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Analytical decoupling control strategy using a unity feedback control structure for MIMO processes with time delays

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Abstract

An analytical decoupling control method is proposed for multiple-input–multiple-output (MIMO) processes with multiple time delays. The desired diagonal system transfer matrix is proposed first in terms of the H_2 optimal performance specification, resulting in the ideal desired decoupling controller matrix derived within the framework of a unity feedback control structure. It is demonstrated that dead-time compensators must be enclosed in the decoupling controller matrix to realize absolute decoupling for MIMO processes with multiple time delays. To alleviate the difficulties associated with the implementation, the ideal desired decoupling controller matrix is transformed into a practical form using an analytical approximation approach. Correspondingly, the stability of the resultant control system is assessed, together with its robust stability in the presence of process uncertainties. An on-line tuning rule for the single adjustable parameter of each column controllers in the decoupling controller matrix is given to cope with the process unmodeled dynamics. Finally, illustrative examples are given to show the superiority of the proposed method over the recently improved decoupling control methods.

Keywords: MIMO process; Time delay; Decoupling; H_2 optimal performance specification; Analytical approximation; Robust stability

1. Introduction

Effective control of multivariable processes is a difficult issue in the context of process control. Because of loop interactions, well established control methods for single-input–single-output (SISO) systems can hardly be extended to multiple-input–multiple-output (MIMO) systems [1,2]. Besides, due to the existence of multiple time delays in such a multivariable process, high gains are kept from being used in the individual control loops, resulting in sluggish system response and degraded decoupling regulation [3,4]. Many different control strategies have been developed to overcome the aforementioned obstacles. On the basis of successful application of the Smith predictor (abbr. SP) for SISO systems, earlier literature [3,5] applied SP to MIMO processes with time delays to obtain a delay-free characteristic equation of such a system transfer function matrix, and then extended some previous decoupling/decentralized control methods developed for linear multivariable systems without time delay. Improved tuning capacity within the framework of a multivariable SP structure have been reported [6,7] in terms of frequency response specifications such as ultimate frequency and magnitude/phase margins. In contrast, based on the internal model control (IMC) structure [8], essentially similar to an SP structure, enhanced decoupling control methods have been proposed [9–11] using some numerical optimization algorithms in frequency domain. Subsequently, an analytical tuning method [12] was derived to relieve such a computation effort. About the same time, on-line sequential tuning methods using frequency response data obtained from relay feedback tests have been reported [13–16], while the simultaneous auto-tuning methods [17–19] have also been

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presented for some industrial multivariable processes with time delays. Owing to the primary requirement of decoupling regulation for many industrial MIMO processes, a large number of existing methods utilized a decoupler augmented to such a MIMO process to procure diagonal dominance of the system transfer matrix, and then configured the decentralized controllers by means of some up-to-date decentralized/multiloop control methods. For example, recent Refs. [20–23] presented some decoupling control strategies based on dynamic decouplers, while the static decoupler, i.e. the inverse of the process static gain transfer matrix, was adopted in the recent literature [24–26]. However, the requirements of properness and causality for implementation make it difficult for dynamic decouplers to be configured precisely, especially for MIMO processes with high dimension or large time delays [13,20]. A static decoupler, on the other hand, does not affect dynamic decoupling for the resultant system. It should be noted that although recently enhanced multiloop/decentralized control methods, e.g. Refs. [27–30], can achieve remarkable improvement in system performance, tuning of the loop controllers aims at the compromise between achievable system performance and cross-interaction level among individual loops. This, inevitably, leads to performance degradation when compared to a MIMO control system with a full controller matrix [4,8]. Therefore, when high performance of both system response and decoupling regulation is required, decoupling control strategies are preferred in engineering practice.

Recently, Wang et al. [31] presented a decoupling controller matrix design method within the framework of a unity feedback control structure. By proposing a desirable system response transfer matrix, the executable decoupling controller matrix was derived using a recursive-least-square (RLS) optimization algorithm. Obvious improvement in both system response and decoupling regulation can be found over other decoupling control methods developed recently. However, the numerical computation effort seems burdensome for practical implementation, and evaluation of the control system robust stability was left open despite that it is common to be faced with process uncertainties in practice. Based on the analytical controller design developed recently [12,32], this paper proposes a new analytical design method for the decoupling controller matrix. As a result, the computation effort can be relieved significantly and the resultant decoupling controller matrix can be conveniently tuned on line to cope with process uncertainties. Besides, an intuitive approach in terms of the multivariable spectral radius criterion is presented for robust stability analysis of the resultant control system. The paper is organized as follows: Section 2 briefly introduces the decoupling control preconditions for MIMO processes. In Section 3, the desired system response transfer matrix is proposed according to the H_2 optimal performance specification. The ideal desired decoupling controller matrix and its practical form are derived analytically in Section 4. In Section 5, robust constraints for tuning the adjustable parameters in the decoupling controller matrix are established to check the closed-loop system stability in the presence of process additive/multiplicative uncertainties. A corresponding on-line tuning rule is also provided. Illustrative examples are given in Section 6 to demonstrate the superiority of the proposed method. Finally, conclusions are drawn in Section 7.

2. Decoupling control preconditions

Consider the general transfer matrix form for a MIMO process with time delays as the following:

$$G=\begin{bmatrix} g_{11} & \cdots & g_{1m} \\ \vdots & \vdots & \vdots \\ g_{m1} & \cdots & g_{mm} \end{bmatrix}, \tag{1}$$

where $g_{ij}=g_{0,ij}\mathrm{e}^{-\theta_{ij}s}$; $i,j=1,2,\ldots,m$, of which $g_{0,ij}$ is a delay-free, physically proper and stable transfer function. The application of the widely adopted unity feedback control structure to the process is illustrated in Fig. 1, where d_i and d_o represent respectively load disturbances injected into the process inputs and outputs, and n the system output measurement noises. The closed-loop system transfer matrix can be determined as

$$H=GC(I+GC)^{-1}. \tag{2}$$

The decoupled system response transfer matrix, ideally, should be in the form of

$$H=\begin{bmatrix} h_{11} & 0 & \cdots & \cdots & 0 \\ 0 & h_{22} & 0 & \cdots & 0 \\ 0 & \ddots & \ddots & \ddots & 0 \\ 0 & \cdots & \cdots & 0 & h_{mm} \end{bmatrix}, \tag{3}$$

where h_{ii} is a physically proper and stable transfer function, and $h_{ij}=0$ for $i\neq j$, $i,j=1,2,\ldots,m$. That is, H should be a non-singular diagonal transfer matrix, i.e., $H=\mathrm{diag}[h_{ii}]_{m\times m}$ and $\det(H)\neq 0$.

Combining Eqs. (2) and (3), the fundamental decoupling precondition can be ascertained as $\det[G(0)]\neq 0$, i.e., the multivariable process to be regulated must be non-singular in essence, or in other words, not ill-conditioned. Many existing decoupling control methods had based exactly on the process static gain matrix $G(0)$ for the decoupler design. This paper focuses on MIMO processes with $\det[G(0)]\neq 0$, that is, the process output responses deserve to be decoupled from each other in essence.

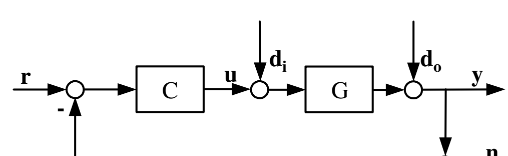

Fig. 1. Conventional unity feedback control structure.

It can be seen from Eq. (2) that the controller matrix C should be non-singular and bear the responsibility for keeping $(I+GC)^{-1}$ stable. Furthermore, to make system operation easier, there should be no cross-interaction between tuning each column controllers of C since each column controllers have the same input signal and there exists a mathematical postmultiplication relationship between G and C. Thereby, decoupling regulation for individual system output variables can be conveniently implemented on line.

3. Desired system response transfer matrix

For a multivariable process, the achievable system output responses are constrained by the process time delays and right-half-plane (RHP) zeros of the process transfer matrix determinant [4,8]. What is the optimal desired system response transfer matrix corresponding to the achievable system output responses? If this problem can be explicitly ascertained in the first place, the ideal desired decoupling controller matrix can then be inversely derived within the framework of a unity feedback control structure.

Note that the inverse of H in Eq. (3) is also a diagonal transfer matrix. Substituting Eq. (3) into Eq. (2), we obtain the following controller matrix:

$$C = G^{-1}(H^{-1}-I)^{-1} = \frac{\mathrm{adj}(G)}{\det(G)}\mathrm{diag}\left[\frac{h_{ii}}{1-h_{ii}}\right]_{m\times m}, \tag{4}$$

where $\mathrm{adj}(G)=[G^{ij}]^{\mathrm{T}}_{m\times m}$ is the adjoint of the process transfer matrix G, and G^{ij} denotes the complement minor corresponding to each transfer element g_{ij} of G. Denote the controller matrix $C=[c_{ij}]_{m\times m}$. According to the postmultiplication relationship between a square matrix and a diagonal matrix, each column controllers of C can be derived as

$$c_{ji} = \frac{G^{ij}}{\det(G)}\cdot\frac{h_{ii}}{1-h_{ii}}, \quad i,j=1,2,\ldots,m. \tag{5}$$

Let

$$p_{ij} = \frac{G^{ij}}{\det(G)} = p_{0,ij}\mathrm{e}^{L_{ij}s}, \quad i,j=1,2,\ldots,m, \tag{6}$$

where $p_{0,ij}$ represents the 'delay-free' part of p_{ij}, that is, at least one term in either of the nominator and denominator polynomials of $p_{0,ij}$ does not include any time delay and thus is rational. It can be seen from Eqs. (4) and (6) that $G^{-1}=[p_{ij}]^{\mathrm{T}}_{m\times m}$.

Define the 'inverse relative degree' of $p_{0,ij}$ to be n_{ij} $(i,j=1,2,\ldots,m)$, that is, the largest integer that satisfies

$$\lim_{s\to\infty}\frac{s^{n_{ij}-1}}{p_{0,ij}} = 0 \tag{7}$$

and let

$$N_i = \max\{n_{ij};\ j=1,2,\ldots,m\}, \quad i=1,2,\ldots,m, \tag{8}$$

$$\theta_i = \max\{L_{ij};\ j=1,2,\ldots,m\}, \quad i=1,2,\ldots,m. \tag{9}$$

It can be seen from Eq. (5) that each column controllers of C are related to the same diagonal element of H, i.e., all of c_{ji} $(j=1,2,\ldots,m)$ are corresponding to the same diagonal transfer function h_{ii} for $i=1,2,\ldots,m$. Note that θ_i in Eq. (9) is positive, which can be identified through Eq. (6) using the algebra of linear matrix. Some or even all of the ith column controllers c_{ji} $(j=1,2,\ldots,m)$ may not be physically realizable if the desired diagonal transfer function h_{ii} for the ith system output response does not include an equivalent time delay to offset θ_i. Also, it can be seen from Eq. (5) that if the relative degree of the delay-free part of h_{ii} were lower than N_i, some or even all of c_{ji} $(j=1,2,\ldots,m)$ would not be proper and thus cannot be physically implemented. In addition, $\det(G)$ may contain RHP zeros and if h_{ii} does not include these RHP zeros, each of c_{ji} $(j=1,2,\ldots,m)$ would be bundled with unstable poles, which is not allowed in practice.

Considering the H_2 optimal performance specification of IMC theory [8] with the above implementation constraints, the practical desired diagonal elements of the system response transfer matrix are proposed in the form of

$$h_{ii} = \frac{\mathrm{e}^{-\theta_i s}}{(\lambda_i s+1)^{N_i}}\prod_{k=1}^{q_i}\frac{-s+z_k}{s+z_k^*}, \quad i=1,2,\ldots,m, \tag{10}$$

where λ_i is an adjustable parameter set for obtaining the desirable response performance for the ith process output variable, and z_k $(k=1,2,\ldots,q_i)$ are the RHP zeros of $\det(G)$ excluding those canceled by the common RHP zeros of G^{ij} $(j=1,2,\ldots,m)$, and q_i is their number and z_k^* the complex conjugate of z_k.

With the above practical desired diagonal elements shown in Eq. (10), it can be ascertained from Eqs. (5)–(9) that at least one of each column controllers of C can be physically implemented in a proper and rational form, while the others of the corresponding column controllers can be practically implemented in series with some specified dead-time compensators. Thus, the desired diagonal system response transfer matrix shown in Eq. (3) can be realized, resulting in decoupling regulation for the individual system output variables.

The RHP zero number of $\det(G)$ can be ascertained by observing its Nyquist curve. For the case of $\det(G)$ with no RHP pole, the number of its Nyquist curve encircling the origin is equal to its RHP zero number according to the Nyquist stability criterion. Alternatively, the RHP zeros of $\det(G)$ can be explicitly computed using numerical solutions or any mathematical software package.

For MIMO processes with multiple time delays, $\det(G)$ may has infinite many RHP zeros due to multiple time delay terms involved. In the case that $\det(G)$ has infinite many RHP zeros but finite left-half-plane (LHP) zeros, the desired system transfer matrix are proposed with the following diagonal elements

$$h_{ii} = \frac{e^{-\theta_i s}}{(\lambda_i s+1)^{N_i}} \cdot \frac{\phi(s) e^{(\theta_{\max}-\theta_{\min})s}}{\phi(-s)} \prod_{k=1}^{q_i} \frac{-s-z_k}{s-z_k^*},$$
$$i = 1, 2, \ldots, m, \tag{11}$$

where z_k ($k = 1, 2, \ldots, q_i$) denote the finite LHP zeros of $\det(G)$ excluding those equal to the complex conjugates of the common RHP zeros of G^{ij} ($j = 1, 2, \ldots, m$), and $\theta_{\min}$ is the minimum of all the time delay factors involved in $\det(G)$ and $\theta_{\max}$ is the corresponding maximum. $\phi(s)$ is defined from the following reformulation of $\det(G)$, i.e.

$$\det(G) = \frac{\phi(s) e^{-\theta_{\min} s}}{\psi(s)},$$

where $\psi(s)$ is the least common denominator of all terms of $\det(G)$, and $\phi(s)$ is the corresponding numerator polynomial, in which there exists at least one term that does not contain any time delay and thus is rational. Apparently, $\det(G)$ has the same zeros with $\phi(s)$.

Note that $\phi(-s)$ in Eq. (11) is the complex conjugate of $\phi(s)$ and all the zeros of $\phi(-s)$ are located at the mirror points of $\phi(s)$ across the imaginary axis in the complex plane. In fact, it can be seen that $\phi(-s)$ may include time prediction factors that are not allowed in a physical all-pass filter, of which $\theta_{\max} - \theta_{\min}$ is the maximal time prediction length. The second part of h_{ii} shown in Eq. (11),

$$\frac{\phi(s) e^{(\theta_{\max}-\theta_{\min})s}}{\phi(-s)} \prod_{k=1}^{q_i} \frac{-s-z_k}{s-z_k^*}$$

thus can be ideally viewed as an all-pass filter, contributing to achieving the H_2 optimal performance specification for system output response. It should be noted that there inevitably exists RHP zero-pole cancellation in this filter, which, however, cannot be removed directly from the expression. A transformation is therefore needed to approximate it for implementation. For this reason, an analytical approximation method will be presented in Section 4. Besides, it should be noted that although $\det[G(s)]/\det[G(-s)]$ can be directly utilized to configure the all-pass part of these diagonal elements, an additional all-pass filter, $\psi(-s)/\psi(s)$, will be unfavorably introduced, which tends to degrade the achievable system performance and thus is not recommended.

For the case that $\det(G)$ has infinite number of RHP and LHP zeros, it is suggested to use those dominant RHP zeros of $\det(G)$ to construct the desired system response transfer matrix. This will facilitate the decoupling controller matrix to be analytically derived in a simple way, but at the cost of certain system performance. Hence, a balance needs to be made by the users between the achievable control system performance and the calculation complexity for deriving the corresponding controller matrix and its cost of implementation. According to the frequency domain control theory, e.g. [4], off-dominant zeros of a control system characteristic equation actually have little impact on the achievable system performance.

To sum up, the desired diagonal system response transfer matrix forms are listed in Table 1 according to four possible cases of the RHP zero distribution of $\det(G)$ in the complex plane.

4. Decoupling controller matrix design

According to the proposed diagonal system response transfer matrix listed in Table 1, the ideal desired decoupling controller matrix C can be derived by using Eq. (5). For instance, with Case 2 that $\det(G)$ has finite RHP zeros, each column controllers of the ideal desired decoupling controller matrix can be derived as

$$c_{\text{ideal},ji} = \frac{G^{ij}}{\det(G)} \cdot \frac{\frac{e^{-\theta_i s}}{(\lambda_i s+1)^{N_i}} \prod_{k=1}^{q_i} \frac{-s+z_k}{s+z_k^*}}{1 - \frac{e^{-\theta_i s}}{(\lambda_i s+1)^{N_i}} \prod_{k=1}^{q_i} \frac{-s+z_k}{s+z_k^*}},$$
$$i, j = 1, 2, \ldots, m. \tag{12}$$

For a MIMO process with multiple time delays, it can be seen from Eq. (6) that the first part of Eq. (12) is not a rational transfer function and thus difficult to be implemented in practice. In addition, the RHP zeros of $\det(G)$ will cause RHP zero-pole cancellation in Eq. (12), triggering the decoupling controller matrix to behave in an unstable manner. A practical form is therefore required to transform the ideal desired form of Eq. (12) for implementation.

Using Eqs. (6)–(9), we rearrange Eq. (12) in the form of

$$c_{ji} = \frac{D_{ij} e^{-(\theta_i - L_{ij})s}}{(\lambda_i s+1)^{N_i} \prod_{k=1}^{q_i}(s+z_k^*)} \cdot \frac{1}{1 - \frac{e^{-\theta_i s}}{(\lambda_i s+1)^{N_i}} \prod_{k=1}^{q_i} \frac{-s+z_k}{s+z_k^*}},$$
$$i, j = 1, 2, \ldots, m, \tag{13}$$

where λ_i becomes the common adjustable parameter of each column controllers in C and

$$D_{ij} = p_{0,ij} \prod_{k=1}^{q_i}(-s+z_k). \tag{14}$$

Note that the second part of c_{ji} has the following properties:

$$\lim_{s\to\infty} \frac{1}{1 - \frac{e^{-\theta_i s}}{(\lambda_i s+1)^{N_i}} \prod_{k=1}^{q_i} \frac{-s+z_k}{s+z_k^*}} = 1, \tag{15}$$

$$\lim_{s\to 0} \frac{1}{1 - \frac{e^{-\theta_i s}}{(\lambda_i s+1)^{N_i}} \prod_{k=1}^{q_i} \frac{-s+z_k}{s+z_k^*}} = \infty. \tag{16}$$

Thus it can be viewed as a special integrator with relative degree of zero that is capable of eliminating the steady-

Table 1
Ideal desired forms of system response transfer matrix and decoupling controller matrix

	det(G)	h_{ii} $(i=1,2,\ldots,m)$	c_{ji} $(i,j=1,2,\ldots,m)$
Case 1	No RHP zero	$\frac{e^{-\theta_i s}}{(\lambda_i s+1)^{N_i}}$	$\frac{D_{ij}e^{-(\theta_i-L_{ij})s}}{(\lambda_i s+1)^{N_i}}\cdot\frac{1}{1-\frac{e^{-\theta_i s}}{(\lambda_i s+1)^{N_i}}}$, $D_{ij}=p_{0,ij}$.
Case 2	Finite RHP zeros [z_k $(k=1,2,\ldots,q_i)$ = the RHP zeros excluding those canceled by the common RHP zeros of $G^{ij}(j=1,2,\ldots,m)$]	$\frac{e^{-\theta_i s}}{(\lambda_i s+1)^{N_i}}\prod_{k=1}^{q_i}\frac{-s+z_k}{s+z_k^*}$	$\frac{D_{ij}e^{-(\theta_i-L_{ij})s}}{(\lambda_i s+1)^{N_i}\prod_{k=1}^{q_i}(s+z_k^*)}\cdot\frac{1}{1-\frac{e^{-\theta_i s}}{(\lambda_i s+1)^{N_i}}\prod_{k=1}^{q_i}\frac{-s+z_k}{s+z_k^*}}$, $D_{ij}=p_{0,ij}\prod_{k=1}^{q_i}(-s+z_k)$.
Case 3	Infinite RHP and LHP zeros [z_k $(k=1,2,\ldots,q_i)$ = the dominant RHP zeros excluding those canceled by the common RHP zeros of G^{ij} $(j=1,2,\ldots,m)$]		
Case 4	Infinite RHP but finite LHP zeros [z_k $(k=1,2,\ldots,q_i)$ = the LHP zeros excluding those equal to the complex conjugates of the common RHP zeros of G^{ij} $(j=1,2,\ldots,m)$]	$\frac{e^{-\theta_i s}}{(\lambda_i s+1)^{N_i}}\cdot\frac{\phi(s)e^{(\theta_{\max}-\theta_{\min})s}}{\phi(-s)}\times\prod_{k=1}^{q_i}\frac{-s-z_k}{s-z_k^*}$	$\frac{G^{ij}D_{ij}\psi(s)e^{(\theta_{\min}-\theta_i)s}}{(\lambda_i s+1)^{N_i}\prod_{k=1}^{q_i}(s-z_k^*)}\cdot\frac{1}{1-\frac{D_{ij}\phi(s)e^{-\theta_i s}}{(\lambda_i s+1)^{N_i}\prod_{k=1}^{q_i}(s-z_k^*)}}$, $D_{ij}=\frac{e^{(\theta_{\max}-\theta_{\min})s}}{\phi(-s)}\prod_{k=1}^{q_i}(-s-z_k)$.

state system output offset, which in fact can be implemented using a positive feedback control unit as shown in Fig. 2.

For D_{ij} in Eq. (13), the linear fractional Padé expansion has been shown to be able to approximate the similar function with high accuracy [12,32], so it is adopted here for D_{ij} by

$$D_{U/V}=\frac{\sum_{k=0}^{U}a_k s^k}{\sum_{k=0}^{V}b_k s^k}, \tag{17}$$

where U and V are the user-specified orders to achieve the desirable system response performance specification, and the constant coefficients a_k and b_k are determined by the following two matrix equations:

$$\begin{bmatrix} a_0 \\ a_1 \\ \vdots \\ a_U \end{bmatrix}=\begin{bmatrix} d_0 & 0 & 0 & \cdots & 0 \\ d_1 & d_0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \cdots & \vdots \\ d_U & d_{U-1} & d_{U-2} & \cdots & d_{U-V} \end{bmatrix}\begin{bmatrix} b_0 \\ b_1 \\ \vdots \\ b_V \end{bmatrix}, \tag{18}$$

$$\begin{bmatrix} d_U & d_{U-1} & \cdots & d_{U-V+1} \\ d_{U+1} & d_U & \cdots & d_{U-V+2} \\ \vdots & \vdots & \ddots & \vdots \\ d_{U+V-1} & d_{U+V-2} & \cdots & d_U \end{bmatrix}\begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_V \end{bmatrix}=-\begin{bmatrix} d_{U+1} \\ d_{U+2} \\ \vdots \\ d_{U+V} \end{bmatrix}, \tag{19}$$

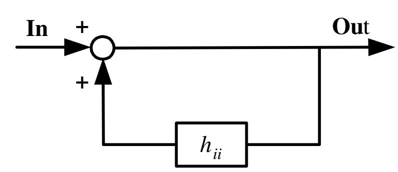

Fig. 2. Positive feedback control unit.

where d_k $(k=0,1,\ldots,U+V)$ are the constant coefficients of each term in the Maclaurin expansion series of D_{ij} shown in Eq. (14), i.e.,

$$d_k=\frac{1}{k!}\lim_{s\to 0}\frac{d^k D_{ij}}{ds^k},\quad k=0,1,\ldots,U+V \tag{20}$$

and b_0 should be chosen as

$$b_0=\begin{cases} 1, & b_k\geqslant 0, \\ -1, & b_k<0. \end{cases} \tag{21}$$

It should be noted that Eqs. (18) and (19) can be obtained by substituting Eq. (17) into the Maclaurin expansion series of D_{ij} and then comparing the constant coefficients of the complex variable at both sides of the equation.

Remark. Note that a physical constraint for specifying U and V, $U-V\leqslant N_i+q_i$, is required so that c_{ji} obtained by substituting Eq. (17) into Eq. (13) can be proper and realizable. Generally, V may be specified first and then U can be taken by $U = V + N_i + q_i$ so as to obtain the best approximation level. From a mathematical point of view, it is preferred to reformulate $p_{0,ij}$ in Eq. (6) first in the form of

$$p_{0,ij} = \frac{\alpha(s)[1 + \eta_1(s)\mathrm{e}^{-\sigma_1 s} + \cdots + \eta_{m-\mu}(s)\mathrm{e}^{-\sigma_{m-\mu}s}]}{\beta(s)[1 + \xi_1(s)\mathrm{e}^{-\delta_1 s} + \cdots + \xi_{m-v}(s)\mathrm{e}^{-\delta_{m-v}s}]},$$

where $\alpha(s)$ and $\beta(s)$ are rational polynomials, $\sigma_k > 0$ $(k = 1, 2, \ldots, m - \mu)$, $\delta_k > 0$ $(k = 1, 2, \ldots, m - v)$, $\mu < m$ and $v < m$. U then can be taken initially as the order of $\alpha(s)$ and V the order of $\beta(s)$, in view of that those terms with time delays in the nominator and denominator decay much faster than $\alpha(s)$ and $\beta(s)$ as $s \to \infty$. It is obvious that increasing the orders of U and V will result in better approximation level, but at the cost of higher computation effort and implementation complexity.

As for the choice of b_0 in Eq. (21), the purpose here is to keep all of b_k $(k = 0, 1, \ldots, V)$ the same sign so as to exclude any possibility of RHP zeros in the denominator of $D_{U/V}$ in Eq. (17). This is a necessary but not sufficient condition. The existence of such RHP zeros may be identified by the Routh–Hurwitz stability criterion. It is therefore suggested to utilize the Routh–Hurwitz criterion (or its simplified version [34]) to check the stability of such a high order approximation before being used in practice. Nevertheless, the proposed approximation in terms of $V \leqslant 2$ can be directly utilized without such exercise, and thus is recommended in engineering practice for simplicity.

For other cases of the RHP zero distribution of $\det(G)$ as categorized in Section 3, the desired decoupling controller matrix can be derived analytically following a similar design procedure as the above. They are summarized in Table 1, in which D_{ij} of each case can be approximated in a rational form for implementation using Eqs. (17)–(21).

5. Control system stability analysis

As analytical approximation is utilized to transform the ideal desired decoupling controller matrix in Table 1 for implementation, the stability of the resultant control system needs to be checked. Besides, there exist always the process unmodeled dynamics in practice. Evaluation of the control system robust stability needs to be conducted in the presence of process uncertainties, and correspondingly, on-line tuning of the adjustable parameters in the decoupling controller matrix needs to be studied to cope with the process uncertainties.

For the nominal control system, it can be derived from Fig. 1 that the transfer matrix from the system inputs r, d_{i}, d_{o} and n to the outputs y and u is

$$\begin{bmatrix} y \\ u \end{bmatrix} = \begin{bmatrix} GC(I+GC)^{-1} & (I+GC)^{-1}G & I - GC(I+GC)^{-1} & -GC(I+GC)^{-1} \\ C(I+GC)^{-1} & -C(I+GC)^{-1}G & C(I+GC)^{-1} & -C(I+GC)^{-1} \end{bmatrix} \begin{bmatrix} r \\ d_{\mathrm{i}} \\ d_{\mathrm{o}} \\ n \end{bmatrix}. \tag{22}$$

It can be seen that r, d_{o} and n have similar impact on y and u. Hence, the stability analysis for the nominal system can be limited to the submatrix connecting r and d_{i} to y and u. In view of that G has been assumed to be non-singular and stable, and that there exists an equivalent transformation $GC(I + GC)^{-1} = I - (I + GC)^{-1}$, the sufficient and necessary condition for holding the nominal system stability can be concluded as that $(I + GC)^{-1}$ must be stable, which may be checked graphically by using the Nyquist curve criterion, or numerically by computing whether $\det(I + GC)$ has any RHP zeros.

In the presence of process uncertainties, the transfer matrix in Eq. (22) could become very complex and the closed-loop control system may lose stability in an intangible manner. How to assess all of the stabilizing set of C for various process uncertainties is difficult and has remained as an open issue in the process control community [4,33]. Here, the robust stability analysis is focused on the process additive, multiplicative input and output uncertainties, which are commonly encountered in engineering practice. Usually, the process additive uncertainties, shown in Fig. 3(a), can be viewed as parameter perturbation to the process transfer matrix and the actual process family may be described as $\Pi_{\mathrm{A}} = \{\widehat{G}_{\mathrm{A}}(s) : \widehat{G}_{\mathrm{A}}(s) = G(s) + \Delta_{\mathrm{A}}\}$, where Δ_{A} is assumed to be stable. The process multiplicative input uncertainties, shown in Fig. 3(b), can be loosely interpreted as the process input actuator uncertainties and the actual process family may be described as $\Pi_{\mathrm{I}} = \{\widehat{G}_{\mathrm{I}}(s) : \widehat{G}_{\mathrm{I}}(s) = G(s)(I + \Delta_{\mathrm{I}})\}$, where Δ_{I} is assumed to be stable. The process multiplicative output uncertainties, shown in Fig. 3(c), can be practically viewed as the process output measurement uncertainties and the actual process family may be described as $\Pi_{\mathrm{O}} = \{\widehat{G}_{\mathrm{O}}(s) : \widehat{G}_{\mathrm{O}}(s) = (I + \Delta_{\mathrm{O}})G(s)\}$, where Δ_{O} is assumed to be stable. Note that many other types of process unstructured or structured uncertainties may be incorporated into the above-mentioned types of process uncertainties in practice [4]. Hence, the robust stability analysis presented in the following, without loss of generality, can be applied to a wide variety of process uncertainties.

By reorganizing the perturbed control system in the form of the standard $M - \Delta$ structure for robustness analysis [33], the transfer matrix from the outputs to inputs of Δ_{A}, Δ_{I} and Δ_{O} can be derived respectively as

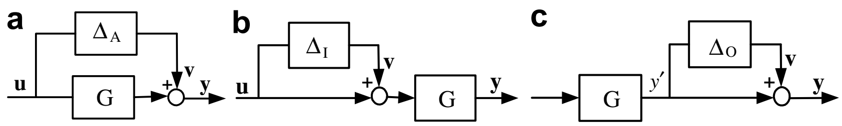

Fig. 3. The process additive (a), multiplicative input (b), and output (c) uncertainties.

$$M_A = -C(I + GC)^{-1}, \tag{23}$$

$$M_I = -C(I + GC)^{-1}G, \tag{24}$$

$$M_O = -GC(I + GC)^{-1}. \tag{25}$$

Note that M_A, M_I and M_O hold stability provided that the nominal control system has been conducted stable, i.e., the transfer matrix in Eq. (22) has been guarded stable.

Then using the small gain theorem, the robust stability constraints can be obtained as

$$\|C(I + GC)^{-1}\|_\infty < \frac{1}{\|\Delta_A\|_\infty}, \tag{26}$$

$$\|C(I + GC)^{-1}G\|_\infty < \frac{1}{\|\Delta_I\|_\infty}, \tag{27}$$

$$\|GC(I + GC)^{-1}\|_\infty < \frac{1}{\|\Delta_O\|_\infty}. \tag{28}$$

The robust stability constraints shown in Eqs. (26)–(28), however, are not analytical and the computation effort for H infinity norm is considerably large, especially for MIMO processes with multiple time delays. To relieve the computation burden, the equivalent relationship between the small gain theorem and the multivariable spectral radius stability criterion [4] can be explored, i.e.,

$$\|M\Delta\|_\infty < 1 \iff \rho(M\Delta) < 1 \quad \forall\omega \in [0, \infty).$$

Thereby, the above robust stability constraints can be reformulated respectively as

$$\rho(C(I + GC)^{-1}\Delta_A) < 1 \quad \forall\omega \in [0, \infty), \tag{29}$$

$$\rho(C(I + GC)^{-1}G\Delta_I) < 1 \quad \forall\omega \in [0, \infty), \tag{30}$$

$$\rho(GC(I + GC)^{-1}\Delta_O) < 1 \quad \forall\omega \in [0, \infty). \tag{31}$$

Correspondingly, the spectral radius stability constraints shown in Eqs. (29)–(31) can be checked graphically by observing whether the magnitude plots of the left sides of Eqs. (29)–(31) fall below the unity for all $\omega \in [0, +\infty)$. In this way, the admissible tuning range of the adjustable parameters of the decoupling controller matrix C can be numerically ascertained. With a given bound of Δ_A, Δ_I or Δ_O, Eqs. (29)–(31) may be employed to evaluate the control system robust stability. Even though the precise forms of these uncertainties are usually not available in practice, they may be characterized in the forms similar to those illustrated in Section 6.

Combined with Eq. (10), it can be seen that with small adjustable parameter λ_i in the decoupling controller matrix C, the corresponding ith system output response becomes faster, but the output energy of the ith column controllers of C and their corresponding actuators grows larger, tending to surpass their output capacities in practice. Besides, more aggressive dynamic behavior of the ith system output response is likely to occur in the presence of process uncertainties. On the contrary, increasing λ_i will slow down the corresponding ith system output response, but the output energy of the ith column controllers of C and their corresponding actuators will be required smaller. Consequently, less aggressive dynamic behavior of the ith system output response will be yield in the presence of process uncertainties. Therefore, tuning the adjustable parameters λ_i ($i = 1, 2, \ldots, m$) is a trade-off between the achievable system response performance and the output capacities of C and its corresponding actuators.

Based on the robust stability analysis and our simulation experience, it is suggested to set the adjustable parameters λ_i ($i = 1, 2, \ldots, m$) within the range of $(1.0\text{–}10)\theta_i$ initially, and then adjust them monotonously on line to achieve a desirable specification of system output responses.

To cope with the process uncertainties, it is suggested to increase monotonously the adjustable parameters λ_i ($i = 1, 2, \ldots, m$) of C on line, so that the nominal system response will be gradually slowed down for better system robust stability. If by doing so, the control system performance and robust stability are still not acceptable, the process re-identification will need to be conducted to obtain a better process model for the derivation of C, so that the process unmodeled dynamics can be effectively reduced to

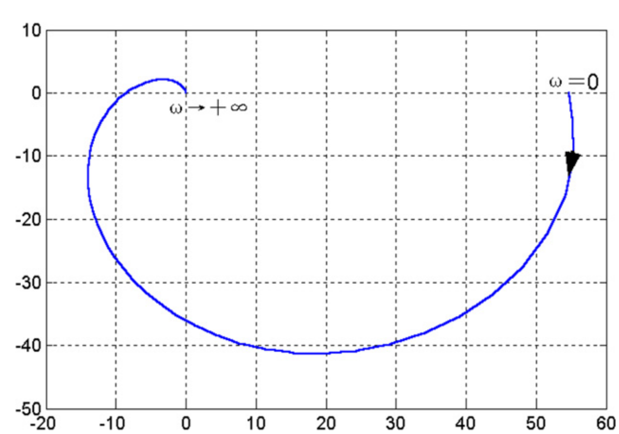

Fig. 4. Nyquist curve of the transfer matrix determinant of Example 1.

achieve better nominal system performance and robust stability.

6. Illustrative examples

Two widely studied examples are employed here to demonstrate the effectiveness and superiority of the proposed decoupling control method. The first example is used for the case that the process transfer matrix determinant has no RHP zero, and the second example is for one of the opposite cases.

Example 1. Consider the widely studied 3×3 industrial distillation column [35]

$$G=\begin{bmatrix} \frac{1.986e^{-0.71s}}{66.7s+1} & \frac{-5.24e^{-60s}}{400s+1} & \frac{-5.984e^{-2.24s}}{14.29s+1} \\ \frac{-0.0204e^{-0.59s}}{(7.14s+1)^2} & \frac{0.33e^{-0.68s}}{(2.38s+1)^2} & \frac{-2.38e^{-0.42s}}{(1.43s+1)^2} \\ \frac{-0.374e^{-7.75s}}{22.22s+1} & \frac{11.3e^{-3.79s}}{(21.74s+1)^2} & \frac{9.811e^{-1.59s}}{11.36s+1} \end{bmatrix}.$$

The Nyquist curve of the process transfer matrix determinant is drawn in Fig. 4. It is seen that the Nyquist curve does not encircle the origin, so there is no RHP zero in $\det(G)$. Using Eq. (6) yields $L_{11}=0.71$, $L_{12}=0.8$, $L_{13}=-1.4$. Thus, $\theta_1=0.8$ can be obtained from Eq. (9).

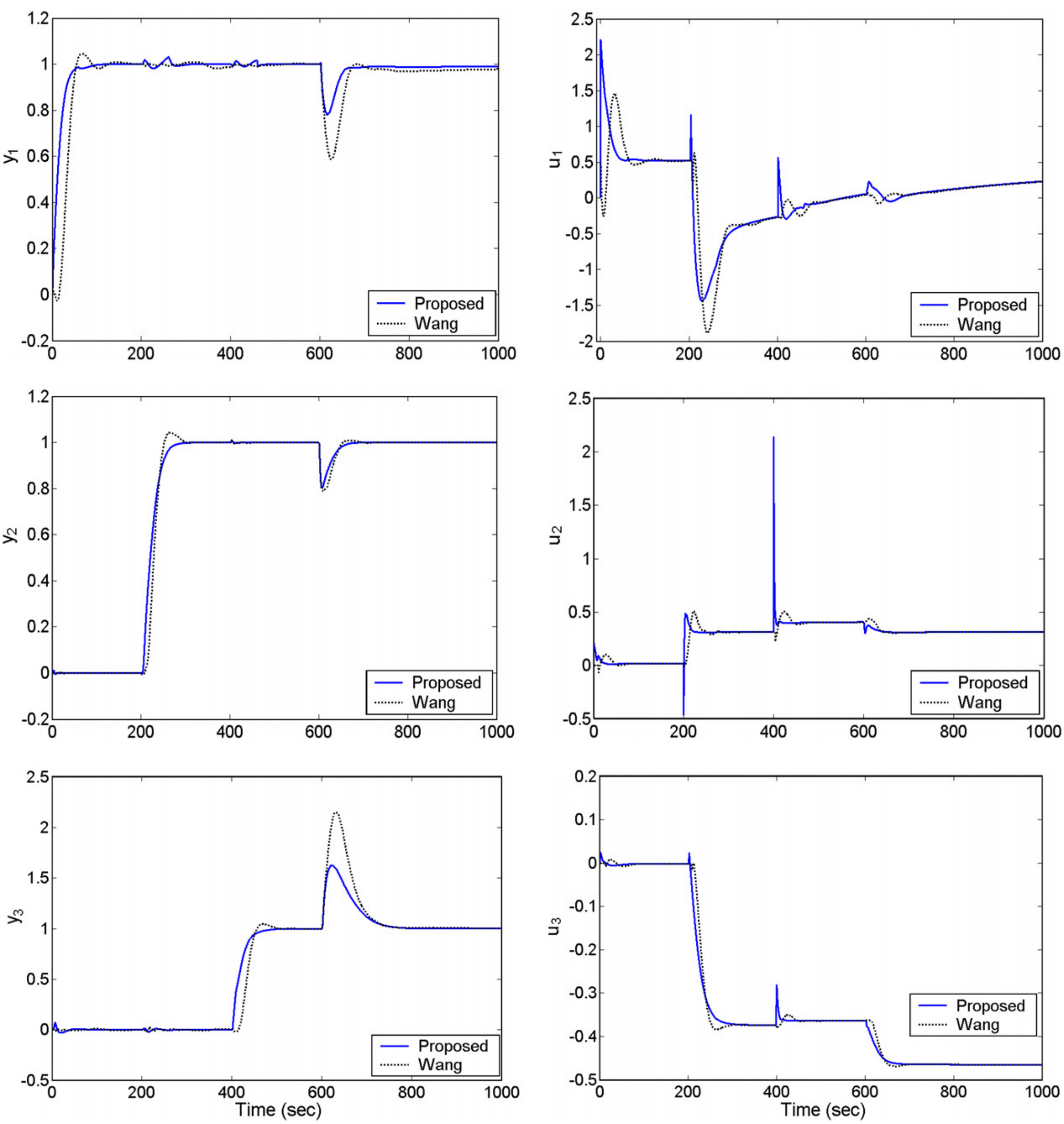

Fig. 5. Nominal system responses for Example 1.

Then, Eq. (7) gives $n_{11}=1$, $n_{12}=1$, $n_{13}=0$, so $N_1=1$ can be derived from Eq. (8). Similarly, the use of Eqs. (6)–(9) can yield $\theta_2=0.68$, $\theta_3=1.85$ and $N_2=2$, $N_3=1$. According to the design formula for Case 1 in Table 1, the diagonal elements of the desired system response transfer matrix are obtained as

$$h_{11}=\frac{\mathrm{e}^{-0.8s}}{\lambda_1 s+1},\quad h_{22}=\frac{\mathrm{e}^{-0.68s}}{(\lambda_2 s+1)^2},\quad h_{33}=\frac{\mathrm{e}^{-1.85s}}{\lambda_3 s+1}.$$

Thereby, the decoupling controller matrix can be derived using the analytical design formulae given in Table 1 and Eqs. (17)–(21). Wang's method [31] had demonstrated its superiority over many existing decoupling control methods for this process, and thus is adopted here for comparison. To obtain the similar controller orders with those of Wang's method for a fair comparison, the following executable controller matrix form is given:

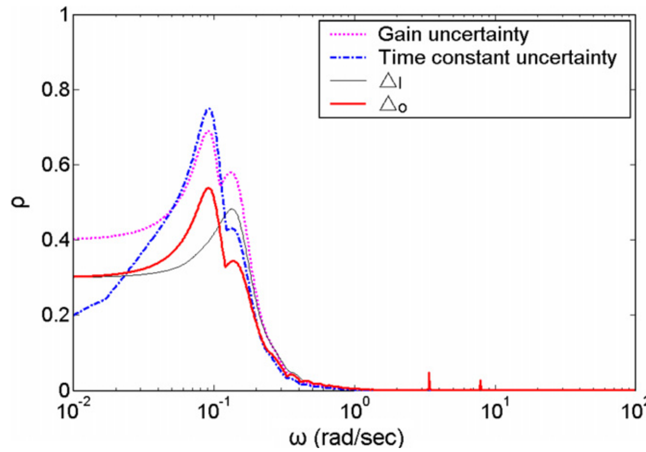

Fig. 6. Magnitude plots of spectral radius for Example 1.

here for benchmark comparison as well as in Wang et al. [31]. By adding a unit step change at $t=0$, $t=200$ and $t=400$ to the ternary setpoint inputs respectively, and a

$$
\begin{aligned}
c_{11}&=f_1\cdot\frac{14543s^2+256.3578s+0.5502}{(\lambda_1 s+1)(438.7353s+1)}\mathrm{e}^{-0.09s},\\
c_{21}&=f_1\cdot\frac{12391s^3+746.2116s^2+9.7508s+0.0199}{(\lambda_1 s+1)(3940.3s^2+447.8424s+1)},\\
c_{31}&=f_1\cdot\frac{1736.5s^3-21.7287s^2-0.8474s-0.002}{(\lambda_1 s+1)(4815.4s^2+449.8302s+1)}\mathrm{e}^{-2.2s},\\
c_{12}&=f_2\cdot\frac{4773900s^6-6620600s^5-3286200s^4-532380s^3-41045s^2-526.1791s-0.296}{(\lambda_2 s+1)^2(611700s^4+109510s^3+12128s^2+465.9313s+1)}\mathrm{e}^{-3.73s},\\
c_{22}&=f_2\cdot\frac{13471000s^6+3306200s^5+892990s^4+117120s^3+6709.9s^2+142.0148s+0.3149}{(\lambda_2 s+1)^2(336570s^4+33465s^3+9959.2s^2+461.3811s+1)},\\
c_{32}&=f_2\cdot\frac{-197040s^5-104730s^4-29099s^3-4024.9s^2-171.9233s-0.374}{(\lambda_2 s+1)^2(257300s^4+55907s^3+10254s^2+461.9346s+1)}\mathrm{e}^{-2.2s},\\
c_{13}&=f_3\cdot\frac{400930s^4+33536s^3+1342.3s^2+31.5279s+0.2638}{(\lambda_3 s+1)(33025s^3+3869.9s^2+447.5041s+1)}\mathrm{e}^{-1.79s},\\
c_{23}&=f_3\cdot\frac{16790s^3+1582.9s^2+39.2646s+0.0885}{(\lambda_3 s+1)(511.4853s^2+440.0233s+1)},\\
c_{33}&=f_3\cdot\frac{2195s^3+212.3057s^2+5.2157s+0.01}{(\lambda_3 s+1)(1319.1s^2+441.8636s+1)}\mathrm{e}^{-0.26s},
\end{aligned}
$$

where

$$f_1=\frac{1}{1-\frac{\mathrm{e}^{-0.8s}}{\lambda_1 s+1}},\quad f_2=\frac{1}{1-\frac{\mathrm{e}^{-0.68s}}{(\lambda_2 s+1)^2}},\quad f_3=\frac{1}{1-\frac{\mathrm{e}^{-1.85s}}{\lambda_3 s+1}}.$$

Note that f_1, f_2 and f_3 can be implemented respectively using the feedback control unit shown in Fig. 2. The adjustable parameters are taken as $\lambda_1=15$, $\lambda_2=12$ and $\lambda_3=18$ to have the similar rising speed of the system output responses with Wang's method. It should be mentioned that although a unit step change of the setpoint inputs is seldom adopted in engineering practice, it is performed

step change of load disturbance with a magnitude of 0.1 to each of the ternary process inputs at $t=600$ simultaneously, we obtain the system responses shown in Fig. 5. It should be noted that the simulation solver option is chosen as ode5 (Dormand-Prince) and the simulation step size is fixed as 0.02 throughout this paper.

From Fig. 5, it is clearly seen that there is no overshoot in the setpoint responses by using the proposed method, and the ternary process output responses are almost decoupled from each other. Moreover, obviously improved load disturbance rejection performance is obtained. It should be noted that better nominal system performance for the set-point tracking and the load disturbance rejection can be conveniently obtained in the proposed method by gradually decreasing the adjustable parameters λ_1, λ_2 and λ_3 on line, or by using a higher order controller matrix form that can be analytically obtained using the design formulae Eqs. (17)–(21). Besides, it should be mentioned that conventional PID controllers are not capable of obtaining acceptable system response performance or even cannot stabilize the system output responses, due to the petty approximation capacity for the ideal desired decoupling controller matrix shown in Table 1. The same conclusion was illustrated in Wang et al. [31] by using the Nyquist curve comparison.

To demonstrate robustness of the proposed method, the same perturbation tests are conducted as in Wang et al. [31], that is, all the static gains of each element in the process transfer matrix are actually 40% larger, and in another case, all the time constants of each element in the process transfer matrix are assumed 40% larger to introduce the unmodeled dynamics. According to the robust stability analysis given in Section 5, the magnitude plots of spectral radius for identifying robust stability of the corresponding perturbed systems are shown in Fig. 6. It can be seen that both of the peak values (dotted and dash dot lines) are much less than the unity, indicating that the proposed control system facilitates good robust stability. Correspondingly, the perturbed system responses are provided

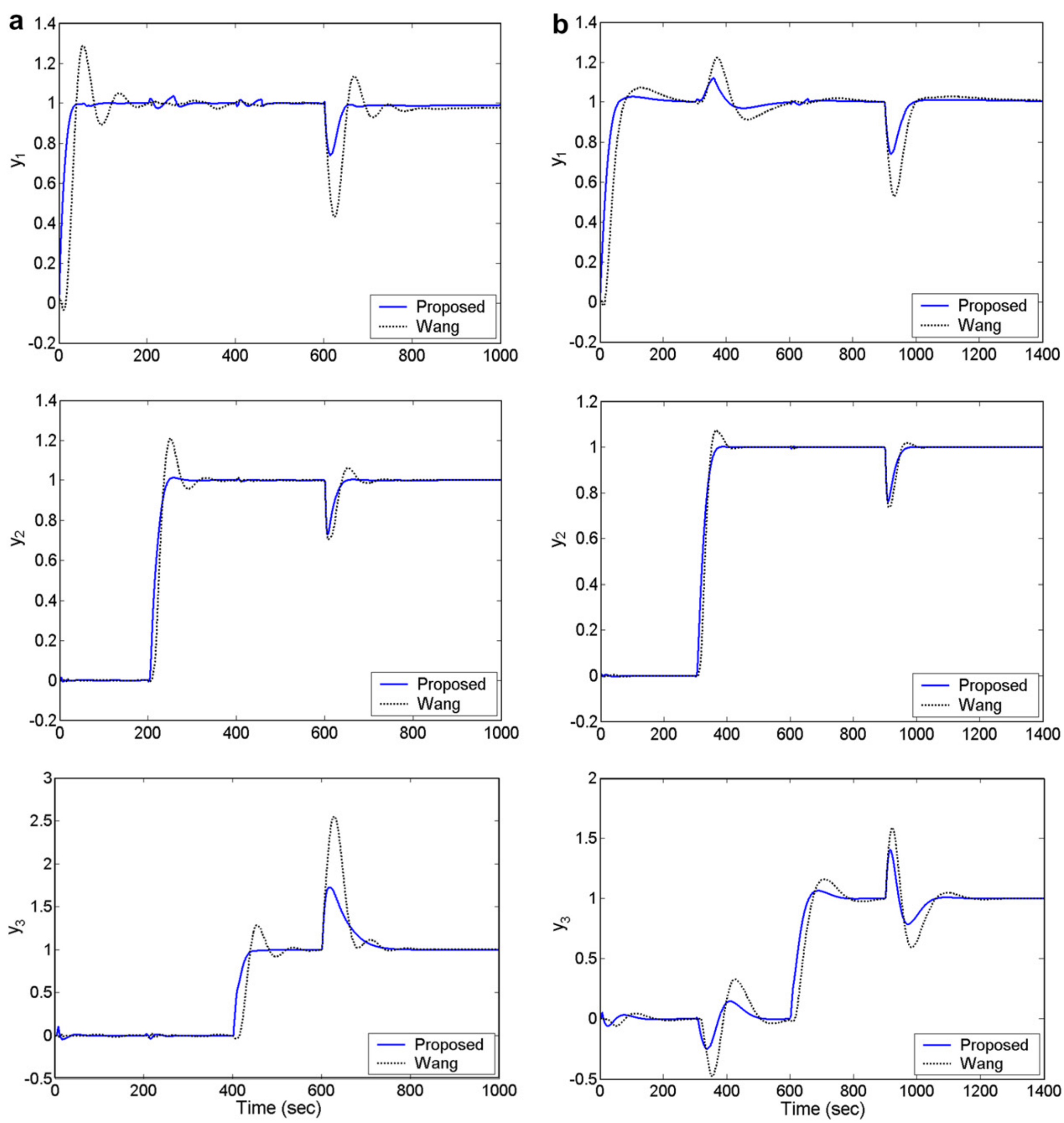

Fig. 7. Perturbed system responses for Example 1 due to the process static gains (a) and time constants (b) variation.

respectively in Fig. 7(a) and (b). Note that Fig. 7(a) has demonstrated also that the process static gains perturbation does not affect decoupling regulation of the system output responses, as can be concluded from the analytical controller matrix design procedure given in Section 4. The corresponding ternary control outputs have varied little compared with those shown in Fig. 5, and thus are omitted for saving space.

To demonstrate robust stability of the proposed control system against the process multiplicative uncertainties, assume that there actually exist the process multiplicative input uncertainties $\Delta_{\mathrm{I}} = \mathrm{diag}[(s+0.3)/(s+1), (s+0.2)/(s+1), (s+0.2)/(s+1)]_{3\times3}$. This can be loosely interpreted as the first process input actuator has up to 100% uncertainty at high frequencies and almost 30% uncertainty in the low frequency range, while the other two process inputs increase by up to 100% uncertainty at high frequencies and by almost 20% uncertainty in the low frequency range. In another case, assume that there exist the process multiplicative output uncertainties $\Delta_{\mathrm{O}} = \mathrm{diag}[-(s+0.2)/(2s+1), -(s+0.2)/(2s+1), -(s+0.3)/(2s+1)]_{3\times3}$, which can be practically viewed as the first two process output measurements obtained from the corresponding sensors decrease by up to 50% uncertainty at high frequencies and by almost 20% uncertainty in the low frequency range, while the third process output measurement decreases by up to 50% uncertainty at high frequencies and by almost 30% uncertainty in the low frequency range. Fig. 6 has shown the corresponding magnitude plots of spectral radius based on the assumed Δ_{I} (thin solid line) and Δ_{O} (thick solid line), both of which indicate that the proposed control system could preserve robust stability well. The corresponding perturbed system responses are shown in Fig. 8.

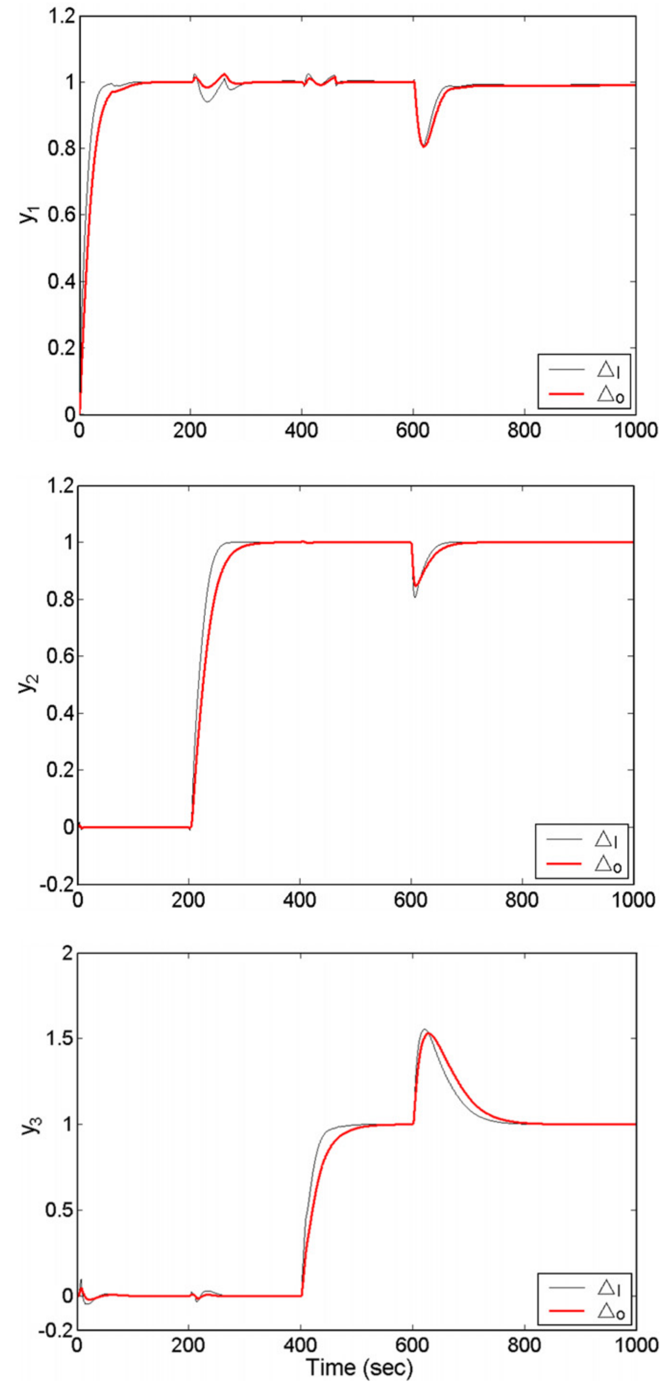

Fig. 8. Perturbed system responses for Example 1 due to the process multiplicative uncertainties.

Example 2. Consider the binary process studied by Jerome and Ray [9]

$$G = \begin{bmatrix} \dfrac{1.05e^{-4.58s}}{1.64s+1} & \dfrac{0.32}{(1.6s+1)(1.61s+1)} \\ \dfrac{1.18e^{-15.2s}}{3.6s+1} & \dfrac{0.9}{(4.5s+1)(4.51s+1)} \end{bmatrix}.$$

It had been ascertained in Jerome's paper that there are infinite many RHP zeros and four LHP zeros in the process transfer matrix determinant, and the four LHP zeros are the approximate roots of the following polynomial:

$$\chi(s) = (1.64s+1)(4.0542s+1)(40.459s^2+11.116s+1).$$

Thus this process belongs to Case 4 in Table 1.

First, we write the process transfer matrix determinant in the form of

$$\det(G) = \frac{[0.945(1.6s+1)(1.61s+1)(3.6s+1) - 0.3776(1.64s+1)(4.5s+1)(4.51s+1)e^{-10.62s}]e^{-4.58s}}{(1.64s+1)(4.5s+1)(4.51s+1)(1.6s+1)(1.61s+1)(3.6s+1)}.$$

Thereby, it follows that $\theta_{\min} = 4.58$, $\theta_{\max} = 15.2$, and $\phi(s)$ is the polynomial within the square bracket of the numerator and $\psi(s)$ the denominator polynomial.

Subsequently, using Eqs. (6)–(9) yields $\theta_1 = \theta_2 = 4.58$ and $N_1 = N_2 = 2$. So the diagonal elements of the desired system response transfer matrix can be obtained using Table 1 as

$$h_{11} = \frac{D\phi(s)\mathrm{e}^{-4.58s}}{\chi(s)(\lambda_1 s + 1)^2}, \quad h_{22} = \frac{D\phi(s)\mathrm{e}^{-4.58s}}{\chi(s)(\lambda_2 s + 1)^2},$$

where

$$D = \frac{(-1.64s + 1)(-4.0542s + 1)(40.459s^2 - 11.116s + 1)}{0.945(-1.6s + 1)(-1.61s + 1)(-3.6s + 1)\mathrm{e}^{-10.62s} - 0.3776(-1.64s + 1)(-4.5s + 1)(-4.51s + 1)}.$$

Note that D cannot be directly implemented due to the RHP zero-pole cancellation. The proposed analytical approximation formula Eq. (17) is therefore used by taking $U = 2$ and $V = 1$ for simplicity, resulting in its rational approximation

$$D_{2/1} = \frac{20.1786s^2 + 10.0787s + 1.7624}{0.5868s + 1}.$$

Then, the use of the design formula for Case 4 in Table 1 results in the decoupling controller matrix

$$C = D_c \cdot \begin{bmatrix} \dfrac{0.9F_1(1.64s + 1)}{(\lambda_1 s + 1)^2} & -\dfrac{0.32F_2(1.64s + 1)(4.5s + 1)(4.51s + 1)}{(\lambda_2 s + 1)^2(1.6s + 1)(1.61s + 1)} \\ -\dfrac{1.18F_1(1.64s + 1)(4.5s + 1)(4.51s + 1)\mathrm{e}^{-15.2s}}{(\lambda_1 s + 1)^2(3.6s + 1)} & \dfrac{1.05F_2(4.5s + 1)(4.51s + 1)\mathrm{e}^{-4.58s}}{(\lambda_2 s + 1)^2} \end{bmatrix},$$

where

$$D_c = \frac{D_{2/1}(1.6s + 1)(1.61s + 1)(3.6s + 1)}{\chi(s)},$$

$$F_1 = \frac{1}{1 - \frac{D_{2/1}\phi(s)\mathrm{e}^{-4.58s}}{\chi(s)(\lambda_1 s+1)^2}}, \quad F_2 = \frac{1}{1 - \frac{D_{2/1}\phi(s)\mathrm{e}^{-4.58s}}{\chi(s)(\lambda_2 s+1)^2}}.$$

Note that both F_1 and F_2 can be practically implemented using the feedback control unit shown in Fig. 2.

It should be noted that Jerome and Ray [9], based on the standard IMC structure, suggested a controller matrix design that seemed to be capable of dumping all of the undesirable response dynamics on a single-output variable to obtain apparently improved response performance of the other output variables. For comparison, the optimization of the second output variable at the cost of severely degraded response performance of the first output variable, is repeated here. In our proposed method, the adjustable parameters are taken as $\lambda_1 = 3.5$ and $\lambda_2 = 3.0$ to obtain the similar rising speed of the system output responses with Jerome's method. By adding a unit step change at $t = 0$ and 150 respectively to the binary setpoint inputs, and an inverse step change of load disturbance with a magnitude of 0.1 to the binary process inputs at $t = 300$ simultaneously, we obtain the system output responses shown in Fig. 9.

From Fig. 9, it can be seen that entirely decoupled binary system output responses have been achieved by using the proposed method (solid line), and the second process output response is comparable with that of Jerome's method, baring from a small time delay that is employed to yield the decoupled output responses. Note that Jerome's method has resulted in severe oscillation in the first process output response and the oscillatory binary control output signals are less likely acceptable from a practical point of view. Therefore, it is demonstrated that the method of sacrificing the dynamic response performance of one process output for improvement of the other process output responses is worthless in contrast to the proposed decoupling control method.

To compare the control system robust stability, assume that all the time constants of the process transfer matrix are actually 20% larger to introduce the unmodeled dynamics. The perturbed system output responses are provided in Fig. 10.

It is obvious again that the proposed decoupling control system holds good robust stability in the presence of these severe process uncertainties. It should be noted that the control outputs of the proposed control system have varied only slightly while those of Jerome's method exhibited much severer oscillation, both of which are omitted for saving space.

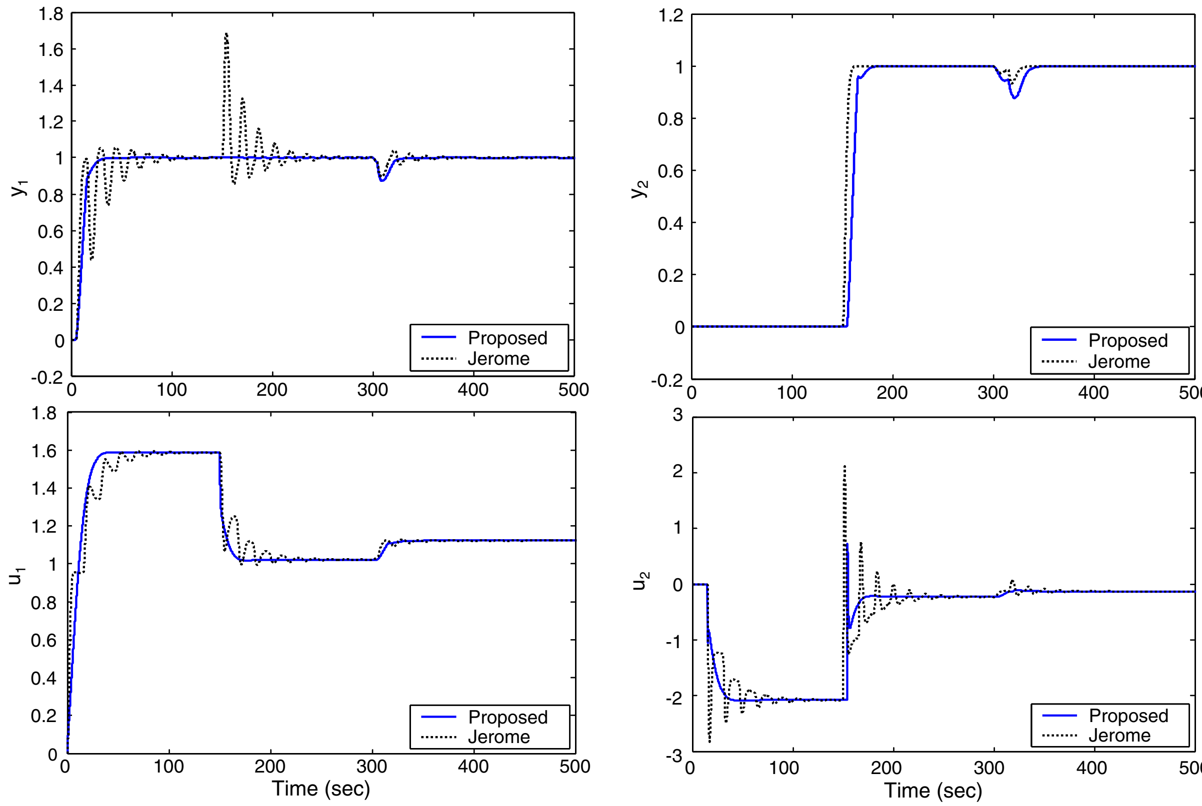

Fig. 9. Nominal system output responses for Example 2.

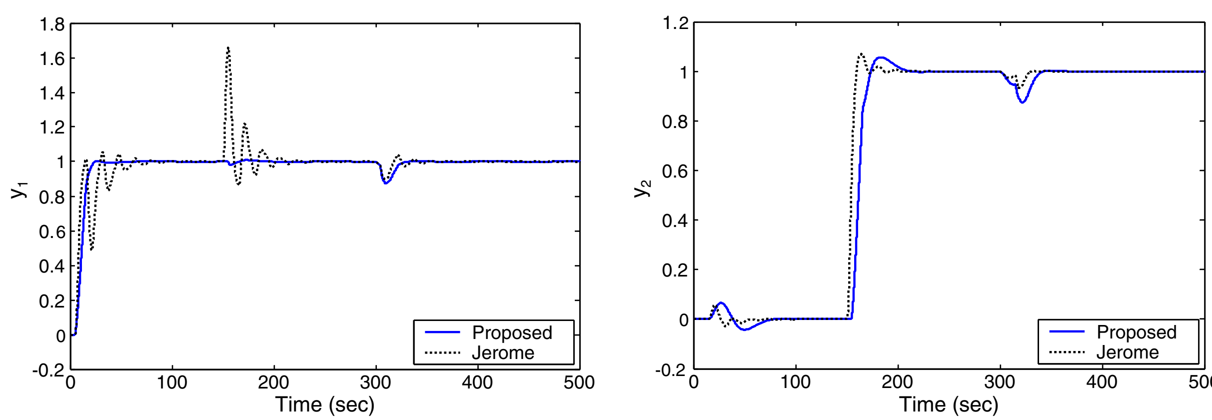

Fig. 10. Perturbed system output responses for Example 2.

7. Conclusions

An analytical decoupling controller matrix design has been proposed within the framework of a conventional unity feedback control structure that is widely adopted in engineering practice. It has been demonstrated that the proposed method can realize significant or even absolute decoupling regulation for the nominal system. The key lies with the formulation of a practical desired closed-loop system transfer matrix in terms of the H_2 optimal performance specification and analysis of the non-minimum-phase (NMP) characteristic of the process inverse transfer matrix (i.e. G^{-1}). New concepts of 'inverse relative degree' and 'time delay' have been introduced and defined in this paper for each transfer element of G^{-1}. The resultant controller design can be conducted with much reduced computation effort compared with recently improved decoupling control methods based on some numerical optimization algorithms. The stability has been analyzed for both the nominal system and the perturbed system with process additive, multiplicative input and output uncertainties that are often encountered in practice. Tuning of the decoupling controller matrix for balance between the nominal system performance and its robust stability can be conveniently performed on line, owing to that each column controllers of the proposed decoupling controller matrix can be tuned in common by a single adjustable parameter in a monotonous manner.

Acknowledgements

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