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Optimal safety stock levels of subassemblies and manufacturing components

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Abstract

In order to control the time to market and manufacturing costs, companies produce and purchase many parts and components before receiving customer orders. Consequently, demand forecasting is a critical decision process. Using modular product design and super bills of materials are two effective strategies for developing a reliable demand forecasting process. They reduce the probability of stockouts in diversified production contexts. Furthermore, managing and controlling safety stocks for pre-assembled modules provide an effective solution to the problem of minimizing the effects of forecast errors. This paper develops, evaluates, and applies innovative cost-based analytical models so that the optimal safety stock of modular subassemblies and components in assembly to order and manufacturing to order systems, respectively, can be rapidly quantified. The implementation of the proposed models in two industrial case applications demonstrates that they significantly reduce the safety stock inventory levels and the global logistical cost.
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1. Introduction

The principal operating conditions that affect the performance of a fulfillment process in modular production systems are: demand variability, variation of supply lead times, variation of production capacity, and availability of manufacturing and distribution resources (e.g. facilities, equipment, machines, and tools). These different kinds of variability create uncertainty, which can be managed by using flexible production systems capable of guaranteeing a high degree of customer satisfaction. The following is a crucial question for managers operating production systems suffering from strongly changing operating conditions: what level of customer service and safety stock will minimize manufacturing and logistical costs?

The production strategy adopted in response to customer demand determines the locations of safety stocks along the chain in the logistics system. Safety stocks are located at the most appropriate point in the supply chain where the pulling action of market demand starts affecting the management of materials (Randal and Urlich, 2001). As a result, in make-to-stock (MTS) production systems, storage quantities and safety stocks of products must be managed and controlled. Furthermore, in assembly to order (ATO) production systems the safety stocks

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of pre-assembled modules must also be managed, while in manufacturing to order (i.e. make-to-order MTO) production systems the safety stocks are mainly composed of components and raw materials.

The aim of this paper is to develop and apply a set of cost-based analytical models capable of optimizing the customer service level in ATO and MTO production systems. The proposed models are based on determining the level of optimal safety stock (OSS) for pre-assembled modules and manufacturing components used in final products.

Section 2 presents a review of the literature regarding the main approaches to determining safety stock. Section 3 reviews well-known analytical models used in determining safety stocks for product parts and components produced in many varieties. Section 4 discusses the optimization of the service level for so-called "traditional" products, whose structure is described by usual bills of materials (BOM) and where the demand for each part is based on decomposition of the demand for the final product, which is also known as "father product". Production of traditional products is not module based.

Section 5 introduces innovative cost-based optimization models for modular products, while Section 6 presents two industrial applications of the proposed models. Lastly, Section 7 presents conclusions and suggestions for further research.

2. Review of the literature

Manzini et al. (2004, 2006a) and Ferrari et al. (2001) both discuss the importance of flexibility, i.e. the ability of a system to adapt to changing market demand in terms of both product variation or change (capability flexibility) and product quantity (capacity flexibility). In particular, recent growth and strong development of e-commerce has brought a new focus on flexibility in material handling systems and warehousing facilities (Manzini et al., 2006b, 2007).

Because of the great economic impact that different approaches to material management and control have on company investment and operating costs, in depth studies of both warehousing systems and storage/retrieval management have been carried out during recent decades. In particular, determination of the best stock levels and their locations in an industrial system influences both flexibility and the customer service level, i.e. the ability of an industrial company to deal with fluctuations in production and supply rates so that manufacturing and logistical costs can be minimized (Bonney, 1994; Manzini et al., 2005, 2007).

Safety stock is an effective management tool for protecting the company against the uncertainty and variability of product demand and raw materials supply (Whybark and Williams, 1976). This instrument can simultaneously improve the customer service level and reduce the instability of production planning and scheduling (De Bodt and Wassenhove, 2001). Significant discussions of safety stock utilization are presented by several authors, such as Inderfurth (1994), Hoshino (1996), and Maia and Qassim (1999).

Furthermore, product structure and fulfillment system both have a significant influence on OSS levels (Collier, 1982; McClain et al., 1984; Baker et al., 1986; Hiller, 2002; Caridi and Cigolini, 2002; Manzini et al., 2004). In particular, the standardization of common components and subassemblies applied to various final products means stock levels and production costs can be reduced thanks to the increase in batch production sizes and the use of more efficient and advanced technologies. In addition, component purchase costs can be reduced because discounts can now be obtained (Randal and Urlich, 2001). Hiller (2002) demonstrates that the use of a common standardized component instead of different components having the same function is convenient, even if the purchase cost is higher (10–20% more).

The literature presents three main approaches to determining the best safety stock level. The first is based on the variation of demand (Mentzer and Krishnan, 1985; Benton, 1991; Alstrom, 2001; Charnes et al., 1995; Vollmann et al., 2005), the second on the variation of the forecasting errors (Eppen and Martin, 1988; Zinn and Marmorstein, 1990; Riva et al., 1992; Krupp, 1997; Gardner and Diaz-Saiz, 2002), and finally the third approach is based on product structure and standardization of products and components (Collier, 1982; McClain et al., 1984; Baker et al., 1986; Hiller, 2002).

In the first approach the hypothesis of a normal distribution of demand of products is usually adopted (Benton, 1991). When reorder point inventory management is adopted, the safety stock level must correlate with reorder quantity levels. Benton (1991) presents a simulation model on this subject and discusses the main relationships between different purchase policies (Economic Order Quantity, Mc Laren's Order Moment, and Least Unit Cost), safety stocks, customer service levels, and capital investments. Alstrom (2001) presents a model that minimizes a total cost function (sum of inventory costs, shortage costs, and order costs) in which the service levels and safety stocks are based on production order sizes.

The second approach is based on the assumption that safety stocks are proportional to the forecast errors, and can be applied in cases of easily foreseen demand. This approach can be also applied when demand is highly variable (Gardner and Diaz-Saiz, 2002). Eppen and Martin (1988) propose and compare two models for calculating safety stocks. These models are based on the variability of demand and purchase lead time, as well as on the forecast errors. In particular, they propose a single exponential smoothing model and treat demand and component purchase lead times as stochastic variables. Furthermore, Krupp (1997) proposes a useful model for cases with significant trend and seasonality in demand. Molinder (1997) compares the use of safety stocks with the adoption of safety lead times, i.e. the variability of both demand and supply lead times.

In considering the third approach to safety stock determination, the literature discusses the impact of the product structure (Molinder, 1997) and the level of component standardization (Collier, 1982; McClain et al., 1984; Baker et al., 1986; Hiller, 2002) on safety stock level determination, but only a few manuscripts deal with OSS level determination in cases of products produced in a wide variety of different models.

Several contributions in the literature demonstrate the convenience of using super bills of materials (SBOM) once parts and products have been modularized (Persona et al., 2003). The SBOM is capable of improving both the accuracy of the forecasting process (Vollmann et al., 2005) and the flexibility of the production planning process (King and Benton, 1988). In fact, when a wide variety of products is offered, the forecasting activity of pre-assembled modules (in ATO systems) and raw materials or components (in MTO systems) can reduce the production lead time of final products and improve the customer service level. In particular, the determination of the correct inventory level for the pre-assembled modules is a strategic issue in an ATO system. On the other hand, in MTO production systems, the definition and management of the safety stocks of so-called "phantom modules" are required in order to meet market requirements and expectations. The phantom modules, also called "ghost modules", are modules which have not been physically assembled yet: the generic MTO module is not a subassembly but a kit of parts, i.e. components and raw materials, which could simultaneously belong to different modules, and similarly more than one "ghost module" could require the same component C.

Potamianos et al. (1997) propose a model to calculate inventory levels for products based on modular structures in ATO and MTO environments. The model is based on the evaluation of several parameters such as the function of the module, the module cost, the trend of final product demand, and the accuracy of the forecasting model. However, the proposed approach is limited by its qualitative nature, and requires various types of data which are seldom easy to collect from industrial applications.

Persona et al. (2003) propose four different analytical models to calculate the safety stock levels for subassemblies and components in different ATO and MTO operating contexts, and in the presence or absence of a demand forecasting process. The main advantages of the proposed models are the following:

1. *quantitative nature*, based on pre-assigned values of the customer service level;
2. *easy implementation*, based on simple data collection;
3. *safety stock reduction*. A lower average safety stock level is obtained compared to traditional methods based on the same target values of the customer service level;
4. *correlation based*, i.e. the proposed models are based on the correlation and mutual relationships between the optional modules of the final products (the so-called "basic products").

The models proposed by Persona et al. (2003) are based on the pre-assignment of the value of the standardized parameter k, which has a considerable effect on safety stock determination.

The aim of this paper is to determine the best value of k by minimizing a total cost function that measures the trade off between the inventory costs of managing safety stocks and the production losses.

3. Safety stock determination in ATO and MTO systems

The use of SBOM in ATO and MTO systems means the forecasting errors collected on different products that share modules can be balanced and compensated. Consequently, the forecasting activity can be simplified by managing the data in the optional modules (e.g. consumption, lead time, etc.) instead of directly managing the data relating to the final product, i.e. the so-called "average product" (or "basic product").

The order quantity for module M in the generic unit period of time t can be stated as

$$M_t = m_t \cdot F_t, \tag{1}$$

where M_t is the order quantity of module M in the period of time t, m_t the module use coefficient in the period t, and F_t the order quantity for basic product in t.

By designing and managing SBOM, the consumption of the generic module can be planned by determining forecasts for the average product. In fact, the planning of module orders is based on the multiplication of the module use coefficient and the forecast quantity of the basic product: as a result, in order to evaluate the variability of the production mix, the variability of the basic product needs to be extrapolated.

The innovative models proposed by the authors in this manuscript are based on the analytical models illustrated by Persona et al. (2003): they measure the safety stocks of parts in modular production systems quantifying the order quantity of the basic products and the standard deviation of the modular options. A brief presentation of the main results discussed by Persona et al. (2003) now follows.

3.1. ATO production systems

The models proposed by Persona et al. (2003) for quantifying the safety stock in an ATO context without predicting values of the module consumption are based on the following equation:

$$SS_t^M = k\sigma_{\%}^M F_t \sqrt{\mathrm{LT}_{\mathrm{Supply}}^M}, \tag{2}$$

where SS_t^M is the safety stock for module M in the period of time t, $\sigma_{\%}^M$ the standard deviation of module M use coefficient, F_t the forecasted order quantity of the basic product, to which module M belongs, during period t, $\mathrm{LT}_{\mathrm{supply}}^M$ the average lead time for the assembly activities of module M, and k the non-dimensional parameter which relates to the customer service level. The meaning of this parameter is made clear in the following sections.

The generic module M is a subassembly that performs a specific operational function and in general belongs to different basic products, represented by a unique "average" product. The proposed model is capable of reacting efficiently to the short-term variability of product mix because the average product's volume F_t does not change in the short-term, but only during a long planning horizon after reconfiguration of the production system. Therefore, from Eq. (2), the safety stock level of module M is proportional to the forecasted quantities of the basic product. The value of k is related to the customer service level in order to guarantee the consumption requested of module M during each period of time t.

3.2. MTO production systems

Alternative option modules can be used to provide a product function in an MTO production system. As a consequence they are generally available simultaneously, and their demands are correlated. The generic part (e.g. component or raw material) C could belong to more than one option module. Consequently, when a customer asks for a final product configuration that includes a specific module M_i, all the other available modules are automatically excluded: considering a period of time t, an increase in the coefficient use of a specific module M_i influences the value of coefficient use of different and alternative modules, which assume lower values than was previously the case.

Fig. 1 shows an example of product structure of various manufacturing products (*Product 1, Product 2, Product k*, etc.) based on a set of interdependent modules (*Module 1, Module 2, Module n*). *Component 2* belongs to more than one module (i.e. *Module 1* and *Module i*).

The following equation quantifies the safety stock level of a part C (raw material or component of a modular product) which belongs to different interdependent modules.

$$\mathrm{SS}_t^C = k \cdot \sqrt{\sum_{i=1}^{m} (c_{Mi}\sigma_{\%}^{M_i})^2 + 2\sum_{i=1}^{m}\sum_{j=i+1}^{m} c_{M_j} c_{M_i} \rho_{\%}^{M_i,M_j} \sigma_{\%}^{M_i} \sigma_{\%}^{M_j}} \times F_t \cdot \sqrt{\mathrm{LT}_{\mathrm{Supply}}^C}, \tag{3}$$

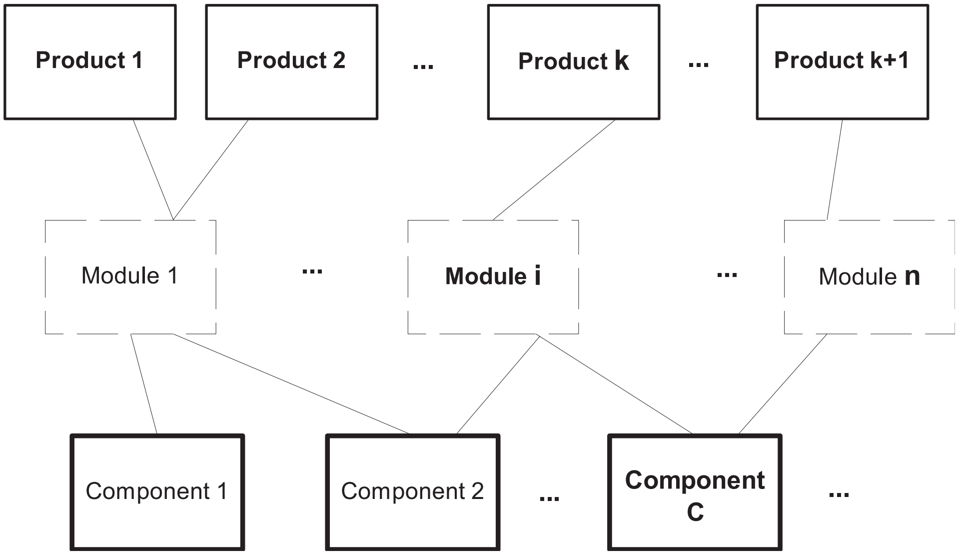

Fig. 1. Structure of modular products in MTO.

where F_t is the forecasted demand for the basic product in t, m the number of modules in the product mix of final products, c_{M_i} the use coefficient for the component C in module M_i, $\sigma_{\%}^{M_i}$ the standard deviation of the use coefficient of module M_i, $\rho_{\%}^{M_i,M_j}$ correlation coefficient of the use coefficient of the two modules M_i and M_j, and $\mathrm{LT}_{\mathrm{Supply}}^{C}$ the average lead time for purchasing component C.

The standard deviation of the component C use coefficient is based on the correlation between the generic modules i and j which belong to the final product of demand F_t.

A brief MTO example is now presented to clarify the application of the previously introduced analytical expressions. Table 1 presents the historical orders of three optional modules of a product mix, whose average product orders are reported in the second column of the table. The c_{M_i} use coefficient of each module is calculated for each unit period of time t (e.g. a month or a week). If a component C is applied in both *Module 1* (M_1) and *Module 2* (M_2), the correlation coefficient of the use coefficient of the two modules $\rho_{\%}^{M_1,M_2}$ is approximately -0.25. The values of safety stock are calculated in accordance with Eq. (3) and are reported assuming $k = 1.65$ (i.e. customer service level LS equal to 95%) and $\mathrm{LT}_{\mathrm{Supply}} = 1$ in the last column of Table 1.

The value of the service level in Eqs. (2) and (3), and consequently of parameter k, is not optimized but predefined according to the agreement between the producer and the customer. The innovative contribution of this study is to present a set of analytical cost-based models capable of optimizing the customer service level LS in accordance with minimization of the global production and logistical costs.

4. Service level optimization model for "traditional" product structure

Analytical models to determine the safety stock for a generic product traditionally assume that demand is stochastic, independent, and described by a normal distribution (Vollmann et al., 2005). By also assuming these hypotheses, a model is presented which quickly determines the best safety stock level for a product described by usual BOM and whose structure is "traditional", i.e. the demand for each part is based on a decomposition process of final products. Analytical expressions for service level optimization in ATO and MTO environments described in Sections 5 and 6 are based on this model.

The following expression measures the probability that demand D, defined for a period of time t (e.g. a day, a week, etc.), is lower than a predefined maximum admissible value of demand D_M. It is the customer service level associated with inventory level D_M (i.e. the value of the related standardized parameter k):

$$\mathrm{LS}(D_M) = \int_{-\infty}^{D_M} f(D)\,\mathrm{d}D = \mathrm{LS}(k) = \int_{-\infty}^{k} g(z)\,\mathrm{d}z, \tag{4}$$

Table 1
Orders and use coefficients for modules and average product

Period	Average product	Orders			Use coefficient (%)			Safety stock
		Module A	Module B	Module C	Module A	Module B	Module C	Module A
1	2000	500	800	700	25.00	40.00	35.00	66.6
2	2300	550	850	900	23.91	36.96	39.13	76.6
3	2400	560	880	960	23.33	36.67	40.00	79.9
4	2360	610	830	920	25.85	35.17	38.98	78.6
5	2190	590	820	780	26.94	37.44	35.62	72.9
6	2095	450	795	850	21.48	37.95	40.57	69.8
7	1955	450	790	715	23.02	40.41	36.57	65.1
8	2305	545	850	910	23.64	36.88	39.48	76.8
9	2390	565	875	950	23.64	36.61	39.75	79.6
10	2340	600	830	910	25.64	35.47	38.89	77.9
11	2171	596	805	770	27.45	37.08	35.47	72.3
12	2090	450	795	845	21.53	38.04	40.43	69.6
13	2005	500	800	705	24.94	39.90	35.16	66.8
14	2280	540	860	880	23.68	37.72	38.60	75.9
15	2385	555	880	950	23.27	36.90	39.83	79.4
16	2330	590	820	920	25.32	35.19	39.48	77.6
17	2170	590	810	770	27.19	37.33	35.48	72.3
18	2095	460	795	840	21.96	37.95	40.10	69.8
19	2010	495	800	715	24.63	39.80	35.57	66.9
20	2235	550	840	845	24.61	37.58	37.81	74.4
21	2375	555	870	950	23.37	36.63	40.00	79.1
22	2280	590	800	890	25.88	35.09	39.04	75.9
23	2180	580	820	780	26.61	37.61	35.78	72.6
24	2080	450	790	840	21.63	37.98	40.38	69.3
Mean	2209.2	538.4	825.2	845.6	0.243551	0.374313	0.382136	
Variance	2217.9	2995.5	942.3	7237.6	0.000319	0.000221	0.000405	
Standard dev.	2214.5	54.7	30.7	85.1	0.017869	0.014867	0.020125	

where $f(D)$ is the normal probability function of the demand variable D, and $g(z)$ the normal standard distribution associated with variable D.

Consequently, Eq. (4) measures the probability that there is not a stockout in the period of time t. The unit period of time t can belong to a horizon of time T_0.

The product safety stock level for the unit period of time t is

$$D_M - D_m, \tag{5}$$

where D_m is the average value of the variable D.

The average value of the unfulfilled demand D_{OUTu} in a unit period of time t and when demand D is greater than the maximum admissible inventory level D_M, i.e. in presence of a stockout, is

$$D_{\mathrm{OUT}u} = D_{m\mathrm{OUT}} - D_M = (D_{m\mathrm{OUT}} - D_m) - (D_M - D_m) = \sigma_D(z_{m\mathrm{OUT}} - k), \tag{6}$$

where $D_{m\mathrm{OUT}}$ is the average requested demand in case of stockout, σ_D the standard deviation of demand D, and $z_{m\mathrm{OUT}}$ the standardized average requested demand in case of stockout.

Assuming a desired value of service level LS, the value of the standardized variable $z_{m\mathrm{OUT}}$ is

$$z_{m\mathrm{OUT}} = \frac{\int_k^\infty z g(z)\,\mathrm{d}z}{\int_k^\infty g(z)\,\mathrm{d}z} = \frac{\int_k^\infty [z(1/\sqrt{2\pi})\mathrm{e}^{-z^2/2}]\,\mathrm{d}z}{1-\mathrm{LS}} = \frac{(1/\sqrt{2\pi})\mathrm{e}^{-k^2/2}}{1-\mathrm{LS}}. \tag{7}$$

The average value of unfulfilled demand in cases of stockout for a single reorder period is

$$D_{\mathrm{OUT}} = \sigma_D\sqrt{\mathrm{LT_{Supply}}}\left[\frac{(1/\sqrt{2\pi})\mathrm{e}^{-k^2/2}}{1-\mathrm{LS}} - k\right], \tag{8}$$

where safety stock is calculated using the analytical model based on the measure of the supply lead time ($\mathrm{LT_{Supply}}$) defined for the component. It is assumed that the value of $\mathrm{LT_{Supply}}$ is known and deterministic. Assuming n to be the number of reorders in the analyzed period of time T_0, the number n_{OUT} of stockouts is

$$n_{\mathrm{OUT}} = n \cdot (1 - \mathrm{LS}). \tag{9}$$

4.1. Average safety stock consumption

Various situations may arise when there is no stockout between two consecutive orders:

Case A. $D < D_m$, i.e. the demand in the reorder period is less than the average value, so the safety stock is not used.

Case B. $D_m < D < D_M$, i.e. the demand in the reorder period ranges from the average demand to the maximum demand value D_M, which assures the service level LS. As a result, the safety stock is partially used.

In Case A, the average storage quantity G_{A} is equal to the safety stock:

$$G_{\mathrm{A}} = \mathrm{SS} = k\sigma_D\sqrt{\mathrm{LT_{Supply}}}, \tag{10}$$

where SS is the safety stock.

G_{A} refers to a single reorder period of time as demonstrated by the presence of the factor $\sqrt{\mathrm{LT_{Supply}}}$.

Therefore, by definition of the safety stock and of the previously introduced parameter n, the number of cycles n_{A}, where safety stock is not used during a period of time T_0, is

$$n_{\mathrm{A}} = \frac{n}{2}. \tag{11}$$

In Case B, the average demand is equal to D_B, as illustrated in Fig. 2. Consequently, the average quantity of consumed safety stock D_{pSS} (pSS = partial safety stock consumption) for the unit period of time t is

$$\begin{cases} D_{\mathrm{pSS}} = D_B - D_m = z_{\mathrm{pSS}}\sigma_D, \\ z_{\mathrm{pSS}} = \dfrac{\int_0^k z g(z)\,\mathrm{d}z}{\int_0^k g(z)\,\mathrm{d}z} = \dfrac{(1/\sqrt{2\pi})(1 - \mathrm{e}^{-k^2/2})}{\mathrm{LS} - \frac{1}{2}}, \end{cases} \tag{12}$$

where z_{pSS} is the standardized value of the average quantity of consumed SS.

Therefore the expected average level of safety stock not consumed and referring to a single reorder period is

$$G_{\mathrm{B}} = \mathrm{SS} - z_{\mathrm{pSS}}\sigma_D\sqrt{\mathrm{LT_{Supply}}} = \sigma_D\sqrt{\mathrm{LT_{Supply}}}\left[k - \frac{(1/\sqrt{2\pi})(1 - \mathrm{e}^{-k^2/2})}{\mathrm{LS} - \frac{1}{2}}\right]. \tag{13}$$

Lastly, the number of cycles n_{B} in which safety stock is partially used in the period of time T_0 is

$$n_{\mathrm{B}} = n \cdot (\mathrm{LS} - \tfrac{1}{2}). \tag{14}$$

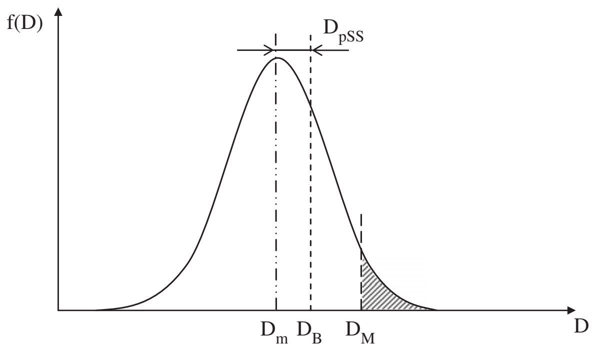

Fig. 2. Demand values in the case of partial consumption of safety stock.

4.2. Total cost of safety stock management

The total cost of safety stock management can be defined as the sum of inventory holding and shortage costs. The inventory holding cost during the period of time T_0 can be calculated using

$$C_{m\mathrm{STK}} = \frac{C_{\mathrm{uSTK}}(G_{\mathrm{A}} \cdot n_{\mathrm{A}} + G_{\mathrm{B}} \cdot n_{\mathrm{B}})}{n} = C_{\mathrm{uSTK}}\sigma_D\sqrt{\mathrm{LT_{Supply}}}\left[k \cdot \mathrm{LS} - \frac{1}{\sqrt{2\pi}}(1 - \mathrm{e}^{-k^2/2})\right], \tag{15}$$

where C_{uSTK} is the unit inventory holding cost of the generic part during the period of time T_0 (e.g. it provides the annual inventory holding cost if T_0 is one year).

The stockout (i.e. shortage) cost in the period of time T_0 can be expressed by

$$C_{\mathrm{mOUT}} = C_{\mathrm{uOUT}} \cdot n_{\mathrm{OUT}} \cdot D_{\mathrm{OUT}} = nC_{\mathrm{uOUT}}(1 - \mathrm{LS})\sigma_D \times \sqrt{\mathrm{LT_{Supply}}}\left(\frac{(1/\sqrt{2\pi})\mathrm{e}^{-k^2/2}}{1 - \mathrm{LS}} - k\right), \tag{16}$$

where C_{uOUT} is the unit shortage cost for the generic part.

By Eqs. (15) and (16) the global cost for the period of time T_0 is

$$C_{\mathrm{TOT}} = C_{m\mathrm{OUT}} + C_{m\mathrm{STK}} = C_{\mathrm{uOUT}}\sigma_D\sqrt{\mathrm{LT_{Supply}}}\left[\frac{\mathrm{e}^{-k^2/2}(n + \alpha)}{\sqrt{2\pi}} + \alpha\left(-\frac{1}{2\pi} + k \cdot \mathrm{LS}\right) - nk(1 - \mathrm{LS})\right], \tag{17}$$

where

$$\alpha = \frac{C_{\mathrm{uSTK}}}{C_{\mathrm{uOUT}}}.$$

Fig. 3 shows that the optimal service level LS, which minimizes Eq. (17), is a function of ratio α and of the number of deliveries n during a period of time T_0.

If parameter α decreases, the optimal service level increases. In fact decreasing the value of α means that stockout cost is increasingly more important than the inventory holding cost which implies that managers keep higher stock levels to avoid stockout. Moreover, the inventory level of safety stock is lower for components that are supplied with low frequency n (i.e. in presence of long periods of time between two consecutive reorders).

For each value of n the value of the parameter α which corresponds to a customer service level equal to 50% can be identified: it is not worth using the safety stock for higher values of α.

Fig. 4 shows a more detailed analysis of the optimal values of k (i.e. LS) when the value of α is less than 1, i.e. for components having a low annual inventory holding cost to unitary shortage cost ratio.

5. Service level optimization in ATO and MTO production systems

The purpose of this section is to present an analytical model that rapidly determines the most economic level of safety stock in the case of products produced in a wide variety of models.

In ATO production systems the safety stock level of a singular option module M is proportional to the forecast quantity of the basic product and to the variability of the consumed option module, as shown in Eq. (2). Based on Eq. (16), the expression of the shortage cost for module M during the period of time T_0 is

$$C^M_{m\mathrm{OUT}} = C^M_{\mathrm{uOUT}} \cdot n^M_{\mathrm{OUT}} \cdot D^M_{\mathrm{OUT}} = \sum_{p=1}^{n^M} \left[C^M_{\mathrm{uOUT}} (1 - \mathrm{LS}) F_p \sigma^M_{\%} \sqrt{\mathrm{LT}^M_{\mathrm{Supply}}} \times \left(\frac{(1/\sqrt{2\pi}) \mathrm{e}^{-k^2/2}}{1 - \mathrm{LS}} - k \right) \right], \quad (18)$$

where C^M_{uOUT} is the unit shortage cost for module M, p the generic reorder period, F_p the forecasted order quantity for the basic product in the reorder period p, n^M the number of reorder periods in T_0 for the module M, n^M_{OUT} the number of stockouts during T_0 for module M, and D^M_{OUT} the average value of unfulfilled demand for module M in case of stockout.

The amount of inventory holding cost during the period of time T_0 is

$$C^M_{m\mathrm{STK}} = \sum_{p=1}^{n^M} \frac{C^M_{\mathrm{uSTK}} F_p}{n^M} \sigma^M_{\%} \sqrt{\mathrm{LT}^M_{\mathrm{Supply}}} \times \left[k \cdot \mathrm{LS} - \frac{1}{\sqrt{2\pi}} (1 - \mathrm{e}^{-k^2/2}) \right], \quad (19)$$

where C^M_{uSTK} is the unit inventory holding cost of module M during the period of time T_0 (e.g. annual inventory holding cost).

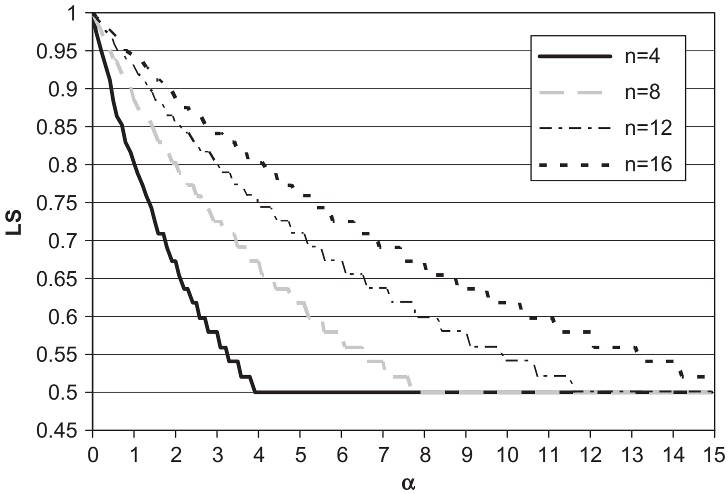

Fig. 3. Determination of the optimal LS related to n [deliveries/year] and α.

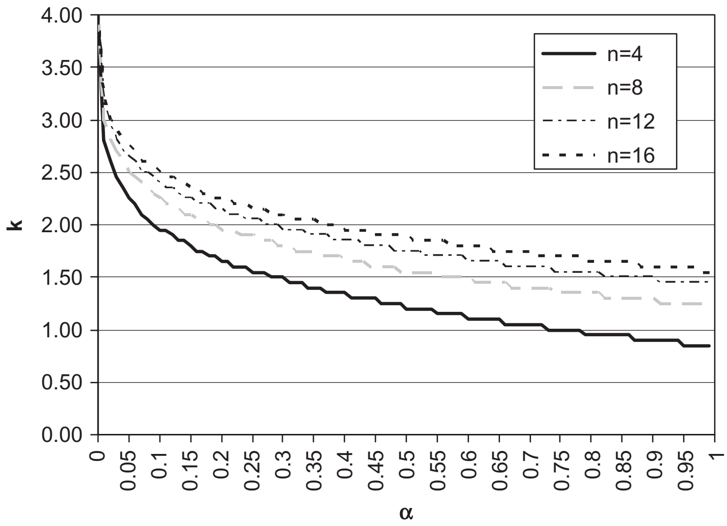

Fig. 4. Determination of the optimal k value. $\alpha < 1$, n [deliveries/year].

The global logistical cost for the period of time T_0 is the sum of inventory holding and shortage costs, and is based on Eq. (17):

$$C^M_{\text{TOT}} = \sum_{p=1}^{n^M} F_p \sigma^M_{\%} \sqrt{\text{LT}^M_{\text{Supply}}} \times \left(\frac{e^{-k^2/2}\left(C^M_{\text{uOUT}} + (C^M_{\text{uSTK}}/n^M)\right)}{\sqrt{2\pi}} + \frac{C^M_{\text{uSTK}}}{n^M}\left(-\frac{1}{\sqrt{2\pi}} + k \cdot \text{LS}\right) - C^M_{\text{uOUT}} k(1-\text{LS}) \right). \quad (20)$$

In an MTO production system the safety stock of a singular option module M_i is related to the forecasts of the basic products to variation in the option module consumption (i.e. the variability), and finally, to the correlation of demand between the optional modules (the so called "options") that refer to the same product function.

Consequently, the following analytical expression quantifies the global logistical cost for the generic component C during the period of time T_0:

$$C^C_{\text{TOT}} = \sum_{p=1}^{n^C} F_p \sigma^C_{\%} \sqrt{\text{LT}^C_{\text{Supply}}} \times \left(\frac{e^{-k^2/2}(C^C_{\text{uOUT}} + (C^C_{\text{uSTK}}/n^C))}{\sqrt{2\pi}} + \frac{C^C_{\text{uSTK}}}{n^C}\left(-\frac{1}{\sqrt{2\pi}} + k \cdot \text{LS}\right) - C^C_{\text{uOUT}} k(1-\text{LS}) \right), \quad (21)$$

$$\sigma^C_{\%} = \sqrt{\sum_{i=1}^{m} (c_{M_i} \sigma^{M_i}_{\%})^2 + 2 \sum_{i=1}^{m} \sum_{j=i+1}^{m} c_{M_j} c_{M_i} \rho^{M_i,M_j}_{\%} \sigma^{M_i}_{\%} \sigma^{M_j}_{\%}}, \quad (22)$$

where $\sigma^C_{\%}$ is the standard deviation of the component C use coefficient, C_{uSTK} the unit inventory holding cost of component C during the period of time T_0, C^C_{uOUT} the unit shortage cost for component C, F_p the forecasted order quantity for the basic product in the reorder period p, and n^C the number of reorder periods for the component C in T_0.

Analysis of Eqs. (20) and (21) reveals that if the value of F_p is constant throughout the period of time T_0, the customer service level optimization process and the sensitivity analysis discussed and proposed in Sections 4.1 and 4.2 can be applied effectively. On the other hand, in order to minimize the expressions of the global logistical costs, in cases of variable F_p values the service level optimization requires a what-if analysis to be applied to different operating scenarios.

The coherence between the physical dimensions of the units of measurement (e.g. between σ_D, n, and $\text{LT}_{\text{Supply}}$) in the equations previously introduced has to be guaranteed. For example, if the period of reference for T_0 is a year, n is the number of reorders during a year. Furthermore, if the unit of measurement for demand D is the quantity of products to be manufactured in a unit of time (such as a day, week, or month) the lead time must be measured in the same unit of time (i.e. number of days, weeks, months).

The next section presents two significant case studies of customer service level optimization for Italian companies using an MTO strategy.

6. Industrial applications

The proposed models to determine safety stock levels were applied in two Italian companies operating in different industrial sectors. The first company manufactures air conditioning systems characterized by high standardization of products, modular design, and seasonal demand. This company operates an MTO production strategy.

The second company provides personalized design, production, and service for the worldwide professional catering sector and operates in conditions of low and variable product demands.

Eq. (3) was applied to the first case study in order to quantify the safety stocks, after which the value of the best k was quantified by estimating the previously introduced parameter α. In the second case study the simultaneous application of Eqs. (3), (20), and (21) results in a significant reduction of safety stock levels and logistical costs.

Case study 1. Fig. 5 shows customer orders for one basic product (called "Alpha") and its principal modules during a period of 24 months.

Table 2shows some of the air conditioning product components C analyzed (e.g. filters, valves, burning system, etc.) and the specific option modules M_i (module 1, module 2, etc.) that they belong to. Table 3presents the collected values of component parameters for the application of proposed analytical models: the percentage standard deviation $\sigma_{\%}^{C}$, the delivery lead time LT_{Supply}, and the inventory holding cost C_{uSTK}^{C} (in Euro per year).

The stockout cost C_{uOUT}^{C} is represented by two parameters a and a' which reflect different customer behaviors. In particular, a represents "late delivery" cost and takes the following three factors into consideration: the capital immobilization, the increase in assembly cost caused by the inefficiency of an out of line product achievement, and any related penalties.

The second parameter a' also quantifies the extra cost of "lost sale" which is composed of all direct industrial costs and any possible penalties agreed with the customer. As a result, a' is greater than a.

Fig. 6 presents the average inventory levels of a sample of four components during a period of time T_0. In particular, the generic inventory level based on the application of the proposed models (the so-called TO-BE configuration, i.e. the "new solution" in Fig. 6) is compared to the inventory level of the so called AS-IS configuration (the "before approach" in Fig. 6). The AS-IS configuration reflects the operating scenario of the company before the implementation of the proposed set of original

Table 2
Components and modules

Component-C	Module 1	Module 2	Module 3	Module 4
Filter		X	X	
Valve	X		X	X
Burning system		X		X
Heat exchanger		X	X	X

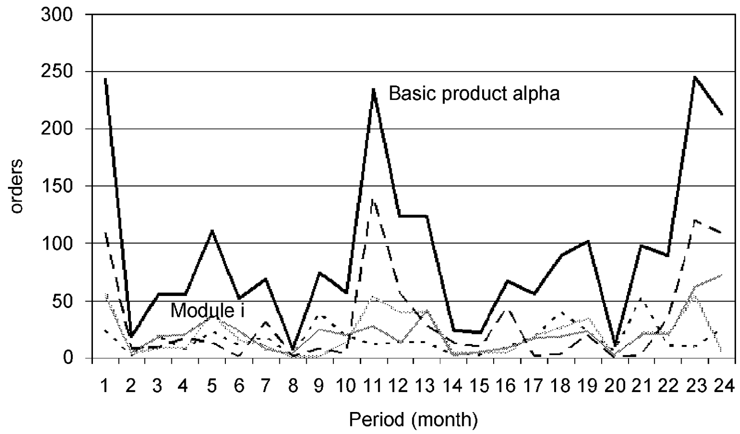

Fig. 5. Orders for basic product and consumption of main modules during a 24 month period.

Table 3
Data collection for component safety stock determination

Component-C	$\sigma^C_{\%}$ (%)	LT_{Supply} [months]	C_{uSTK} [€/year]	a Delay	a' Lost sale
Filter	13.2	1.25	1.644	0.00822	1.3700×10^{-04}
Valve	9.3	0.75	33.954	0.16977	2.8295×10^{-03}
Burning system	20.0	2.00	440.000	2.20000	3.6666×10^{-02}
Heat exchanger	15.7	1.00	100.066	0.50033	8.3388×10^{-03}

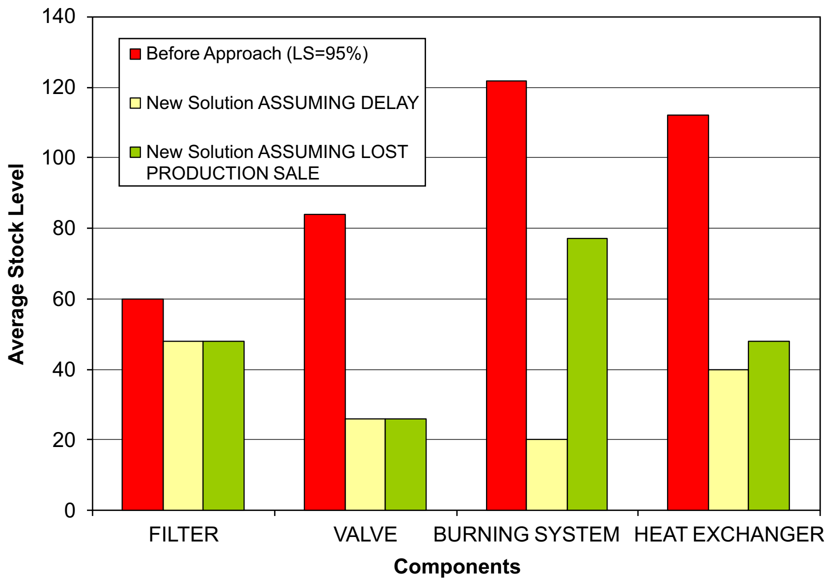

Fig. 6. Comparison between the average inventory levels.

models: service level LS is assumed to be equal to 95% and the inventory requirement for safety stock management of each part is based on the traditional approach (discussed in Section 4) which is coherent with a product structure described by usual BOM.

For example, if C_{uOUT} is € 200 per day in the case of "late delivery", and € 12,000 per day in the case of "lost sale", the two proposed optimizing solutions provide a significant reduction in the total "late delivery" annual costs of approximately 77.2% and of approximately 43.1% in "lost sale" costs. In the first case the optimal solution results in stockout costs and inventory holding costs, whereas in the second case the value of parameter a' means that the warehouse stockout cost is negligible.

Considering different parts and components the amount of safety stock reduction is different. In particular the service level can pass from 95% (AS-IS) to values such as 75% or 65% (TO-BE), and for some components the safety stock has been deleted (i.e. LS = 50%).

Case study 2. In the second industrial case study the proposed optimizing models were applied to a set of 2744 purchase components by designing a database for the company and linking it to the ERP system. The parameters of the proposed models have been calculated from the collection of the large set of historical data illustrated in Fig. 7: item consumption (pieces/year), standard deviation and percentage standard deviation of item consumption, annual inventory holding cost C_{uSTK} (€/piece), time delay in the delivery of basic product caused by a stockout (estimated in working days), and lastly, the stockout cost C_{uOUT} associated with generation of penalties (€/piece). The table computes the values of both parameter α ($C_{uSTK}C_{uOUT}$) and n (reorder cycles/year) for the generic item.

The value of k was optimized by querying the database for different values of the parameters n

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Code	Descr	UM	Leadtime	Cost	Cosumption	DEV-ST	DEV_ST%	Cg	Delay	CURS	a	n
131AA12	TUBE X H²O 17X10 (U.M. CM)	CM	15	0,01410	180793	2652,04	1,47%	0,00352	1,0	127,45	2,7658E-05	4,2
131AA29	TUBE GOMMA TEL. 43X35	CM	15	0,05370	16914	949,09	5,61%	0,01342	1,0	127,45	1,0534E-04	4,2
131AA30	SHEATH	MT	39	0,75290	92	2,76	2,99%	0,18823	1,0	127,45	1,4769E-03	1,8
131AA31	TUBE 48X40	CM	35	0,06460	3192	196,17	6,15%	0,01615	1,0	127,45	1,2672E-04	0,8
131AA32	TUBE	MT	35	0,00590	26260	1476,14	5,62%	0,00148	1,0	127,45	1,1573E-05	6,6
131AA34	TUBE PVC §10	CM	35	0,00560	34754	2508,62	7,22%	0,00140	1,0	127,45	1,0985E-05	13,9
131AA35	TUBE	MT	25	0,02000	36295	4867,47	13,41%	0,00500	1,0	127,45	3,9231E-05	4,2
131AA36	ELETRIC CARD	NR	56	209,05650	51	5,95	11,66%	52,26413	0,2	27,45	1,9040E+00	4,2
131AA37	ELETRIC CARD	NR	56	210,65000	67	5,65	8,43%	52,66250	0,2	27,45	1,9185E+00	4,2
131AA46	TUBE	NR	39	0,05160	16628	1950,78	11,73%	0,01290	1,0	127,45	1,0122E-04	4,2
137AA12	GASKET PROF."E"	MT	39	0,16110	119	12,46	10,45%	0,04027	1,0	127,45	3,1601E-04	0,1
138AA01	GASKET	MT	25	0,49580	300	8,15	2,72%	0,12395	1,0	127,45	9,7254E-04	4,2
157AA01	GLASS FABRIC	CM	35	0,00400	152778	3038,80	1,99%	0,00100	0,2	27,45	3,6430E-05	15,5
157AA21	FASTON	GR	39	0,00930	249104	5381,25	2,16%	0,00232	1,0	127,45	1,8242E-05	4,2
170WR01	INSULATION	NR	39	0,04750	542	128,07	23,63%	0,01188	1,0	127,45	9,3174E-05	0,5
171WR01	INSULATION	NR	39	0,06550	351	42,73	12,17%	0,01638	1,0	127,45	1,2848E-04	0,4
17CBB01	FASTON	NR	39	0,01000	12380	1456,06	11,76%	0,00250	1,0	127,45	1,9616E-05	2,5
1C9AA10	CABLE §1	MT	28	0,07230	3319	488,44	14,72%	0,01808	1,0	127,45	1,4182E-04	4,8
1GA1151	GAS BURNER	NR	49	66,80620	141	15,07	10,69%	16,70155	1,0	127,45	1,3104E-01	7,0

Fig. 7. Determination of component safety stock.

and α, with the result that the safety stock for approximately 230 components was found to be unnecessary and so was removed. Eqs. (3) and (21) were applied to 1097 components produced in an MTO strategy in order to optimize the safety stocks and quantify their consumption. In agreement with the definition and models introduced in Section 4, the "traditional safety stock modeling" was applied to the rest of the components. The main result obtained by applying the proposed optimizing models is a reduction of more than 29% in the safety stock values of purchase items and approximately 32% in the total annual logistic costs compared to the AS-IS inventory management system. The AS-IS management is related to safety stock levels that are quantified by applying the traditional product structure as described by the usual BOM.

7. Conclusions and further research

Accurate demand forecasts in ATO and MTO production systems can be obtained from two practices: the design of modular products and the availability of SBOM. In particular, effective solutions reducing the occurrence of stockouts can be found by introducing safety stocks of pre-assembled modules or components. Several studies propose mathematical models to determine the safety stock levels for modular products, but input data are generally hard to find and also to manage.

A set of analytical models to determine safety stock levels of subassemblies or components according to different operating contexts was presented previously (Persona et al., 2003). The proposed models require the definition of a parameter k, which relates to customer service level, and has a considerable influence on the results obtained.

This manuscript is based on the results presented and applied by Persona et al. (2003) and illustrates an efficient and original methodology to optimize k quickly by minimizing inventory holding and stock-out costs (i.e. delay or shortage). The effectiveness of the proposed models is demonstrated by the results obtained in applying them to several industrial case studies, two of which have been discussed in depth in the paper.

Recommendations for further research include adapting and applying the proposed analytical models to stochastic delivery lead times and different demand probability distributions. Finally, recent industrial experiences of the authors have led to further development of models and methods capable of defining, managing, and measuring the Down Time Costs generated by stockouts and system break-downs in production plants.

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