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Selection of optimum maintenance strategies based on a fuzzy analytic hierarchy process

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Abstract

This paper aims to evaluate different maintenance strategies (such as corrective maintenance, time-based preventive maintenance, condition-based maintenance, and predictive maintenance) for different equipment. An optimal maintenance strategy mix is necessary for increasing availability and reliability levels of production facilities without a great increasing of investment. The selection of maintenance strategies is a typical multiple criteria decision-making (MCDM) problem. To deal with the uncertain judgment of decision makers, a fuzzy modification of the analytic hierarchy process (AHP) method is applied as an evaluation tool, where uncertain and imprecise judgments of decision makers are translated into fuzzy numbers. In order to avoid the fuzzy priority calculation and fuzzy ranking procedures in the traditional fuzzy AHP methods, a new fuzzy prioritization method is proposed. This fuzzy prioritization method can derive crisp priorities from a consistent or inconsistent fuzzy judgment matrix by solving an optimization problem with non-linear constraints. A specific example of selection of maintenance strategies in a power plant with the application of the proposed fuzzy AHP method is given, showing that the predictive maintenance strategy is the most suitable for boilers. As demonstrated by this case study, the fuzzy AHP method proposed in this paper is a simple and effective tool for tackling the uncertainty and imprecision associated with MCDM problems, which might prove beneficial for plant maintenance managers to define the optimum maintenance strategy for each piece of equipment.

Keywords: Maintenance strategies; Multiple criteria decision-making; Fuzzy analytic hierarchy process; Prioritization method; Power plant

1. Introduction

Manufacturing firms face great pressure to reduce their production costs continuously. One of the main expenditure items for these firms is maintenance cost which can reach 15–70% of production costs, varying according to the type of industry (Bevilacqua and Braglia, 2000). The amount of money spent on maintenance in a selected group of companies is estimated to be about 600 billion dollars in 1989 (Wireman, 1990, cited by Chan et al., 2005). On the other hand, maintenance plays

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an important role in keeping availability and reliability levels, product quality, and safety requirements. Unfortunately, unlike production and manufacturing problems which have received tremendous interest from researchers and practitioners, maintenance received little attention in the past. This is one of the reasons that results in low maintenance efficiency in industry at present. As indicated by Mobley (2002), one third of all maintenance costs is wasted as the result of unnecessary or improper maintenance activities. Today, research in this area is on the rise. Moreover, the role of maintenance is changing from a "necessary evil" to a "profit contributor" and towards a "partner" of companies to achieve world-class competitiveness (Waeyenbergh and Pintelon, 2002). Therefore, research on maintenance represents an opportunity for making significant contribution by academics.

In the literature, maintenance can be classified into two main types: corrective and preventive (Li et al., 2006; Waeyenbergh and Pintelon, 2004). Corrective maintenance is the maintenance that occurs after systems failure, and it means all actions resulting from failure; preventive maintenance is the maintenance that is performed before systems failure in order to retain equipment in specified condition by providing systematic inspections, detection, and prevention of incipient failure (Wang, 2002). Based on the development of preventive maintenance techniques, three divisions of preventive maintenance are considered in this paper, i.e. time-based preventive maintenance, condition-based maintenance, and predictive maintenance. These maintenance strategies will be introduced in detail in the next section.

Most plants are equipped with various machines, which have different reliability requirements, safety levels, and failure effect. Therefore, it is clear that a proper maintenance program must define different maintenance strategies for different machines. Thus, the reliability and availability of production facilities can be kept in an acceptable level, and the unnecessary investment needed to implement an unsuitable maintenance strategy may be avoided. For example, for the pump with a standby, the corrective/time-based maintenance may be more cost-effective than the condition-based/predictive maintenance strategy in a production environment with a relatively low reliability requirement.

Although the selection of the suitable maintenance strategy for each piece of equipment is important for manufacturing companies, few studies have been done on this problem. Luce (1999), Okumura and Okino (2003) showed the methods to select the most effective maintenance strategy based on different production loss and maintenance costs incurred by different maintenance strategies. Although the calculation theories for the related costs presented by them are reasonable, the money spent on maintenance is only one of the factors that should be taken into account when choosing maintenance strategies in many cases. Azadivar and Shu (1999) presented the method to select a suitable maintenance strategy for each class of systems in a just-in-time environment, exploring 16 characteristic factors that could play a role in maintenance strategy selection. But this method is not applicable to process plants because of the difference between discrete manufacturing plants and process plants. In the report of Bevilacqua and Braglia (2000), the original method for the selection of maintenance strategies in an important Italian oil refinery was given, and the application of the analytic hierarchy process (AHP) for selecting the best maintenance strategy was described. The criteria they considered seem sufficient, but a crisp decision-making method as the traditional AHP is not appropriate because many of the maintenance goals taken as criteria are non-monetary and difficult to be quantified. Al-Najjar and Alsyouf (2003), Sharma et al. (2005) assessed the most popular maintenance strategies using the fuzzy inference theory and fuzzy multiple criteria decision-making (MCDM) evaluation methodology. The application of the fuzzy theory for this problem is a good solution. However, only a few failure causes were considered as the criteria in their studies. In Mechefske and Wang (2003), the authors proposed to evaluate and select the optimum maintenance strategy and condition monitoring technique making use of fuzzy linguistics. The fuzzy methodology based on qualitative verbal assessment inputs is more practical than the formers, because many of the overall maintenance objectives of the organization are intangible. However, the method of Mechefske and Wang (2003) is very subjective to directly assess the importance of each maintenance goal and the capability of each strategy to achieve each maintenance goal. Considering the shortcomings of the existing methods above, it is necessary to develop a new evaluation scheme for maintenance strategies. This scheme should include different aspects of maintenance goals, be able to model uncertainty and imprecise judgments of decision makers (i.e. maintenance managers and engineers), and be easy to use.

While selecting the suitable maintenance strategies for different machines in manufacturing firms, many maintenance goals or comparing criteria must be taken into consideration, e.g. safety and cost. Therefore, the MCDM theory should be used for the maintenance strategy selection. Several MCDM methods have been developed, such as the weighted-sum model (WSM), the weighted-product model (WPM), the TOPSIS method, and the AHP (Triantaphyllou and Lin, 1996). The AHP is one of the most popular MCDM methods. It has the following advantages (Triantaphyllou et al., 1997; Bevilacqua and Braglia, 2000): (1) it is the only known MCDM model that can measure the consistency in the decision makers' judgments; (2) the AHP can help the decision makers to organize the critical aspects of a problem into a hierarchical structure similar to a family tree, making the decision process easy to handle; (3) pairwise comparisons in the AHP are often preferred by the decision makers, allowing them to derive weights of criteria and scores of alternatives from comparison matrices rather than quantify weights/scores directly. Despite its popularity, this MCDM method is often criticized for its inability to adequately deal with the uncertainty and imprecision associated with the mapping of the decision-makers' perception to crisp numbers (Deng, 1999). For example, when constructing comparison judgment matrices, it is difficult for maintenance managers to exactly quantify the statements such as "what is the relative importance of safety in terms of cost, considering the selection of the suitable maintenance strategy for a boiler in a power plant". The answer may be "between three and five times more important", not "three times more important exactly". Consequently, it is desirable to evaluate maintenance strategies based on the fuzzy AHP methods which use fuzzy data.

The aim of this paper is twofold. One is to evaluate maintenance strategies with the application of the fuzzy AHP method, allowing better modeling of the uncertain judgments with the help of triangular fuzzy numbers. The other is to propose a new fuzzy prioritization method, which can derive exact priorities from fuzzy judgment matrices of pairwise comparisons, in order to avoid the fuzzy priorities calculation and fuzzy ranking procedures as in traditional fuzzy AHP methods. The presented modification of the fuzzy AHP might be beneficial for plant managers to select maintenance strategies as well as other MCDM problems.

The rest of the paper is organized as follows. Section 2 describes the possible alternative maintenance strategies in this study. In Section 3, the comparing criteria for the selection of maintenance strategies are presented. Section 4 introduces the basic concept of the AHP. The new fuzzy prioritization method is given in Section 5. Section 6 describes the application of the proposed evaluation method for the selection of maintenance strategies in a thermal power plant, and conclusion finally.

2. Alternative maintenance strategies

Four alternative maintenance strategies considered in this paper are introduced as following:

(1) *Corrective maintenance*: This alternative maintenance strategy is also named as fire-fighting maintenance, failure based maintenance or breakdown maintenance. When the corrective maintenance strategy is applied, maintenance is not implemented until failure occurs (Swanson, 2001). Corrective maintenance is the original maintenance strategy appeared in industry (Waeyenbergh and Pintelon, 2002; Mechefske and Wang, 2003). It is considered as a feasible strategy in the cases where profit margins are large (Sharma et al., 2005). However, such a fire-fighting mode of maintenance often causes serious damage of related facilities, personnel and environment. Furthermore, increasing global competition and small profit margins have forced maintenance managers to apply more effective and reliable maintenance strategies.

(2) *Time-based preventive maintenance*: According to reliability characteristics of equipment, maintenance is planned and performed periodically to reduce frequent and sudden failure. This maintenance strategy is called time-based preventive maintenance, where the term "time" may refer to calendar time, operating time or age. Time-based preventive maintenance is applied widely in industry. For performing time-based preventive maintenance, a decision support system is needed, and it is often difficult to define the most effective maintenance intervals because of lacking sufficient historical data (Mann et al., 1995). In many cases when time-based maintenance strategies are used,

most machines are maintained with a significant amount of useful life remaining (Mechefske and Wang, 2003). This often leads to unnecessary maintenance, even deterioration of machines if incorrect maintenance is implemented.

(3) *Condition-based maintenance*: Maintenance decision is made depending on the measured data from a set of sensors system when using the condition-based maintenance strategy. To date a number of monitoring techniques are already available, such as vibration monitoring, lubricating analysis, and ultrasonic testing. The monitored data of equipment parameters could tell engineers whether the situation is normal, allowing the maintenance staff to implement necessary maintenance before failure occurs. This maintenance strategy is often designed for rotating and reciprocating machines, e.g. turbines, centrifugal pumps and compressors. But limitations and deficiency in data coverage and quality reduce the effectiveness and accuracy of the condition-based maintenance strategy (Al-Najjar and Alsyouf, 2003).

(4) *Predictive maintenance*: In the literature, predictive maintenance often refers to the same maintenance strategy with condition-based maintenance (Sharma et al., 2005; Mobley, 2002). In this paper, considering the recent development of fault prognosis techniques (Bengtsson, 2004; Byington et al., 2002), predictive maintenance is used to represent the maintenance strategy that is able to forecast the temporary trend of performance degradation and predict faults of machines by analyzing the monitored parameters data. Fault prognostics is a young technique employed by maintenance management, which gives maintenance engineers the possibility to plan maintenance based on the time of future failure and coincidence maintenance activities with production plans, customers' orders and personnel availability. Recently, the intelligent maintenance system was described by Djurdjanovic et al. (2003), focusing on fault prognostic techniques and aiming to achieve near-zero-downtime performance of equipment.

It is worth mentioning that equipment failure and corrective actions of maintenance cannot be avoided completely when the preventive maintenance strategies (including the time-based, condition-based, and predictive maintenance) are applied. This is due to the stochastic nature of equipment failure. However, generally speaking, the amount of equipment failure can be reduced if the preventive maintenance strategies are correctly selected, especially the condition-based/predictive maintenance.

3. Comparing criteria

When different maintenance strategies are evaluated for different machines, the manufacturing firms must set maintenance goals taken as comparing criteria first. Different manufacturing companies may have different maintenance goals. But in most cases, these goals can be divided into four aspects analyzed as follows:

(1) *Safety*: Safety levels required are often high in many manufacturing factories, especially in chemical industry and power plants. The relevant factors describing the Safety are:
   (a) *Personnel*: The failure of many machines can lead to serious damage of personnel on site, such as high pressure vessels in chemical plants.
   (b) *Facilities*: For example, the sudden breakdown of a water-feeding pump can result in serious damage of the corresponding boiler in a power plant.
   (c) *Environment*: The failure of equipment with poisonous liquid or gas can damage the environment.

(2) *Cost*: Different maintenance strategies have different expenditure of hardware, software, and personnel training.
   (a) *Hardware*: For condition-based maintenance and predictive maintenance, a number of sensors and some computers are indispensable.
   (b) *Software*: Software is needed for analyzing measured parameters data when using condition-based maintenance and predictive maintenance strategies.
   (c) *Personnel training*: Only after sufficient training can maintenance staff make full use of the related tools and techniques, and reach the maintenance goals.

(3) *Added-value*: A good maintenance program can induce added-value, including low inventories of spare parts, small production loss, and quick fault identification.
   (a) *Spare parts inventories*: Generally, corrective maintenance need more spare parts than

other maintenance strategies. Spare parts for some machines are really expensive.
   (b) *Production loss*: The failure of more important machines in the production line often leads to higher production loss cost. Selecting a suitable maintenance strategy for such machines may reduce production loss.
   (c) *Fault identification*: Fault diagnostic and prognostic techniques involved in the condition-based and predictive maintenance strategies aim to quickly tell maintenance engineers where and why fault occurs. As a result, the maintenance time can be reduced, and the availability of the production system may be improved.

(4) *Feasibility*: The feasibility of maintenance strategies is divided into acceptance by labors and technique reliability.
   (a) *Acceptance by labors*: Managers and maintenance staff prefer the maintenance strategies that are easy to implement and understand.
   (b) *Technique reliability*: Still under development, condition-based maintenance and predictive maintenance may be inapplicable for some complicated production facilities.

4. Fuzzy AHP

The AHP was developed first by Satty (Zuo, 1991). It is a popular tool for MCDM by structuring a complicated decision problem hierarchically at several different levels. Its main steps include:

(1) *Organizing problem hierarchically*: The problem is structured as a family tree in this step. At the highest level is the overall goal of this decision-making problem, and the alternatives are at the lowest level. Between them are criteria and sub-criteria.
(2) *Development of judgment matrices by pairwise comparisons*: The judgment matrices of criteria or alternatives can be defined from the reciprocal comparisons of criteria at the same level or all possible alternatives. Pairwise comparisons are based on a standardized evaluation schemes (1 = equal importance; 3 = weak importance; 5 = strong importance; 7 = demonstrated importance; 9 = absolute importance).
(3) *Calculating local priorities from judgment matrices*: Several methods for deriving local priorities (i.e. the local weights of criteria and the local scores of alternatives) from judgment matrices have been developed, such as the eigenvector method (EVM), the logarithmic least squares method (LLSM), the weighted least squares method (WLSM), the goal programming method (GPM) and the fuzzy programming method (FPM), as summarized by Mikhailov (2000). Consistency check should be implemented for each judgment matrix.
(4) *Alternatives ranking*: The final step is to obtain global priorities (including global weights and global scores) by aggregating all local priorities with the application of a simple weighted sum. Then the final ranking of the alternatives are determined on the basis of these global priorities.

The above process of the AHP method is similar to the process of human thinking, and turns the complex decision-making process into simple comparisons and rankings. However, decision makers often face uncertain and fuzzy cases when considering the relative importance of one criterion or alternative in terms of another. Therefore, it is difficult to determine the ratios based on the standard scheme in the second step above. For this reason, the fuzzy AHP was proposed, in which the uncertain comparisons ratios are expressed as fuzzy sets or fuzzy numbers, such as "between three and five times less important" and "about three times more important". The triangular fuzzy number, because of its popularity, is used to represent the fuzzy relative importance in this paper. The membership function of triangular fuzzy numbers can be described as:

$$\mu_{\tilde{N}}(x) = \begin{cases} \dfrac{x-l}{m-l}, & l \leqslant x \leqslant m \\ \dfrac{u-x}{u-m}, & m < x \leqslant u, \\ 0 & \text{otherwise,} \end{cases} \tag{1}$$

where l, m, and u are also considered as the lower bound, the mean bound, and the upper bound, respectively. The triangular fuzzy number $\tilde{N}$ is often represented as (l, m, u).

After pairwise comparisons are finished at a level, a fuzzy reciprocal judgment matrix $\tilde{A}$ can be established as

$$\widetilde{A} = \{\widetilde{a}_{ij}\} = \begin{pmatrix} \widetilde{a}_{11} & \widetilde{a}_{12} & \dots & \widetilde{a}_{1n} \\ \widetilde{a}_{21} & \widetilde{a}_{22} & \dots & \widetilde{a}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \widetilde{a}_{n1} & \widetilde{a}_{n2} & \dots & \widetilde{a}_{nn} \end{pmatrix}, \quad (2)$$

where n is the number of the related elements at this level, and $\widetilde{a}_{ij} = 1/\widetilde{a}_{ji}$.

After constructing $\widetilde{A}$, fuzzy priorities $\widetilde{w}_i$, $i = 1, 2, \dots, n$, should be calculated in the traditional fuzzy AHP methods. Many fuzzy prioritization approaches have been developed, such as the method based on the fuzzy modification of the LLSM (Boender et al., 1989), the fuzzy geometry mean method (Buckley, 1985), the direct fuzzification of the $\lambda_{\max}$ method of Satty (Csutora and Buckley, 2001), and the fuzzy least square method (Xu, 2000). In these methods, global priorities expressed as fuzzy numbers can be determined by aggregating fuzzy local priorities. However, as pointed out by Mikhailov (2003), the global fuzzy priorities often have large supports and overlap a wide range. After the normalization procedure of the fuzzy global scores, the unreasonable conditions where the normalized upper value < the normalized mean value < the normalized lower value may occur. Furthermore, to compare the global fuzzy scores, a fuzzy ranking procedure must be included in the traditional fuzzy AHP methods. But different ranking procedures for fuzzy numbers often give different ranking conclusions (Li, 2002).

To overcome the shortcomings of the fuzzy prioritization methods above, two new approaches that can derive crisp priorities from fuzzy pairwise comparison judgments are proposed (Mikhailov, 2003; Mikhailov and Tsvetinov, 2004). One is based on α-cut decomposition of the fuzzy numbers into interval comparisons. In this method, the fuzzy preference programming (FPP) method (Mikhailov, 2000) transforming the prioritization procedure into a fuzzy linear programming problem is used to derive optimized exact priorities, and eventually an aggregation of the optimal priorities derived at the different α-levels is needed for obtaining overall crisp scores of the prioritization elements. These steps make this method a little complicated. The other is a non-linear modification of the FPP strategy without applying α-cut transformations. This idea, deriving crisp priorities from fuzzy judgment matrices, shows a new way to deal with the prioritization problem from fuzzy reciprocal comparisons in the fuzzy AHP. A new and simple prioritization method, which can also derive exact priorities from fuzzy pairwise comparisons, is described in the next section.

5. Fuzzy prioritization method

Suppose that a fuzzy judgment matrix is constructed as Eq. (2) in a prioritization problem, where n elements are taken into account. Among this judgment matrix $\widetilde{A}$, the triangular fuzzy number $\widetilde{a}_{ij}$ is expressed as (l_{ij}, m_{ij}, u_{ij}), i and $j = 1, 2, \dots, n$, where l_{ij}, m_{ij}, and u_{ij} are the lower bound, the mean bound, and the upper bound of this fuzzy triangular set, respectively. Furthermore, we assume that $l_{ij} < m_{ij} < u_{ij}$ when $i \neq j$. If $i = j$, then $\widetilde{a}_{ij} = \widetilde{a}_{ii} = (1, 1, 1)$. Therefore, an exact priority vector $w = (w_1, w_2, \dots, w_n)^{\mathrm{T}}$ derived from $\widetilde{A}$ must satisfy the fuzzy inequalities:

$$l_{ij} \,\widetilde{\leqslant}\, \frac{w_i}{w_j} \,\widetilde{\leqslant}\, m_{ij}. \quad (3)$$

where $w_i > 0$, $w_j > 0$, $i \neq j$, and the symbol $\widetilde{\leqslant}$ means "fuzzy less or equal to".

To measure the degree of satisfaction for different crisp ratios w_i/w_j with regard to the double side inequality (3), a function can be defined as:

$$\mu_{ij}\left(\frac{w_i}{w_j}\right) = \begin{cases} \dfrac{m_{ij} - (w_i/w_j)}{m_{ij} - l_{ij}}, & 0 < \dfrac{w_i}{w_j} \leqslant m_{ij} \\ \dfrac{(w_i/w_j) - m_{ij}}{u_{ij} - m_{ij}}, & \dfrac{w_i}{w_j} > m_{ij}, \end{cases} \quad (4)$$

where $i \neq j$. Being different from the membership function (1) of triangular fuzzy numbers, the function value of $\mu_{ij}(w_i/w_j)$ may be larger than one, and is linearly decreasing over the interval $(0, m_{ij}]$ and linearly increasing over the interval $[m_{ij}, \infty)$, as shown in Fig. 1. The less value of $\mu_{ij}(w_i/w_j)$ indicates that the exact ratio w_i/w_j is more acceptable.

To find the solution of the priority vector $(w_1, w_2, \dots, w_n)^{\mathrm{T}}$, the idea is that all exact ratios w_i/w_j should satisfy $n(n-1)$ fuzzy comparison judgments (l_{ij}, m_{ij}, u_{ij}) as possible as they can, i and $j = 1, 2, \dots, n$, $i \neq j$. Therefore, in this study, the crisp priorities assessment is formulated as a

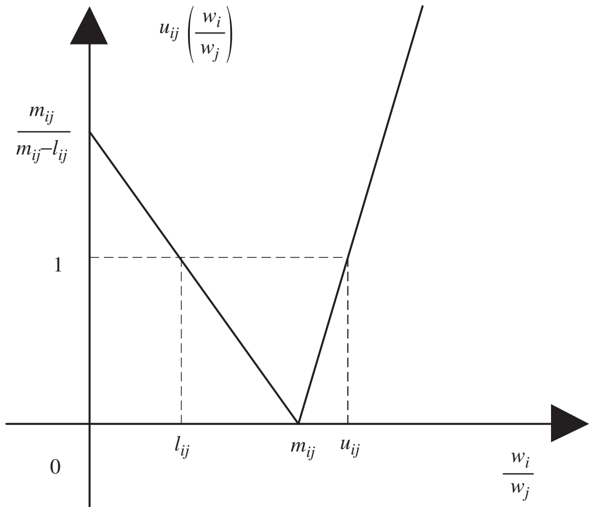

Fig. 1. Function for measuring the satisfaction degree of w_i/w_j.

constrained optimization problem:

$$\begin{aligned}\min J(w_1, w_2, \ldots, w_n) &= \min \sum_{i=1}^{n} \sum_{j=1}^{n} \left[\mu_{ij}^{p}\left(\frac{w_i}{w_j}\right)\right] \\ &= \min \sum_{i=1}^{n} \sum_{j=1}^{n} \left[\delta\left(m_{ij} - \frac{w_i}{w_j}\right)\left(\frac{m_{ij} - (w_i/w_j)}{m_{ij} - l_{ij}}\right)^{p} \right. \\ &\quad \left. + \delta\left(\frac{w_i}{w_j} - m_{ij}\right)\left(\frac{(w_i/w_j) - m_{ij}}{u_{ij} - m_{ij}}\right)^{p}\right]. \qquad (5)\end{aligned}$$

subject to

$$\sum_{k=1}^{n} w_k = 1, \quad w_k > 0, \ k = 1, 2, \ldots, n.$$

where $i \neq j$, $p \in \mathbb{N}$, and

$$\delta(x) = \begin{cases} 0, & x < 0, \\ 1, & x \geqslant 0. \end{cases}$$

The power index p is fixed, and chosen by decision makers in a specific MCDM problem. A larger p is suggested, e.g. 10, as illustrated briefly in the next section.

The function $J(w_1, w_2, \ldots, w_n)$ is non-differentiable. General algorithms for function optimization, limited to convex regular functions, cannot be applied to this optimization problem. Therefore, genetic algorithms, which have great ability to solve difficult optimization problems with discontinuous, multi-modal or non-differentiable objective functions, are chosen in this paper. A toolbox GOAT of genetic algorithms provided by Houck et al. (1995) is utilized in the next section. Because the optimization problem above has non-linear constraints, the penalty techniques (Gen and Cheng, 1996) are combined when employing genetic algorithms for the optimal solution.

In some cases, decision-makers are unable or unwilling to give all pairwise comparison judgments of n elements. However, provided that the known fuzzy set of pairwise comparisons involves n elements, such as $F = \{\tilde{a}_{ij}\} = \{\tilde{a}_{12}, \tilde{a}_{13}, \ldots, \tilde{a}_{1n}\}$ or $\{\tilde{a}_{21}, \tilde{a}_{31}, \ldots, \tilde{a}_{n1}\}$, the solution of priority vector $(w_1, w_2, \ldots, w_n)^{\mathrm{T}}$ will be still able to be derived based on the optimization problem above. Thus, the proposed method can obtain priorities from an incomplete comparison judgment set, which is an interesting advantage comparing with the traditional fuzzy AHP methods.

In order to measure the consistency degree of the fuzzy comparison judgment matrix $\tilde{A}$ as Eq. (2), an index γ can be defined after the optimal crisp priority vector $(w_1^*, w_2^*, \ldots, w_n^*)^{\mathrm{T}}$ is obtained:

$$\gamma = \exp\left\{-\max_{ij}\left\{\mu_{ij}\left(\frac{w_i^*}{w_j^*}\right)\middle| i,\ j = 1, 2, \ldots, n,\ i \neq j\right\}\right\}, \qquad (6)$$

where $\mu_{ij}(w_i^*/w_j^*)$ is the function of (4). The value of γ satisfies $0 < \gamma \leqslant 1$ always. If it is larger than $e^{-1} = 0.3679$, all exact ratios satisfy the crisp inequalities $l_{ij} \leqslant w_i^*/w_j^* \leqslant m_{ij}$, i and $j = 1, 2, \ldots, n$, $i \neq j$, and the corresponding fuzzy judgment matrix has good consistency. $\gamma = 1$ indicates that the fuzzy judgment matrix is completely consistent. In conclusion, the fuzzy judgment matrix with a larger γ value is more consistent.

6. Case study

The Hangzhou Pro-Energy Heat and Power Co., Ltd. (HPEHP) in China is a small thermal power plant with an installed capacity of $2 \times 15 + 1 \times 7.5$ MW. To ensure supplying its users with electricity and heat continuously, the maintenance work for more than 70 pieces of equipment (pumps, fans, and boilers, etc.) is highlighted by this thermal power plant. But the managers are not satisfied with the effect of maintenance activities that depend on corrective maintenance and time-based preventive maintenance mainly, and want to improve their maintenance program without too much increase in investment. Therefore, it is more preferable for them to choose the best mix of maintenance strategies than to make use of the most advanced maintenance strategy for all production facilities. In this section, the revised fuzzy AHP with the proposed prioritization method is applied to the selection of maintenance strategies in HPEHP.

By interviewing the maintenance staff and managers, it is concluded that the criteria in Section 3 can be accepted. Therefore, the AHP hierarchy scheme is constructed correspondingly, shown in Fig. 2. Next, the selection of the optimum maintenance strategy for boilers in HPEHP is presented as an example.

In the following steps of the decision-making process, the fuzzy comparison judgment matrices are decided according to the suggestions of the maintenance staff. The imprecise and uncertain assessments of them are translated into corresponding triangular fuzzy numbers according to Table 1.

The fuzzy comparison judgments of the four main criteria with respect to the overall goal are shown in Table 2. Safety is regarded as the most important criterion, evaluated as being between three and five times more important than Cost, about two times more important than Added-value, and about three times more important than Feasibility. Utilizing the fuzzy prioritization method of Section 5, the exact weights of main criteria are obtained as

$w_1 = 0.4487$ (Safety);
$w_2 = 0.1044$ (Cost);
$w_3 = 0.2783$ (Added-value);
$w_4 = 0.1686$ (Feasibility).

The exact criteria weights above are the results when the power index p in Eq. (5) is chosen as 10. Considering the comparison judgments matrix in Table 2 (as well as other judgment matrices in the following), different values of p lead to different results of crisp weights obtained by solving the optimization problem in Eq. (5). However, by numerical experiments, it is found that the variation of the derived weights is small when p is different. Especially when p is larger than 10, the results are very close. Therefore, the power index p is set to 10 in this case study. Considering the purpose of this

Table 1
Fuzzy judgment scores in the fuzzy analytic hierarchy process

Uncertain judgment	Fuzzy score
About equal	$(1/2, 1, 2)$
About x times more important[a]	$(x-1, x, x+1)$
About x times less important	$(1/(x+1), 1/x, 1/(x-1))$
Between y and z times more important[b]	$(y, (y+z)/2, z)$
Between y and z times less important	$(1/z, 2/(y+z), 1/y)$

[a] $x = 2, 3, \ldots, 9$.
[b] $y, z = 1, 2, \ldots, 9,\ y < z$.

Table 2
Fuzzy comparison matrices at the first level

Goal	Safety	Cost	Added-value	Feasibility
Safety	$(1, 1, 1)$	$(3, 4, 5)$	$(1, 2, 3)$	$(2, 3, 4)$
Cost	$(1/5, 1/4, 1/3)$	$(1, 1, 1)$	$(1/4, 1/3, 1/2)$	$(1/3, 1/2, 1)$
Added-value	$(1/3, 1/2, 1)$	$(2, 3, 4)$	$(1, 1, 1)$	$(1, 2, 3)$
Feasibility	$(1/4, 1/3, 1/2)$	$(1, 2, 3)$	$(1/3, 1/2, 1)$	$(1, 1, 1)$

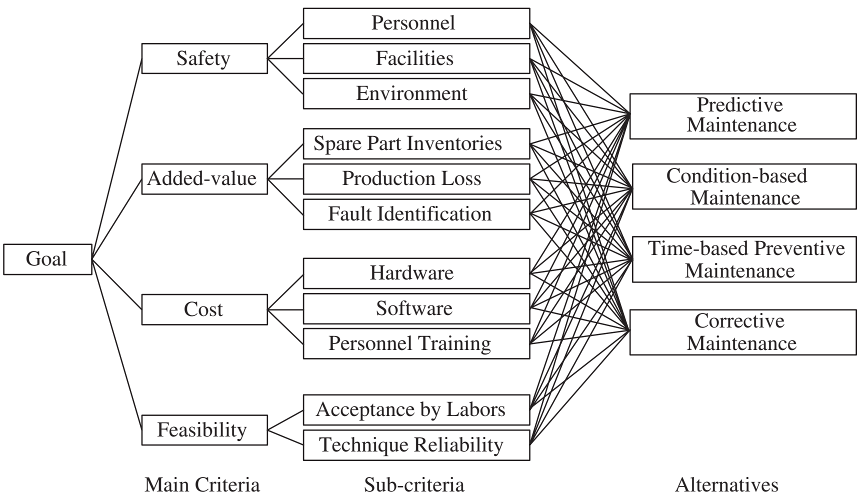

Fig. 2. Hierarchy structure of the fuzzy analytic hierarchy process.

paper and the lack of space, the specific analysis associated with p is not included.

The exact ratios of the obtained weights of four main criteria are given in Table 3, and it can be found that all fuzzy judgments in Table 2 are satisfied approximately. The consistency index γ of the fuzzy judgment matrix shown in Table 2 is 0.6786, indicating good consistency.

Table 3
Exact ratios of weights of main criteria

	w_1	w_2	w_3	w_4
w_1	1	4.2979	1.6123	2.6613
w_2	0.2327	1	0.3751	0.6192
w_3	0.6202	2.6657	1	1.6507
w_4	0.3758	1.6149	0.6058	1

Table 4
Results from fuzzy judgment matrices of sub-criteria

Sub-criteria	Local weights
w_{11} (Personnel)	0.6458
w_{12} (Facilities)	0.2285
w_{13} (Environment)	0.1258
$\gamma_1 = 0.8326$	
w_{21} (Hardware)	0.3141
w_{22} (Software)	0.4808
w_{23} (Personnel training)	0.2052
$\gamma_2 = 0.6255$	
w_{31} (Spare parts inventories)	0.0912
w_{32} (Production loss)	0.6983
w_{33} (Fault identification)	0.2105
$\gamma_3 = 0.6702$	
w_{41} (Acceptance by labors)	0.1250
w_{42} (Technique reliability)	0.8750
$\gamma_4 = 1$	

All sub-criteria are compared at the second level in terms of corresponding main criteria, and the related fuzzy comparison matrices are constructed (the detailed data are presented in Appendix A). By using the same prioritization method, the local weights of sub-criteria are calculated as the results of Table 4.

Four comparison matrices at the second level have good consistency because all γ indexes are larger than $e^{-1} = 0.3679$. It should be noted that the two-dimensional fuzzy comparison matrices are always completely consistent, and their γ must be one (as γ_4 in Table 4).

Four alternative maintenance strategies are also compared in terms of the sub-criteria at the second level. The results of fuzzy judgment matrices can be found in Appendix A. Similarly, the local scores of the alternative maintenance strategies with regard to all sub-criteria are obtained (given in Table 5, and corrective maintenance, time-based preventive maintenance, condition-based maintenance, and predictive maintenance are abbreviated as CM, TM, CBM, and PM, respectively). By multiplying the local weights of sub-criteria in Table 4 by the weights of main criteria, the global weights of all sub-criteria are calculated as shown in the sixth column of Table 5. It is concluded from the global scores that the most suitable maintenance strategy for boilers is predictive maintenance. From the last column of Table 5, the fuzzy judgment matrices of four alternatives in terms of environment, hardware, and acceptance by labors are the most consistent three comparison matrices. On the contrary, the judgment matrices with respect to

Table 5
Results of the fuzzy analytic hierarchy process approach

	CM	TM	CBM	PM	Global weights	γ
Personnel	0.0609	0.1531	0.2535	0.5326	0.2898	0.4070
Facilities	0.0613	0.1475	0.2551	0.5361	0.1025	0.4072
Environment	0.0650	0.1577	0.2267	0.5506	0.0564	0.5645
Hardware	0.4507	0.4146	0.0808	0.0539	0.0328	0.6068
Software	0.4849	0.3707	0.0872	0.0572	0.0502	0.5006
Personnel training	0.5195	0.2857	0.1292	0.0656	0.0214	0.3506
Spare parts inventories	0.0580	0.1926	0.2418	0.5076	0.0254	0.4061
Production loss	0.0479	0.1726	0.3283	0.4511	0.1943	0.2037
Fault identification	0.0590	0.0590	0.3823	0.4997	0.0586	0.5001
Acceptance by labors	0.4480	0.2297	0.1681	0.1042	0.0211	0.6713
Technique reliability	0.4977	0.3354	0.1169	0.0501	0.1475	0.1177
Global scores	0.1749	0.2029	0.2353	0.3858		

Table 6
Results of the standard analytic hierarchy process approach

	CM	TM	CBM	PM	Global weights	CR
Personnel	0.0552	0.1540	0.2500	0.5408	0.3030	0.0218
Facilities	0.0580	0.1420	0.2319	0.5681	0.1073	0.0154
Environment	0.0543	0.1394	0.2423	0.5640	0.0570	0.0178
Hardware	0.4428	0.4215	0.0856	0.0500	0.0297	0.0058
Software	0.5310	0.3266	0.0904	0.0521	0.0471	0.0136
Personnel training	0.5275	0.2865	0.1248	0.0612	0.0187	0.0306
Spare parts inventories	0.0513	0.1649	0.2483	0.5356	0.0265	0.0297
Production loss	0.0436	0.1434	0.3154	0.4977	0.1924	0.0450
Fault identification	0.0573	0.0573	0.3519	0.5334	0.0583	0.0148
Acceptance by labors	0.4673	0.2772	0.1601	0.0954	0.0200	0.0116
Technique reliability	0.5361	0.3007	0.1184	0.0448	0.1401	0.0432
Global scores	0.1716	0.1861	0.2310	0.4113		

personnel training, production loss, and technique reliability are the most inconsistent three ones.

To make sure that the result of the proposed prioritization method can be accepted, the standard AHP method has been also applied to solving the same problem. According to the means of the related fuzzy comparison judgments, the exact comparison matrices are constructed. The weights of criteria are calculated using the eigenvector prioritization method. The result of the standard AHP is given in Table 6. From this table, we can find that the final ranking of four alternatives are the same as the results of the fuzzy AHP, and the global scores are similar. However, compared with the standard AHP, the proposed fuzzy AHP method allows better modeling of the uncertainty and imprecision of decision makers' judgments. For example, the maintenance staff and managers evaluate the criterion Safety as being between three and five times more important than Cost. This uncertain judgment can be represented as the triangular fuzzy number (3,4,5) in the fuzzy AHP, while the standard crisp AHP fails to deal with it.

The seventh column of Table 6 gives the consistency index CR of eleven pairwise comparison matrices of four alternatives with regard to eleven sub-criteria in the standard AHP. If CR is larger than 0.1, the crisp judgment matrix is considered as inconsistent and should be adjusted. The detailed description of CR can be found in Zuo (1991). It is known that a less CR value indicates better consistency. Therefore it can be found from Table 6 that personnel training, production loss, and technique reliability have worse consistency, and hardware, software, and acceptance by labors have better consistency. This conclusion is close to the previous consistency analysis on the results in Table 5, which proves the applicability of γ in Eq. (6) as the consistency index in the proposed fuzzy AHP. However, further research is needed to determine when the fuzzy comparison judgment matrix should be considered inconsistent and unacceptable, and must be adjusted, according to the γ value.

7. Conclusion

In this paper, the selection of maintenance strategies in manufacturing firms is studied. An optimal maintenance strategy mix can improve availability and reliability levels of plants equipment, and reduce unnecessary investment in maintenance. The evaluation of maintenance strategies for each piece of equipment is a multiple criteria decision-making (MCDM) problem. Considering the imprecise judgments of decision makers, the fuzzy AHP is used for the evaluation of different maintenance strategies. The fuzzy AHP models the uncertainty with triangular fuzzy numbers. A new and simple fuzzy prioritization method is proposed to derive crisp priorities from fuzzy comparison judgment matrices, based on an optimization problem with non-linear constraints. The case study shows that the revised fuzzy AHP method is applicable as an evaluation technique for maintenance strategies, and is useful for other similar MCDM problems.

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Appendix A. Detailed data of fuzzy comparison matrices

See Tables 7–9.

Table 7
Fuzzy comparison matrices at the second level

Safety	Personnel	Facilities	Environment
Personnel	(1, 1, 1)	(2, 3, 4)	(4, 5, 6)
Facilities	(1/4, 1/3, 1/2)	(1, 1, 1)	(1, 2, 3)
Environment	(1/6, 1/5, 1/4)	(1/3, 1/2, 1)	(1, 1, 1)
Cost	Hardware	Software	Personnel training
Hardware	(1, 1, 1)	(1/3, 1/2, 1)	(1, 2, 3)
Software	(1, 2, 3)	(1, 1, 1)	(1, 2, 3)
Personnel training	(1/3, 1/2, 1)	(1/3, 1/2, 1)	(1, 1, 1)
Added-value	Spare part inventories	Production loss	Fault identification
Spare part inventories	(1, 1, 1)	(1/9, 1/8, 1/7)	(1/3, 1/2, 1)
Production loss	(7, 8, 9)	(1, 1, 1)	(2, 3, 4)
Fault identification	(1, 2, 3)	(1/4, 1/3, 1/2)	(1, 1, 1)
Feasibility	Acceptance by labors	Technique reliability	
Acceptance by labors	(1, 1, 1)	(1/8, 1/7, 1/6)	
Technique reliability	(6, 7, 8)	(1, 1, 1)	

Table 8
Fuzzy comparison matrices of four alternatives

	CM	TM	CBM	PM
Personnel				
CM	(1, 1, 1)	(1/4, 1/3, 1/2)	(1/6, 1/5, 1/4)	(1/9, 1/8, 1/7)
TM	(2, 3, 4)	(1, 1, 1)	(1/3, 1/2, 1)	(1/4, 1/3, 1/2)
CBM	(4, 5, 6)	(1, 2, 3)	(1, 1, 1)	(1/4, 1/3, 1/2)
PM	(7, 8, 9)	(2, 3, 4)	(2, 3, 4)	(1, 1, 1)
Facilities				
CM	(1, 1, 1)	(1/4, 1/3, 1/2)	(1/6, 1/5, 1/4)	(1/9, 1/8, 1/7)
TM	(2, 3, 4)	(1, 1, 1)	(1/3, 1/2, 1)	(1/5, 1/4, 1/3)
CBM	(4, 5, 6)	(1, 2, 3)	(1, 1, 1)	(1/4, 1/3, 1/2)
PM	(7, 8, 9)	(3, 4, 5)	(2, 3, 4)	(1, 1, 1)
Environment				
CM	(1, 1, 1)	(1/4, 1/3, 1/2)	(1/5, 1/4, 1/3)	(1/9, 1/8, 1/7)
TM	(2, 3, 4)	(1, 1, 1)	(1/3, 1/2, 1)	(1/5, 1/4, 1/3)
CBM	(3, 4, 5)	(1, 2, 3)	(1, 1, 1)	(1/4, 1/3, 1/2)
PM	(7, 8, 9)	(3, 4, 5)	(2, 3, 4)	(1, 1, 1)
Hardware				
CM	(1, 1, 1)	(1/2, 1, 2)	(5, 6, 7)	(7, 8, 9)
TM	(1/2, 1, 2)	(1, 1, 1)	(4, 5, 6)	(7, 8, 9)
CBM	(1/7, 1/6, 1/5)	(1/6, 1/5, 1/4)	(1, 1, 1)	(1, 2, 3)
PM	(1/9, 1/8, 1/7)	(1/9, 1/8, 1/7)	(1/3, 1/2, 1)	(1, 1, 1)
Software				
CM	(1, 1, 1)	(1, 2, 3)	(5, 6, 7)	(7, 8, 9)
TM	(1/3, 1/2, 1)	(1, 1, 1)	(3, 4, 5)	(6, 7, 8)
CBM	(1/7, 1/6, 1/5)	(1/5, 1/4, 1/3)	(1, 1, 1)	(1, 2, 3)
PM	(1/9, 1/8, 1/7)	(1/8, 1/7, 1/6)	(1/3, 1/2, 1)	(1, 1, 1)

Table 9
Fuzzy comparison matrice of four alternatives (Table 8 continued)

	CM	TM	CBM	PM
Personnel training				
CM	(1, 1, 1)	(1, 2, 3)	(4, 5, 6)	(7, 8, 9)
TM	(1/3, 1/2, 1)	(1, 1, 1)	(2, 3, 4)	(3, 4, 5)
CBM	(1/6, 1/5, 1/4)	(1/4, 1/3, 1/2)	(1, 1, 1)	(2, 3, 4)
PM	(1/9, 1/8, 1/7)	(1/5, 1/4, 1/3)	(1/4, 1/3, 1/2)	(1, 1, 1)
Spare part inventories				
CM	(1, 1, 1)	(1/5, 1/4, 1/3)	(1/6, 1/5, 1/4)	(1/9, 1/8, 1/7)
TM	(3, 4, 5)	(1, 1, 1)	(1/3, 1/2, 1)	(1/4, 1/3, 1/2)
CBM	(4, 5, 6)	(1, 2, 3)	(1, 1, 1)	(1/4, 1/3, 1/2)
PM	(7, 8, 9)	(2, 3, 4)	(2, 3, 4)	(1, 1, 1)
Production loss				
CM	(1, 1, 1)	(1/6, 1/5, 1/4)	(1/8, 1/7, 1/6)	(1/9, 1/8, 1/7)
TM	(4, 5, 6)	(1, 1, 1)	(1/4, 1/3, 1/2)	(1/5, 1/4, 1/3)
CBM	(6, 7, 8)	(2, 3, 4)	(1, 1, 1)	(1/3, 1/2, 1)
PM	(7, 8, 9)	(3, 4, 5)	(1, 2, 3)	(1, 1, 1)
Fault identification				
CM	(1, 1, 1)	(1/2, 1, 2)	(1/8, 1/7, 1/6)	(1/9, 1/8, 1/7)
TM	(1/2, 1, 2)	(1, 1, 1)	(1/8, 1/7, 1/6)	(1/9, 1/8, 1/7)
CBM	(6, 7, 8)	(6, 7, 8)	(1, 1, 1)	(1/3, 1/2, 1)
PM	(7, 8, 9)	(7, 8, 9)	(1, 2, 3)	(1, 1, 1)
Acceptance by labors				
CM	(1, 1, 1)	(1, 2, 3)	(2, 3, 4)	(3, 4, 5)
TM	(1/3, 1/2, 1)	(1, 1, 1)	(1, 2, 3)	(2, 3, 4)
CBM	(1/4, 1/3, 1/2)	(1/3, 1/2, 1)	(1, 1, 1)	(1, 2, 3)
PM	(1/5, 1/4, 1/3)	(1/4, 1/3, 1/2)	(1/3, 1/2, 1)	(1, 1, 1)
Technique reliability				
CM	(1, 1, 1)	(1, 2, 3)	(5, 6, 7)	(7, 8, 9)
TM	(1/3, 1/2, 1)	(1, 1, 1)	(2, 3, 4)	(6, 7, 8)
CBM	(1/7, 1/6, 1/5)	(1/4, 1/3, 1/2)	(1, 1, 1)	(3, 4, 5)
PM	(1/9, 1/8, 1/7)	(1/8, 1/7, 1/6)	(1/5, 1/4, 1/3)	(1, 1, 1)

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