_0^t g_i(\tau)N(\zeta_i)\dot{\zeta}_i\, d\tau$ are bounded on $[0, t_f)$. Therefore, $\tilde{\delta}_i$, $\|\tilde{W}_i\|$, $\|\tilde{V}_i\|_F$ and $|s_i|$ are bounded on $[0, t_f)$ for all $t_f > 0$, i.e., all signals in the closed-loop system are bounded on $[0, t_f)$ for all $t_f > 0$. According to the discussion in Ryan (1991), we see that the above conclusion is true for $t_f = +\infty$. Therefore, $\tilde{\delta}_i$, $\|\tilde{W}_i\|$ and $\|\tilde{V}_i\|_F \in L_\infty$. Let $C_{\zeta i}$ be the upper bound of $e^{-\lambda_{i0}t}\int_0^t (g_i(\tau)N(\zeta_i)+1)\dot{\zeta}_i e^{\lambda_{i0}\tau} d\tau$ on $[0, \infty)$, $\mu_i = \mu_{i0}/\lambda_{i0} + V_i(0) + C_{\zeta i}$, then $s_i^2 \leqslant 2b_{i1}V_i(t) \leqslant 2b_{i1}\mu_i$. Similarly, $\|\tilde{W}_i\|^2 \leqslant 2\mu_i/\lambda_{\min}(\Gamma_{wi}^{-1})$, $\|\tilde{V}_i\|_F^2 \leqslant 2\mu_i/\lambda_{\min}(\Gamma_{vi}^{-1})$, and $|\tilde{\delta}_i|^2 \leqslant 2\mu_i\eta_i$.

Case ii: If $|s_i| < c_{s_i}$, then $q_i(s_i|c_{s_i}) = 0$. In this case, the control signal $v_i = 0$, $\dot{\zeta}_i = 0$, $\dot{\hat{W}}_i = 0$, $\dot{\hat{V}}_i = 0$ and $\dot{\hat{\delta}}_i = 0$, i.e., ζ_i, $\hat{W}_i$, $\hat{V}_i$ and $\hat{\delta}_i$ are kept unchanged in bounded values.

Therefore, similar to the discussion in Theorem 1, we can conclude from Cases i and ii that all the closed-loop signals are semi-globally uniformly ultimately bounded and Eq. (96) holds. □

4. Simulation results

To demonstrate the effectiveness of the proposed approach, we consider the following nonlinear system:

$$\begin{cases} \dot{x}_{11}(t) = x_{12}(t), \\ \dot{x}_{12}(t) = x_{21}(t) - 0.3\sin(x_{21}(t)) + 0.1x_{11}^2(t-\tau_1(t)) + (2-\sin^2(x_{11}(t)))u_1(t), \\ \dot{x}_{21}(t) = x_{22}(t), \\ \dot{x}_{22}(t) = x_{11}^2(t)u_1(t) + (x_{22}^2(t) + x_{11}(t) + 0.5\cos(x_{21}(t)))u_1^2(t) + 0.2x_{22}(t-\tau_2(t)) \times \sin(x_{21}(t-\tau_2(t))) + (3+\sin(x_{22}(t)))u_2(t), \\ y_1(t) = x_{11}(t), \quad y_2(t) = x_{22}(t), \end{cases} \quad (110)$$

where u_i $(i = 1, 2)$ are outputs of dead-zones.

Both NNs $\hat{W}_i^T S_i(\hat{V}_i^T \bar{z}_i)$, $i = 1, 2$ contain 10 hidden nodes (i.e., $l_1 = l_2 = 10$) and the coefficients in activation function $s(\cdot)$ are taken as $\gamma_{10} = \gamma_{20} = 3.5$. The desired tracking trajectories $y_{1d}(t) = 0.5[\sin(t) + \sin(0.5t)]$ and $y_{2d} = \sin(0.5t) + 0.5\sin(1.5t)$. The design parameters of the above controller are $c_{s_1} = c_{s_2} = 1.0 \times 10^{-5}$, $\lambda_{11} = 1.5$, $\lambda_{21} = 2$, $k_{11} = k_{21} = 2$, $k_{13} = k_{23} = 0.001$, $\eta_1 = \eta_2 = 0.1$, $\rho_1 = \rho_2 = 0.2$, $\sigma_1 = \sigma_2 = \sigma_{w1} = \sigma_{w2} = \sigma_{v1} = \sigma_{v2} = 0.01$, $\Gamma_{w1} = \Gamma_{w2} = \text{diag}\{2.5\}$, $\Gamma_{v1} = \Gamma_{v2} = \text{diag}\{10\}$. The dead-zones are assumed to have linear functions outside the deadband. We select $g_{ir}(v_i) = k_{ir}(v_i - b_{ir})$ and $g_{il}(v_i) = k_{il}(v_i - b_{il})$ with the parameters of the dead-zones $k_{1l} = 0.5$, $k_{1r} = 1.5$, $k_{2l} = 1.5$, $k_{2r} = 2.5$, $b_{1l} = -0.5$, $b_{1r} = 0.5$, $b_{2l} = -2.5$, $b_{2r} = 2$. The initial conditions: $x_{11}(0)=0$, $x_{12}(0)=0$, $x_{21}(0)=0$, $x_{22}(0)=0$, time-varying delays $\tau_1(t)=0.2(1+\sin(t))$, $\tau_2(t)=1-0.5\cos(t)$, $\tau_{\max}=2$, $\bar{\tau}_{\max}=0.6$, $\zeta_1(0) = \zeta_2(0) = 0$, $\hat{W}_1(0) = \hat{W}_2(0) = 0$, $\hat{V}_1(0)$ and $\hat{V}_2(0)$ are randomly taken in the intervals $[-1, 1]$ and $[-0.5, 0.5]$, respectively, $\hat{\delta}_1(0) = \hat{\delta}_2(0) = 0$, the simulation results are shown in Figs. 3–5. From Fig. 3, it can be seen that fairly good tracking performance is obtained.

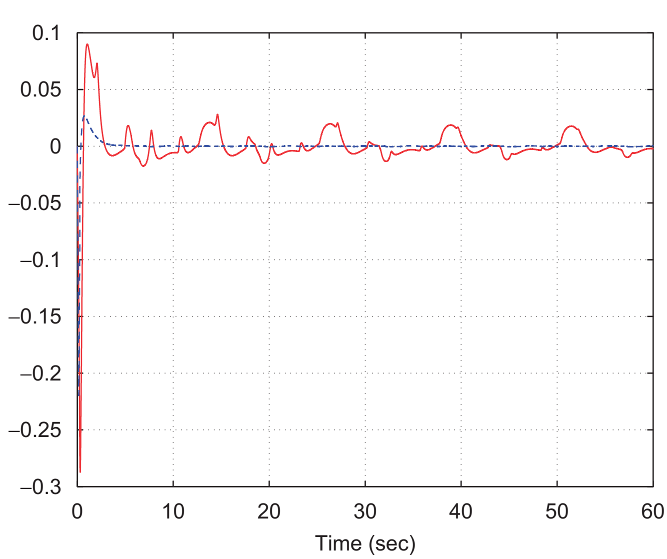

Fig. 3. Tracking errors e_{11} (solid line) and e_{21} (dashed line) with time-varying delays.

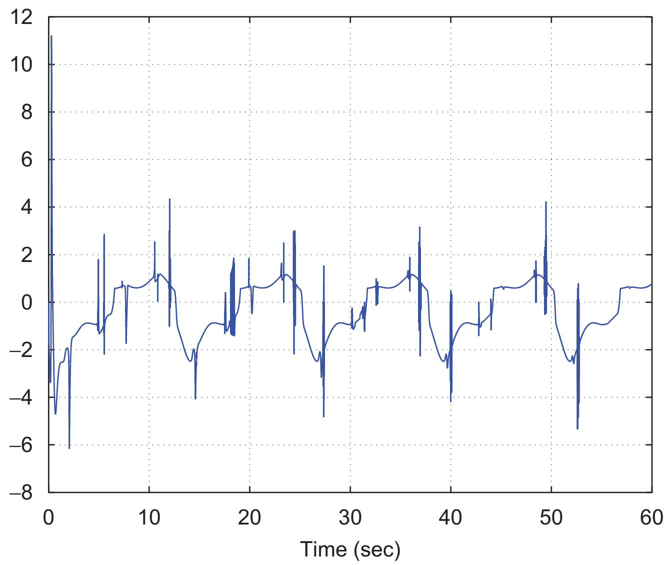

Fig. 4. Control signal v_1 with time-varying delays.

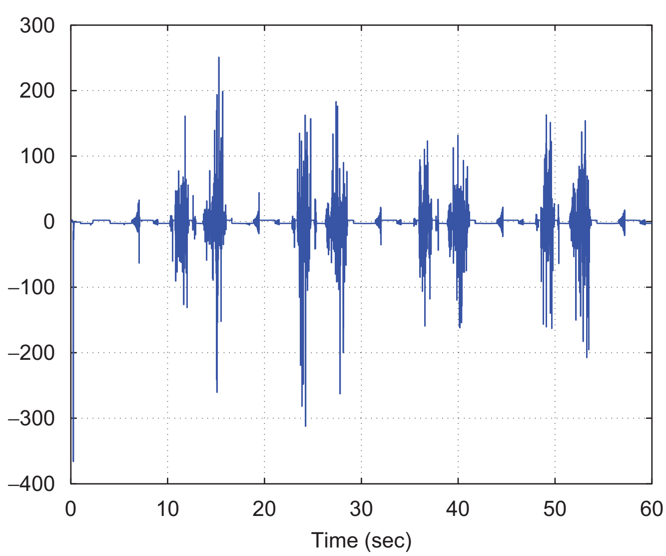

Fig. 5. Control signal v_2 with time-varying delays.

5. Conclusions

A novel adaptive neural control scheme has been presented for a class of uncertain MIMO nonlinear state time-varying delay systems with unknown nonlinear dead-zones and gain signs. The uncertainties from unknown time-varying delays have been compensated for through the use of appropriate Lyapunov–Krasovskii functionals. The controller has been made to be free from singularity problem by utilizing integral Lyapunov function. Based on the intuitive concept and piecewise description of dead-zone and the principle of sliding mode control, the developed controller can guarantee that all signals involved are semi-globally uniformly ultimately bounded.

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