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Robust fuzzy tracking control for robotic manipulators

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Abstract

In this paper, a stable adaptive fuzzy-based tracking control is developed for robot systems with parameter uncertainties and external disturbance. First, a fuzzy logic system is introduced to approximate the unknown robotic dynamics by using adaptive algorithm. Next, the effect of system uncertainties and external disturbance is removed by employing an integral sliding mode control algorithm. Consequently, a hybrid fuzzy adaptive robust controller is developed such that the resulting closed-loop robot system is stable and the trajectory tracking performance is guaranteed. The proposed controller is appropriate for the robust tracking of robotic systems with system uncertainties. The validity of the control scheme is shown by computer simulation of a two-link robotic manipulator.

Keywords: Fuzzy logic; Robot control; Sliding mode control; Adaptive control

1. Introduction

In the recent decades, the tracking control of robot manipulators has received a great of attention. Tracking control is needed to make each joint track a desired trajectory as close as possible. Many control algorithm such as computer torque method [1,2], optimal control [3,4], adaptive control [5,6], variable structure control (VSC) [7–9], neural networks (NNs) [10–12] and fuzzy system [3,4,13–17] have been proposed to deal with this robotic control problem. In [1,2], a computer torque control is developed on the basis of the feedback linearization. However, these designs are possible only the dynamics of the robotic dynamic are well known. The adaptive control schemes can be employed to deal with the unknown robotic dynamics. In these approaches, the linear parameterizations must be assumed, *i.e.* the unknown parameters must be of linear structure. Moreover, the unknown parameters are assumed to be constant or slowly varying. However, as the robotic dynamic systems are nonlinear, highly coupled, and time varying, the linear parameterization property may not be applicable. Also the implementation also requires a precise knowledge of the structure of the dynamic model.

Generally, uncertainties may not be known in practical robotic systems such as changing payload, nonlinear friction, unknown disturbance, and the high-frequency part of the dynamics. Therefore, it is necessary to

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consider these effects containing both structured uncertainties (parametric) and unstructured uncertainties (un-modeled dynamics). Variable structure control (VSC) is one of the robust control strategies to compensate these uncertainties in robotic dynamics. In these robust control design approaches [8,9], a fixed control law based on a priori bound of uncertainty is designed to compensate the effects of system uncertainties. However, the assumptions in these approaches may be restrictive and difficult to be evaluated. On the other hand, extensive approaches have been developed to deal with the adaptive control and robust control of robotics system with uncertainties.

Fuzzy control is a technique of incorporating expert knowledge in designing a controller. Past research of universal approximation theorem [18] shown that any nonlinear function over a compact set can be approximated by a fuzzy system with arbitrary accuracy. There has been significant research work on the adaptive fuzzy control of nonlinear systems [19–22]. In order to eliminate the effect of the modeling errors and disturbance in the system, robust compensators have been developed by H^{∞} control [22] and sliding mode control [20] methods. However, the sliding mode control inherits a discontinuous control action and the undesirable chattering problem will exist in practical application.

In this paper, a novel control algorithm is developed by combining the fuzzy approach with the integral sliding mode control method. The proposed method combines the adaptive fuzzy algorithm and robust control technique to guarantee a robust tracking performance for uncertain robotic system. In the proposed algorithm, the adaptive fuzzy systems are used to cancel the nonlinear robot dynamics, which do not need to have a linear parameterized structure as in the case of conventional adaptive control scheme's assumption. Moreover, by combining the integral variable structure control (IVSC) [23] with uncertainties bound estimation, the proposed control scheme becomes a new robust fuzzy control algorithm of robot manipulators. It is proved that the closed-loop system is globally stable in the Lyapunov sense if all the signals are bounded and the system output can track the desired reference output asymptotically with modeling uncertainties and disturbances.

This paper is organized as follows. A description of fuzzy system is included in Section 2. In Section 3, the robot dynamics, its property and control design is described. A robust fuzzy control with bound estimation is developed in Section 4. Simulation results for the proposed control algorithm are included in Section 5. Finally, the paper is concluded in Section 6.

2. Functional approximation using fuzzy logic system

The fuzzy logic system [13] consists of four parts, the fuzzifier, the knowledge base, the inference engine and the defuzzifier. The fuzzy knowledge base comprises a collection of fuzzy IF–THEN rules in the following form.

$$R^{(l)}: \text{IF } x_1 \text{ is } A_1^l \quad \text{and} \quad \cdots \quad \text{and} \quad x_n \text{ is } A_n^l \text{ Then } y \text{ is } B^l \tag{1}$$

The fuzzy logic system performs a mapping from $U = U_1 \times \cdots \times U_n \subseteq R^n$ to $V \subseteq R$, where $x = [x_1, \cdots, x_n]^{\mathrm{T}} \in U$ and $y \in V \subseteq R$ are the input and output of the fuzzy logic system, respectively. A_i^l and B^l denote the linguistic variables of the input and output of the fuzzy set in U and V, respectively. The variable $i = 1, \ldots, n$ and n denotes the number of input for the fuzzy logic system and $l = 1, \ldots m$, m denotes the number of the fuzzy IF–THEN rules. Based on the fuzzy IF–Then rules in the knowledge base and the compositional rules of the inference engine, the fuzzy inference engine performs a mapping from fuzzy sets in U to fuzzy sets in V. The defuzzifier maps fuzzy sets in U to a crisp point in V. In general, there are many different choices for the design of fuzzy system if the mapping is static. More detailed information of these fuzzy systems can be found in [18].

The fuzzy logic systems with singleton fuzzifier, product inference engine, center average defuzzifier are in the following form:

$$y(x) = \frac{\sum_{l=1}^{m} y^l \left(\prod_{i=1}^{n} \mu_{A_i^l}(x_i) \right)}{\sum_{l=1}^{m} \prod_{i=1}^{n} \mu_{A_i^l}(x_i)} \tag{2}$$

where $\mu_{A_i^l}(x_i)$ is the membership function of the linguistic variable x_i , and y^l represents a crisp value at which the membership function μ_{B^l} for output fuzzy set reaches its maximum. As a usual practice, we assume that $\mu_{B^l}(y^l) = 1$. By introducing the concept of fuzzy basis function vector or the antecedent function vector. (2) can be rewritten as

$$y(x) = \theta^{\mathrm{T}}\xi(x) \tag{3}$$

$$\xi^l(x) = \frac{\prod_{i=1}^{n}\mu_{A_i^l}(x_i)}{\sum_{l=1}^{m}\prod_{i=1}^{n}\mu_{A_i^l}(x_i)} \tag{4}$$

where $\theta = [y^1, \ldots, y^m]^{\mathrm{T}} \in R^m$ is called the parameter vector and $\xi(x) = [\xi^1(x), \ldots, \xi^m(x)]^{\mathrm{T}} \in R^m$ is called the fuzzy basis function vector. One of the most important advantages of fuzzy logic system is that the fuzzy logic system has the capability to approximate nonlinear mappings. More precisely, the universal approximation theorem is quoted as follows.

Theorem 2.1 [18]. *For any given real continuous function $f(x)$ on a compact set $U \in R^n$ and arbitrary $\varepsilon > 0$. There exists a fuzzy logic system $f^*(x|\theta)$ is in the form of (3), such that*

$$\sup_{x\in U} |f^*(x|\theta) - f(x)| < \varepsilon \tag{5}$$

Based on this result, the function $f(x)$ can be expressed as

$$f(x) = \theta^{*^{\mathrm{T}}}\xi(x) + \varepsilon \quad \forall x \in U \subseteq R^n \tag{6}$$

$\theta^*$is the optimal parameters of fuzzy logic system

$$\theta^* = \arg\min_{\theta\in\Omega}\left(\sup_{x\in\Omega_x}|\theta^{\mathrm{T}}\xi(x) - f(x)|\right) \tag{7}$$

Ω and Ω_x denote the sets of suitable bounds on θ and x, respectively. The fuzzy logic system described above is for single-output system. However, it is straightforward to show that a multi-output system can always be approximated by a group of single-output approximation systems.

3. Robot manipulator dynamics and control

A robotic manipulator is defined as an open kinematics chain of rigid links. According to the Lagrangian formulation, the dynamic equation of an n-joint robotic manipulator with revolute joints can be formulated as dynamical model [25,26]:

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \tau \tag{8}$$

where $q, \dot{q}, \ddot{q} \in R^n$ is the vectors of joint position, velocities and accelerations; $M(q) \in R^{n\times n}$ is the matrix of the moment inertia; $C(q,\dot{q})\dot{q} \in R^n$ is the vector of centripetal and Coriolis forces; $G(q) \in R^n$ is the vector of gravitational force; and $\tau \in R^n$ is the vector of applied joint torques. In general, a robotic manipulator is always presented of uncertainties such as frictions and disturbances. Then, (8) can be rewritten as

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) + D = \tau \tag{9}$$

where D is the uncertainties of the dynamics, including frictions $F_r(\dot{q})$ and disturbance τ_{d}. Several fundamental properties of the robot model (9) have been obtained as follows:

Property 1. *The inertia matrix $M(q)$ is a positive definite symmetric matrix, e.g. non-singular and bounded by $m_{\min}\|x\|^2 \leqslant x^T M(q)x \leqslant m_{\max}\|x\|^2\ \forall x \in R^n$, where $m_{\min}$ and $m_{\max}$ are minimum and maximum eigenvalues of M.*

Property 2. *$\dot{M}(q) - 2C(q,\dot{q})$ is skew-symmetric matrix, i.e., $x^{\mathrm{T}}(\dot{M} - 2C)x = 0\ \forall x \in R^n$.*

Property 3. *The unknown disturbance τ_{d} are assumed to be unknown but bounded, i.e. $\|\tau_{\mathrm{d}}\| < \beta_{\mathrm{d}}$.*

Property 4. *The friction in the dynamic equation (9) is in the form* $F_r(\dot{q}) = F_v\dot{q} + F_c\mathrm{sgn}(\dot{q})$ *with* F_v *the coefficient matrix of viscous friction and* F_c *a dynamic friction term. The friction is depended on the angular velocity and the bound of the friction terms may be assumed to be in the form of* $\|F_v(\dot{q}) + F_c(\dot{q})\| \leqslant \beta_{r_1}\|\dot{q}\| + \beta_{r_2}$, $\beta_{r_1}, \beta_{r_2} > 0$.

In the following analysis, it will be assumed that the nonlinear dynamic model of the robot manipulator to be controlled is well known and uncertainties are negligible. As a consequence, (8) can be rewritten as

$$\ddot{q} = F(q,\dot{q}) + M^{-1}(q)\tau \tag{10}$$

where $F(q,\dot{q})$ is a $n \times 1$ vector defined by

$$F(q,\dot{q}) = -M^{-1}(q)[C(q,\dot{q})\dot{q} + G(q)] \tag{11}$$

If $F(q,\dot{q},\ddot{q})$ and $M^{-1}(q)$ are known, we can use the state feedback control law

$$\tau = M(q)[-F(q,\dot{q}) + v'] \tag{12}$$

to linearize and decouple the robot dynamic system (8), where v' is an external input vector $v' = [v'_1, \ldots, v'_n]^{\mathrm{T}}$, such that the control law (12) apply to system (8) results in a closed-loop dynamics with $\ddot{q} = v'$. The objective of control is to follow a given continuously differentiable and uniformly bounded trajectory in the joint space q_{d} and the tracking error $e = q - q_{\mathrm{d}}$ should be kept as small as possible.

Define a sliding surface in the space of the error state vector $S = R^n$ as

$$S = \begin{bmatrix} s_1(e_1) \\ \vdots \\ s_n(e_n) \end{bmatrix} = \begin{bmatrix} c_1e_1 + \dot{e}_1 + k_1\int_0^t e_1\,\mathrm{d}t \\ \vdots \\ c_ne_n + \dot{e}_n + k_n\int_0^t e_n\,\mathrm{d}t \end{bmatrix} \tag{13}$$

where e_i are the tracking error defined by $e_i = q_i - q_{\mathrm{d}i}$, q_i and $q_{\mathrm{d}i}$ are the joint and desired output trajectories for each joint. The coefficients c_i and k_i should be chosen such that all the roots of the polynomial $h_i(s) = s^2 + c_is + k_i$ $(i = 1, \ldots, n)$ are in the open left-half plane. The tracking problem of the robot manipulator in the joint space implies that the error states should stay on the sliding surface $S = 0$ as the time goes to infinity. A sufficient condition to achieve this behavior is to select the control strategy such that

$$\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}(s_i) \leqslant -\eta_{\Delta_i}|s_i|, \quad \eta_{\Delta_i} \geqslant 0 \tag{14}$$

If the sliding condition (14) is satisfied, the system is controlled in such a way that the trajectories of the closed-loop system moves towards the sliding surface and hit it. In order to satisfy the sliding reachability condition, the external input vector v' is defined as

$$v'(q,\dot{q}) = \begin{bmatrix} \ddot{q}_{\mathrm{d}1} - c_1\dot{e}_1 - k_1e_1 - \eta_{\Delta_1}\mathrm{sgn}(s_1) \\ \vdots \\ \ddot{q}_{\mathrm{d}n} - c_n\dot{e}_n - k_ne_n - \eta_{\Delta_n}\mathrm{sgn}(s_n) \end{bmatrix} \tag{15}$$

and $\mathrm{sgn}(\cdot)$ is the usual sign function.

As described above, the plant uncertainties are neglected for the controller design. In order to eliminate the influence due to the frictions and disturbance, the positive constant η_{Δ_i} are replaced by $\eta_i^* + \eta_{\Delta_i}$ to guarantee the existence of sliding condition. η_i^* is the upper bound of uncertainties, i.e. $|D_i| \leqslant \eta_i^*$. Hence, differentiate (13) with respect to time, the dynamics of the system (9) can be rewritten in term of S as follows:

$$\dot{S} = \begin{bmatrix} \dot{s}_1 \\ \vdots \\ \dot{s}_n \end{bmatrix} = \begin{bmatrix} k_1e_1 + c_1\dot{e}_1 + \ddot{e}_1 \\ \vdots \\ k_ne_n + c_n\dot{e}_n + \ddot{e}_n \end{bmatrix} = F(q,\dot{q}) + M^{-1}(q)\tau + D' + v(q,\dot{q}) \tag{16}$$

where

$$D' = M^{-1}(q)(F_{\mathrm{v}}(\dot{q}) + \tau_{\mathrm{d}}) \tag{17}$$

and

$$v(q,\dot{q}) = \begin{bmatrix} k_1 e_1 + c_1 \dot{e}_1 - \ddot{q}_{d1} \\ \vdots \\ k_n e_n + c_n \dot{e}_n - \ddot{q}_{dn} \end{bmatrix} \tag{18}$$

If the dynamical model of the robot manipulator to be controlled and the bounded of uncertainties are known, then we can use the control law (12) for the robotic dynamic. However, the dynamics of the robotic dynamic are generally unknown in practice and there are system uncertainties. To solve these problems, the robust fuzzy control algorithm is proposed in Section 4.

4. Adaptive fuzzy control of robot manipulator

In Section 3, the dynamic model of the robot manipulator is assumed to be known, then we can use the control law in (12) to linearize and control the robot dynamic system (9). However, the robotic model is unknown and the control law is unrealizable. In this paper, we propose to use a fuzzy logic system to approximate the unknown system dynamics. Moreover, we employ the integral sliding mode control to compensate both the structured and the unstructured uncertainties. In order to take into account the unknown uncertainties bounds, an adaptive term $\hat{\eta}$ are provided to estimate these parameters online. If the robotic dynamic model is unknown, this implies that the elements of the matrices $F(q,\dot{q})$ and $M(q)$ of (9) are also unknown:

$$F(q,\dot{q}) = [f_i(q,\dot{q})] \tag{19}$$

$$M(q) = [m_{ij}(q)] \tag{20}$$

where $i,j = 1,\ldots,n$. We shall propose the fuzzy logic system, described in Section 2 to model the unknown function $f_i(q,\dot{q})$ and $m_{ij}(q)$, with fuzzy logic system $\hat{f}_i(q,\dot{q}|\theta_i)$ and $\hat{m}_{ij}(q|\theta_{ij})$ for n-link robotic system defined as

$$\hat{f}_i(q,\dot{q}|\theta_i) = \theta_{f_i}^{\mathrm{T}} \xi(q,\dot{q}) \tag{21}$$

$$\hat{m}_{ij}(q|\theta_{ij}) = \theta_{m_{ij}}^{\mathrm{T}} \xi(q) \tag{22}$$

Hence, the control law (12) can be defined as

$$\tau = \hat{M}(q)[-\hat{F}(q,\dot{q}) + v'] \tag{23}$$

where $\hat{M}(q)$ and $\hat{F}(q,\dot{q})$

$$\hat{F}(q,\dot{q}) = \begin{bmatrix} \hat{f}_1(q,\dot{q}|\theta_{f_1}) \\ \hat{f}_2(q,\dot{q}|\theta_{f_2}) \\ \vdots \\ \hat{f}_n(q,\dot{q}|\theta_{f_n}) \end{bmatrix} = \begin{bmatrix} \theta_{f_1}^{\mathrm{T}} \xi(q,\dot{q}) \\ \theta_{f_2}^{\mathrm{T}} \xi(q,\dot{q}) \\ \vdots \\ \theta_{f_n}^{\mathrm{T}} \xi(q,\dot{q}) \end{bmatrix} \tag{24}$$

$$\hat{M}(q) = \begin{bmatrix} \hat{m}_1(q|\theta_{m_1}) \\ \hat{m}_2(q|\theta_{m_2}) \\ \vdots \\ \hat{m}_n(q|\theta_{m_n}) \end{bmatrix} = \begin{bmatrix} \theta_{m_1}^{\mathrm{T}} \xi(q) \\ \theta_{m_2}^{\mathrm{T}} \xi(q) \\ \vdots \\ \theta_{m_n}^{\mathrm{T}} \xi(q) \end{bmatrix} \tag{25}$$

where $\hat{m}_i = [\hat{m}_{1i}(q|\theta_{m_i}),\ldots,\hat{m}_{ni}(q|\theta_{m_i})]^{\mathrm{T}}$ and $\theta_{m_i} = [\theta_{m_{1i}},\ldots,\theta_{m_{ni}}]^{\mathrm{T}}$.

Theorem 4.1. *Consider the control problem of the robotic system (9). If the control law (23) is used, the nonlinear functions $f_i(q,\dot{q})$, $m_{ij}(q)$ are estimated by (21) and (22), the parameters vector $[\theta_{f_1},\ldots,\theta_{f_n}]^{\mathrm{T}}$, $[\theta_{m_1},\ldots,\theta_{m_n}]^{\mathrm{T}}$ and the uncertainties bound estimates $\hat{p}_i$ are adjusted by the adaptive law (26)–(28), the closed-loop system signals will be bounded and the tracking error will converge to zero asymptotically*:

$$\dot{\theta}_{f_i} = \gamma_{f_i} s_i \xi(q, \dot{q}) \tag{26}$$

$$\dot{\theta}_{m_{ij}} = \gamma_{m_{ij}} s_i \xi(q) \tau_j \tag{27}$$

$$\dot{\hat{p}}_i = \gamma_{p_i} |s_i| \tag{28}$$

$i,j = 1, \ldots, n$

Proof. Define the optimal parameters vector $\theta^*_{f_i}, \theta^*_{m_{ij}}$ of fuzzy systems

$$\theta^*_{f_i} = \arg \min_{\theta_{f_i} \in \Omega_{f_i}} \left(\sup_{q,\dot{q} \in R^n} |\hat{f}_i(q, \dot{q}|\theta_{f_i}) - f_i(q, \dot{q})| \right) \tag{29}$$

$$\theta^*_{m_{ij}} = \arg \min_{\theta_{m_{ij}} \in \Omega_{m_{ij}}} \left(\sup_{q \in R^n} |\hat{m}_{ij}(q|\theta_{m_{ij}}) - m_{ij}(q)| \right) \tag{30}$$

where Ω_{f_i} and $\Omega_{m_{ij}}$ are constraint sets for θ_{f_i}, $\theta_{m_{ij}}$ defined as

$$\Omega_{f_i} = \{\theta_{f_i} \in R^n \| \theta_{f_i}| \leqslant M_{f_i}\}, \Omega_{m_{ij}} = \{\theta_{m_{ij}} \in R^n \| \theta_{m_{ij}}| \leqslant M_{m_{ij}}\} \tag{31}$$

where M_{f_i} and $M_{m_{ij}}$ are pre-specified parameters for estimated parameters bound. Assume that the fuzzy parameter vectors θ_{f_i} and $\theta_{m_{ij}}$ never reach the boundaries. Define the minimum approximation error:

$$\omega_i = f_i(q, \dot{q}) - \hat{f}_i(q, \dot{q}|\theta^*_{f_i}) + (m_{ij}(q) - \hat{m}_{ij}(q|\theta^*_{m_{ij}}))\tau_j \tag{32}$$

and assume the approximation errors are upper bounded by $|\omega_i| < \omega_{i\max}$. And define $\tilde{p}_i = p^*_i - \hat{p}_i$, where $p^*_i = |D'_i + w_i|_{\max}$ is the upper bounded of uncertainties. Then, we have

$$\begin{aligned}
\dot{S} &= M^{-1}(q)\tau + F(q, \dot{q}) + D' + v(q, \dot{q}) \\
&= [M^{-1}(q) + \hat{M}^{-1}(q|\theta_{m_{ij}}) - \hat{M}^{-1}(q|\theta_{m_{ij}})] \times [\hat{M}(q|\theta_{m_{ij}})(-\hat{F}(q, \dot{q}|\theta_{f_i}) + v')] + F(q, \dot{q}) + D' + v(q, \dot{q}) \\
&= [M^{-1}(q) - \hat{M}^{-1}(q|\theta_{m_{ij}})] \cdot \hat{M}(q|\theta_{m_{ij}}) \cdot (-\hat{F}(q, \dot{q}|\theta_{f_i}) + v') + F(q, \dot{q}) + v(q, \dot{q}) - \hat{F}(q, \dot{q}|\theta_{f_i}) + v' + D' \\
&= [M^{-1}(q) - \hat{M}^{-1}(q|\theta_{m_{ij}})]\tau + F(q, \dot{q}) - \hat{F}(q, \dot{q}|\theta_{f_i}) + v(q, \dot{q}) - (\hat{p} + \eta)\mathrm{sgn}(S) - v(q, \dot{q}) + D' \\
&= [M^{-1}(q) - \hat{M}^{-1}(q|\theta_{m_{ij}})]\tau + F(q, \dot{q}) - \hat{F}(q, \dot{q}|\theta_{f_i}) - (\hat{p} + \eta)\mathrm{sgn}(S) + D' \\
&= [\hat{M}^{-1}(q|\theta^*_{m_{ij}}) - \hat{M}^{-1}(q|\theta_{m_{ij}})]\tau + F(q, \dot{q}|\theta^*_{f_i}) - \hat{F}(q, \dot{q}|\theta_{f_i}) - (\hat{p} + \eta)\mathrm{sgn}(S) + D' + \omega \\
&= \tilde{\theta}^{\mathrm{T}}_{f_i} \xi(q, \dot{q}) + \tilde{\theta}^{\mathrm{T}}_{m_{ij}} \xi(q)\tau - (\hat{p} + \eta)\mathrm{sgn}(S) + D' + \omega
\end{aligned} \tag{33}$$

where $\tilde{\theta}_{f_i} = \theta^*_{f_i} - \theta_{f_i}, \theta_{m_{ij}} = \theta^*_{m_{ij}} - \theta_{m_{ij}}, \hat{p} = [\hat{p}_1 \hat{p}_2 \ldots \hat{p}_n]^{\mathrm{T}}, \eta = [\eta_{\Delta 1} \eta_{\Delta 2} \ldots \eta_{\Delta n}]^{\mathrm{T}}$.

Now consider the Lyapunov candidate

$$V = \sum_{i=1}^{n} V_i \tag{34}$$

where

$$V_i = \frac{1}{2} s_i^2 + \frac{1}{2\gamma_{f_i}} \tilde{\theta}^{\mathrm{T}}_{f_i} \tilde{\theta}^{+}_{f_i} \sum_{j=1}^{n} \frac{1}{2\gamma_{m_{ij}}} \tilde{\theta}^{\mathrm{T}}_{m_{ij}} \tilde{\theta}_{m_{ij}} + \frac{1}{2\gamma_{p_i}} \tilde{p}^{\mathrm{T}}_i \tilde{p}_i \tag{35}$$

where $\gamma_{f_i}, \gamma_{m_{ij}}$ and γ_{p_i} are design positive constants parameters. The time derivative of V along the error trajectory (33) is

$$\begin{aligned}
\dot{V}_i &= s_i \dot{s}_i + \frac{1}{\gamma_{f_i}} \tilde{\theta}^{\mathrm{T}}_{f_i} \dot{\tilde{\theta}}^{+}_{f_i} \sum_{j=1}^{n} \frac{1}{\gamma_{m_{ij}}} \tilde{\theta}^{\mathrm{T}}_{m_{ij}} \dot{\tilde{\theta}}_{m_{ij}} + \frac{1}{\gamma_{p_i}} \tilde{p}^{\mathrm{T}}_i \dot{\tilde{p}}_i \\
&= s_i \left(\tilde{\theta}^{\mathrm{T}}_{f_i} \xi(q, \dot{q}) + \sum_{j=1}^{n} \tilde{\theta}^{\mathrm{T}}_{m_{ij}} \xi(q, \dot{q}) u_j - (\hat{p}_i + \eta_{\Delta_i})\mathrm{sgn}(s_i) + \omega_i + D'_i \right) + \frac{1}{\gamma_{f_i}} \tilde{\theta}^{\mathrm{T}}_{f_i} \dot{\tilde{\theta}}^{+}_{f_i} \sum_{j=1}^{n} \frac{1}{\gamma_{m_{ij}}} \tilde{\theta}^{\mathrm{T}}_{m_{ij}} \dot{\tilde{\theta}}_{m_{ij}} + \frac{1}{\gamma_{p_i}} \tilde{p}^{\mathrm{T}}_i \dot{\tilde{p}}_i
\end{aligned}$$

$$= s_i\tilde{\theta}_{f_i}^{\mathrm{T}}\xi(q,\dot{q}) + \frac{1}{\gamma_{f_i}}\tilde{\theta}_{f_i}^{\mathrm{T}}\dot{\tilde{\theta}}_{f_i}^{+}\sum_{j=1}^{n} s_i\tilde{\theta}_{m_{ij}}^{\mathrm{T}}\xi(q,\dot{q})u_j + \sum_{j=1}^{n}\frac{1}{\gamma_{m_{ij}}}\tilde{\theta}_{m_{ij}}^{\mathrm{T}}\dot{\tilde{\theta}}_{m_{ij}} - s_i(\hat{p}_i+\eta_{\Delta_i})\mathrm{sgn}(s_i) + s_i\omega_i + s_iD_i' + \frac{1}{\gamma_{p_i}}\tilde{p}_i^{\mathrm{T}}\dot{\tilde{p}}_i$$

$$\leqslant \frac{1}{\gamma_{f_i}}\tilde{\theta}_{f_i}^{\mathrm{T}}(\gamma_{f_i}s_i\xi(q,\dot{q}) + \dot{\tilde{\theta}})_{f_i} + \sum_{j=1}^{n}\frac{1}{\gamma_{m_{ij}}}\tilde{\theta}_{m_{ij}}^{\mathrm{T}}\left(\gamma_{m_{ij}}s_i\xi(q,\dot{q})u_j + \dot{\tilde{\theta}}_{m_{ij}}\right) + s_i\omega_i + s_iD_i' - |s_i|p_i^*$$

$$+ |s_i|(p_i^* - \hat{p}_i) - |s_i|\eta_{\Delta_i} + \frac{1}{\gamma_{p_i}}\tilde{p}_i^{\mathrm{T}}\dot{\tilde{p}}_i$$

$$\leqslant \frac{1}{\gamma_{f_i}}\tilde{\theta}_{f_i}^{\mathrm{T}}(\gamma_{f_i}s_i\xi(q,\dot{q}) + \dot{\tilde{\theta}}_{f_i}) + \sum_{j=1}^{n}\frac{1}{\gamma_{m_{ij}}}\tilde{\theta}_{m_{ij}}^{\mathrm{T}}\left(\gamma_{m_{ij}}s_i\xi(q,\dot{q})u_j + \dot{\tilde{\theta}}_{m_{ij}}\right) + \frac{1}{\gamma_{p_i}}\tilde{p}_i^{\mathrm{T}}(\gamma_{p_i}|s_i| + \tilde{p}_i) - |s_i|\eta_{\Delta_i} \tag{36}$$

where $\dot{\tilde{\theta}}_{f_1} = \dot{\theta}_{f_1}$ and $\dot{\tilde{\theta}}_{m_{ij}} = \dot{\theta}_{m_{ij}}$. Substitute (26)–(28) into (36), then we have

$$\dot{V}_i \leqslant -\eta_{\Delta i}|s_i| < 0 \tag{37}$$

To complete the proof and establish asymptotic convergence of the tracking error, we need to prove that $s_i \to 0$ as $t \to \infty$. Integrating both sides of (37), we have

$$\int_0^\infty |s_i|\mathrm{d}t \leqslant \frac{1}{\eta_{\Delta i}}(V(0) - V(\infty)) < \infty \tag{38}$$

Then, we have shown that $s_i \in L_1$, from (37), we know that $s_i \in L_\infty$, because we have proved that all the variables on the right-hand side of (36) are bounded, we have $\dot{s}_i \in L_\infty$. Using the Collary of Barbalet's Lemma [24], if $s_i, \dot{s}_i \in L_\infty$ and $s \in L_p$, for some $p \in [1,\infty]$. We have $s_i \to 0$ as $t \to \infty$, thus $e_i \to 0$ as $t \to \infty$. □

Remark 1. Since the control (23) contains the sign function, direct application of such control signals to the robotic system (8) may result in chattering caused by the signal discontinuity. To overcome this problem, the control law is smooth out within a thin boundary layer Φ [7] by replacing the sign function by a saturation function defined as

$$\mathrm{sat}\left(\frac{s_i}{\Phi_i}\right) = \begin{cases} \mathrm{sgn}\left(\frac{s_i}{\Phi_i}\right) & \left|\frac{s_i}{\Phi_i}\right| > 1 \\ \frac{s_i}{\Phi_i} & \left|\frac{s_i}{\Phi_i}\right| \leqslant 1 \end{cases} \tag{39}$$

The control law of (23) leads the output trajectory to move along the sliding surface and yields $|s(t)| \leqslant \Phi$. From (13) and (39), it can be noted that the steady-state error due to the boundary layer can be removed and there is no reaching phase problem. Consequently,

$$|s_i(t)| = \left|c_ie + \dot{e}_i + k_i\int_0^t e_i\mathrm{d}t\right| \leqslant \Phi_i, \quad i = 1,\ldots,n \tag{40}$$

Taking the Laplace transform of both side of (40) yields

$$\left|\left(c_i + s + \frac{k_i}{s}\right)e_i(s)\right| \leqslant \frac{\Phi_i}{s} \tag{41}$$

Eq. (41) can be rewritten as

$$|e_i(s)| \leqslant \left|\frac{\Phi_i}{s^2 + c_is + k_i}\right| = \left|\frac{\Phi_i}{(s+\alpha_{i1})(s+\alpha_{i2})}\right| \leqslant \frac{\Phi_i}{|(s+\alpha_{i1})||(s+\alpha_{i2})|} \tag{42}$$

Consider $\alpha_{i\min}$ is the minimum characteristic root of $(s+\alpha_{i1})(s+\alpha_{i2})$.

$$|e_i(s)| \leqslant \frac{\Phi_i}{|(s+\alpha_{i1})||(s+\alpha_{i2})|} \leqslant \frac{\Phi_i}{|(s+\alpha_{i\min})|^2} \tag{43}$$

Take the inverse Laplace transform of both sides of (43) and to fine the extreme value

$$|e_i(t)| \leqslant \Phi_i \cdot t \cdot \exp^{-\alpha_{\min}\cdot t} \tag{44}$$

where exp is the exponential function and for all $t \geqslant 0$, we have

$$\lim_{t\to\infty} \Phi_i \cdot t \cdot \exp^{-\alpha_{\min}\cdot t} = 0 \tag{45}$$

Therefore, $\lim_{t\to\infty}|e_i(t)| = 0$.

Remark 2. The above stability result is achieved under the assumption that all the parameter vectors are within the constraint sets or on the boundaries of the constraint set but moving their interior ($|\theta_{f_i}| = M_{f_i}$, $|\theta_{m_{ij}}| < M_{m_{ij}}$). To guarantee the parameters are bounded. The adaptive laws (26) and (27) can be modified by using the projection algorithm [18]. The modified adaptive laws are given as follows:

For θ_{f_i}, we use

$$\dot{\theta}_{f_i} = \begin{cases} \gamma_{f_i} s_i \xi(q,\dot{q}) & \text{if } (|\theta_{f_i}| < M_{f_i}) \text{ or} \\ & \quad (|\theta_{f_i}| = M_{f_i} \text{ and } s_i\theta_{f_i}^{\mathrm{T}}\xi(q,\dot{q}) \geqslant 0) \\ P_{f_i}[\gamma_{f_i} s_i \xi(q,\dot{q})] & \text{if } (|\theta_{f_i}| = M_{f_i}) \text{ and} \\ & \quad s\theta_{f_i}^{\mathrm{T}}\xi(q,\dot{q}) < 0) \end{cases} \tag{46}$$

For $\theta_{m_{ij}}$, we use

$$\dot{\theta}_{m_{ij}} = \begin{cases} \gamma_{m_{ij}} s_i \xi(q) u_j & \text{if } (|\theta_{m_{ij}}| < M_{m_{ij}}) \text{ or} \\ & \quad (|\theta_{m_{ij}}| = M_{m_{ij}} \text{ and } s_i\theta_{m_{ij}}^{\mathrm{T}}\xi(q)u_j \geqslant 0) \\ P_{m_{ij}}[\gamma_{m_{ij}} s_i \xi(q) u_j] & \text{if } (|\theta_{m_{ij}}| = M_{m_{ij}}) \text{ and} \\ & \quad s_i\theta_{m_{ij}}^{\mathrm{T}}\xi(q)u_j < 0) \end{cases} \tag{47}$$

where the projection operator, $P_{f_i}[*]$ and $P_{m_{ij}}[*]$ are defined as

$$P_{f_i}[\gamma_{f_i} s_i \xi(q,\dot{q})] = \gamma_{f_i} s_i \xi(q,\dot{q}) - \gamma_{f_i} s_i \frac{\theta_{f_i}\theta_{f_i}^{\mathrm{T}}\xi(q,\dot{q})}{|\theta_{f_i}|^2} \tag{48}$$

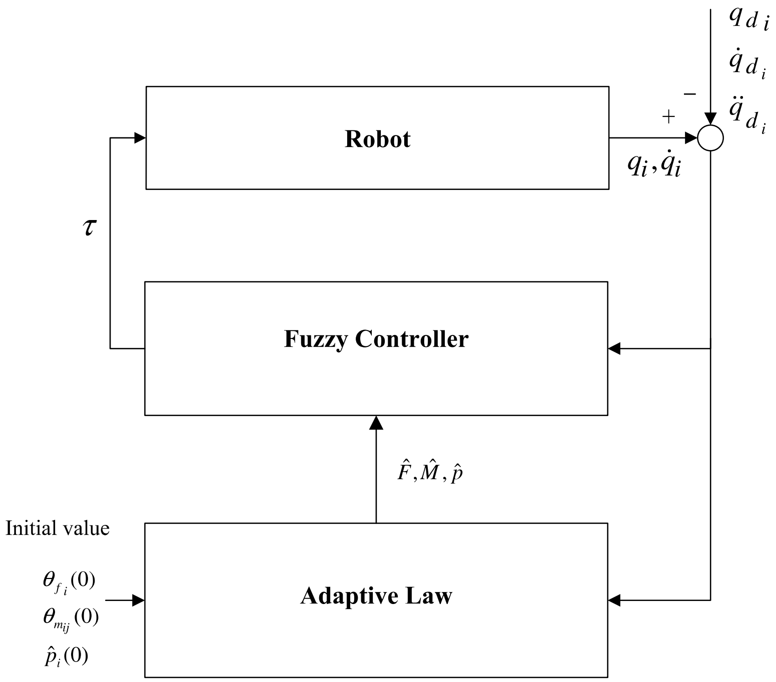

Fig. 1. Overall scheme of the adaptive control system.

$$P_{m_{ij}}[\gamma_{m_{ij}}s_i\xi(q)u_j] = \gamma_{m_{ij}}s_i\xi(q)u_j - \gamma_{m_{ij}}s_i\frac{\theta_{m_{ij}}\theta_{m_{ij}}^{\mathrm{T}}\xi(q)u_j}{|\theta_{m_{ij}}|^2} \tag{49}$$

Then, the overall adaptive fuzzy control scheme is shown in Fig. 1.

To summarize the above analysis, the step-by-step procedures for the adaptive fuzzy control of uncertain robotic system is outlined as follows

Design Procedure:

Step 1. The design parameters M_{f_i}, $M_{m_{ij}}$ are specified based on practical constrains.
Step 2. Specify the desired coefficients $c_1, \cdots, c_n$, $k_1, \cdots, k_n$ in (13).

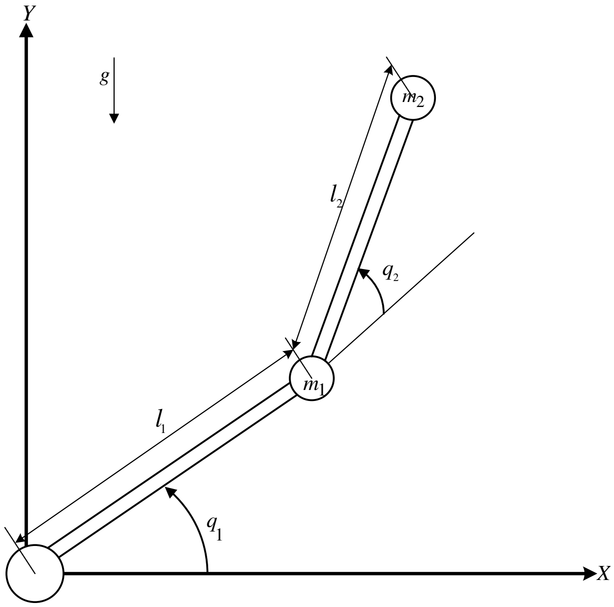

Fig. 2. Two degrees of freedom robot manipulator.

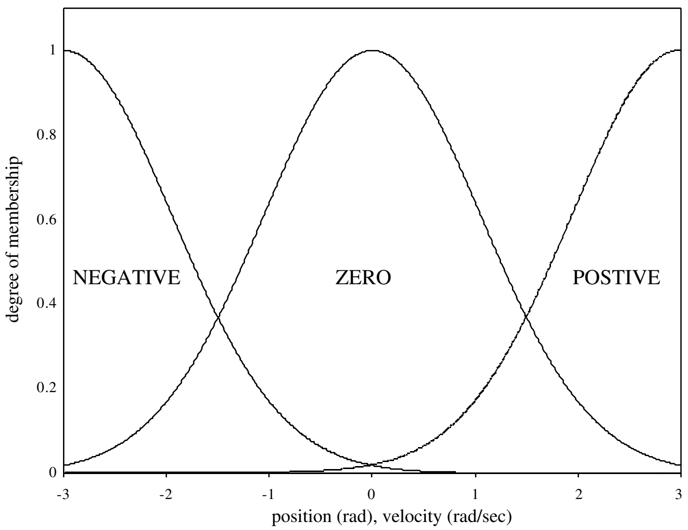

Fig. 3. Membership function of input variables.

Step 3. Select the learning coefficients γ_{f_i}, $\gamma_{m_{ij}}$ and γ_{p_i}.
Step 4. Define fuzzy sets A_i for linguistic variable $q, \dot{q}$ and the membership functions μ_{A_i} is uniformly cover the universe of discourse.
Step 5. Construct the fuzzy rule bases for the fuzzy system $\hat{f}_i(q, \dot{q}|\theta_{f_i})$ and $\hat{m}_{ij}(q|\theta_{m_{ij}})$.
Step 6. Construct the fuzzy systems $\hat{F}(q, \dot{q}) = \theta_{f_i}^{\mathrm{T}}\xi(q, \dot{q})$ and $\hat{M}(q) = \theta_{m_{ij}}^{\mathrm{T}}\xi(q)$ in (24) and (25).
Step 7. Construct the control law (23) with the adaptive law in (28), (46) and (47).
Step 8. Obtain the control and apply to the robot dynamic, then compute the adaptive law (46), (47) and (28) to adjust the parameter vector θ_{f_i},$\theta_{m_{ij}}$ and the estimate bound $\hat{p}_i$.

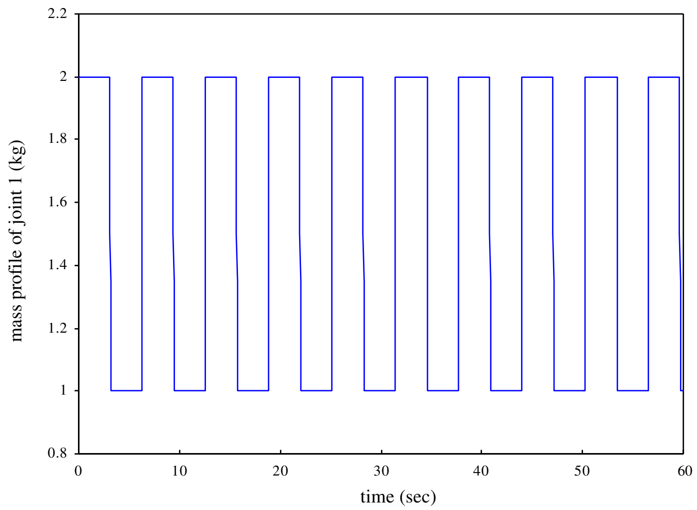

Fig. 4. Mass profile of joint 1.

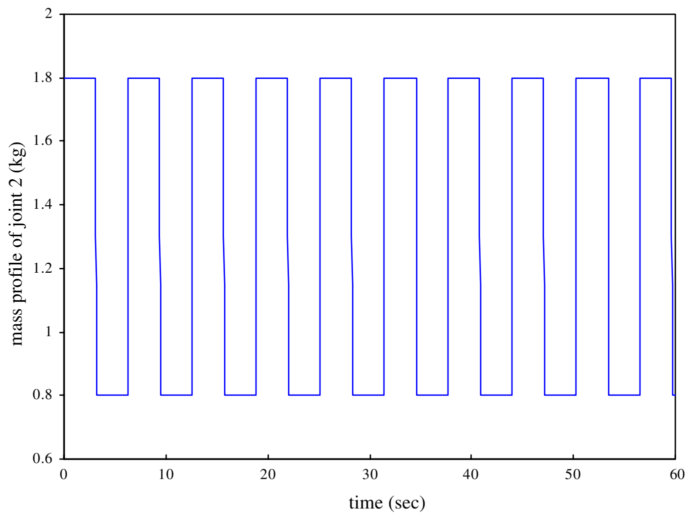

Fig. 5. Mass profile of joint 2.

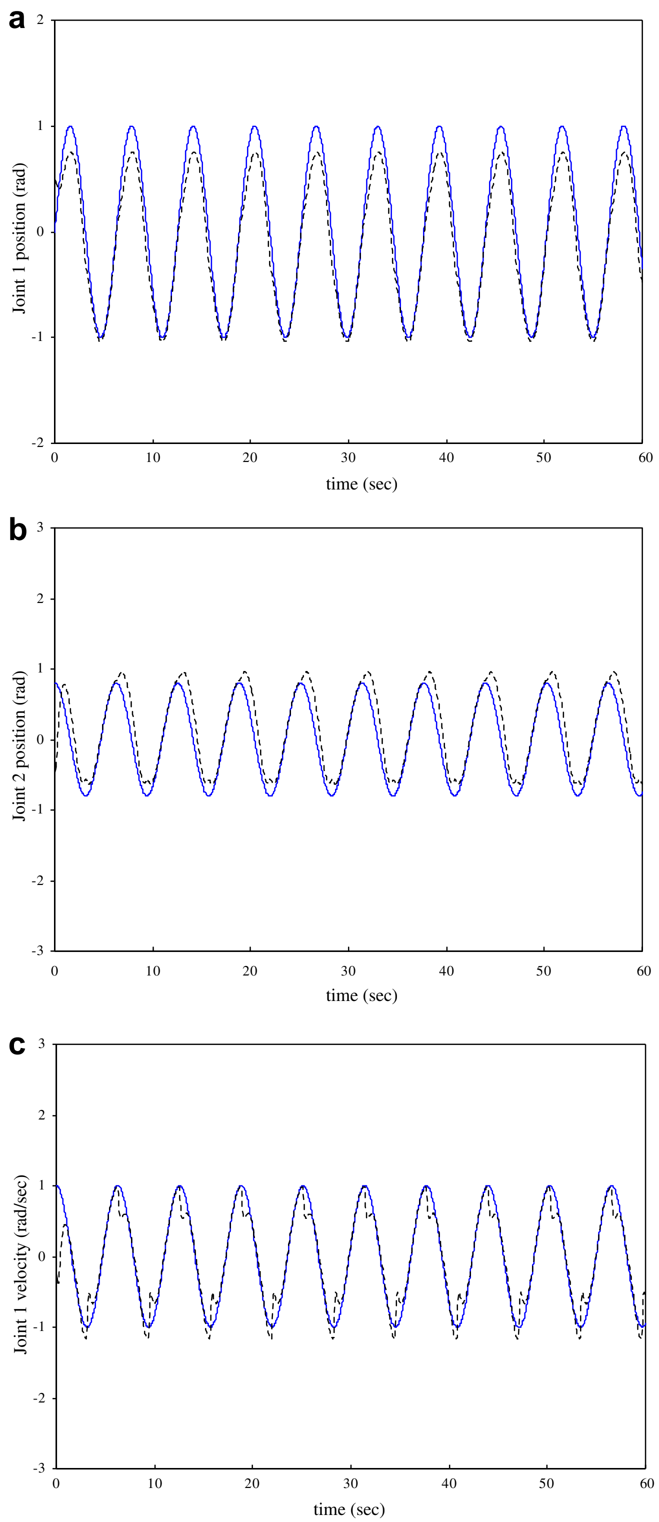

Fig. 6. Simulation results of computed torque control: (a) desired position trajectory of joint 1 q_{d_1} (solid line) and system output trajectory q_1 (dash line), (b) desired position trajectory of joint 2 q_{d_2} (solid line) and system output trajectoryq_2, (c) desired velocity trajectory of joint 1 $\dot{q}_{d_1}$ (solid line) and system output trajectory $\dot{q}_1$ (dash line), (d) desired velocity trajectory of joint 2 $\dot{q}_{d_2}$ (solid line) and system output trajectory $\dot{q}_2$ (dash line), (e) control torque τ_1, and (f) control torque τ_2.

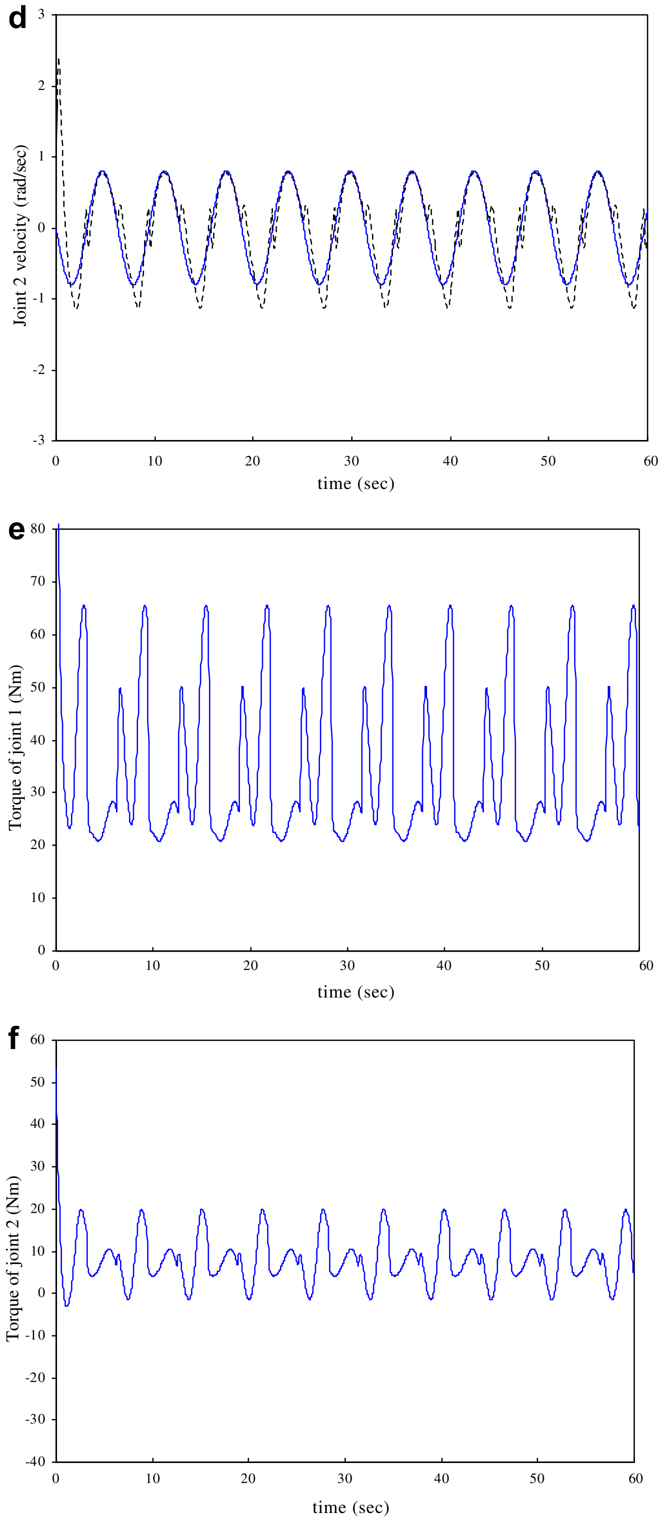

Fig. 6 (*continued*)

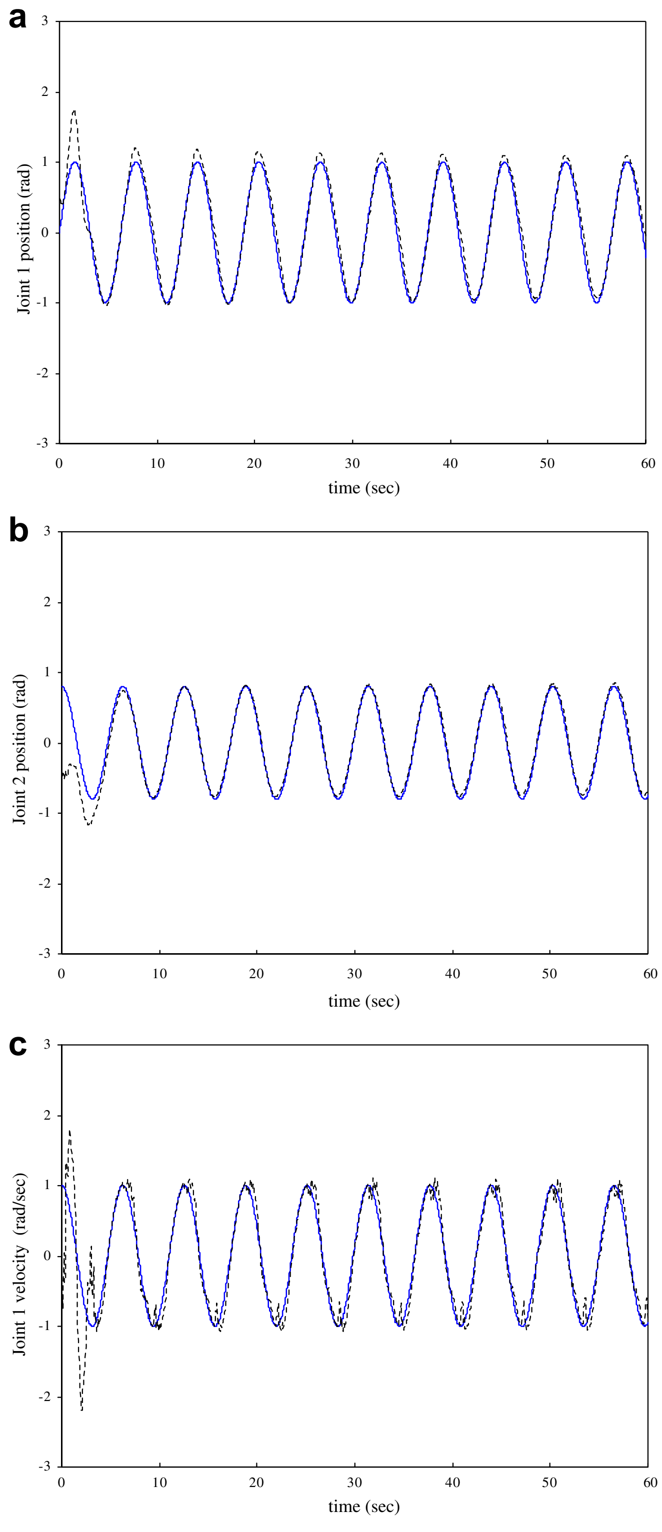

Fig. 7. Simulation results of adaptive fuzzy control: (a) desired position trajectory of joint 1 q_{d_1} (solid line) and system output trajectory q_1 (dash line), (b) desired position trajectory of joint 2 q_{d_2} (solid line) and system output trajectory q_2, (c) desired velocity trajectory of joint 1 $\dot{q}_{d_1}$ (solid line) and system output trajectory $\dot{q}_1$ (dash line), (d) desired velocity trajectory of joint 2 $\dot{q}_{d_2}$ (solid line) and system output trajectory $\dot{q}_2$ (dash line), (e) control torque τ_1, and (f) control torque τ_2.

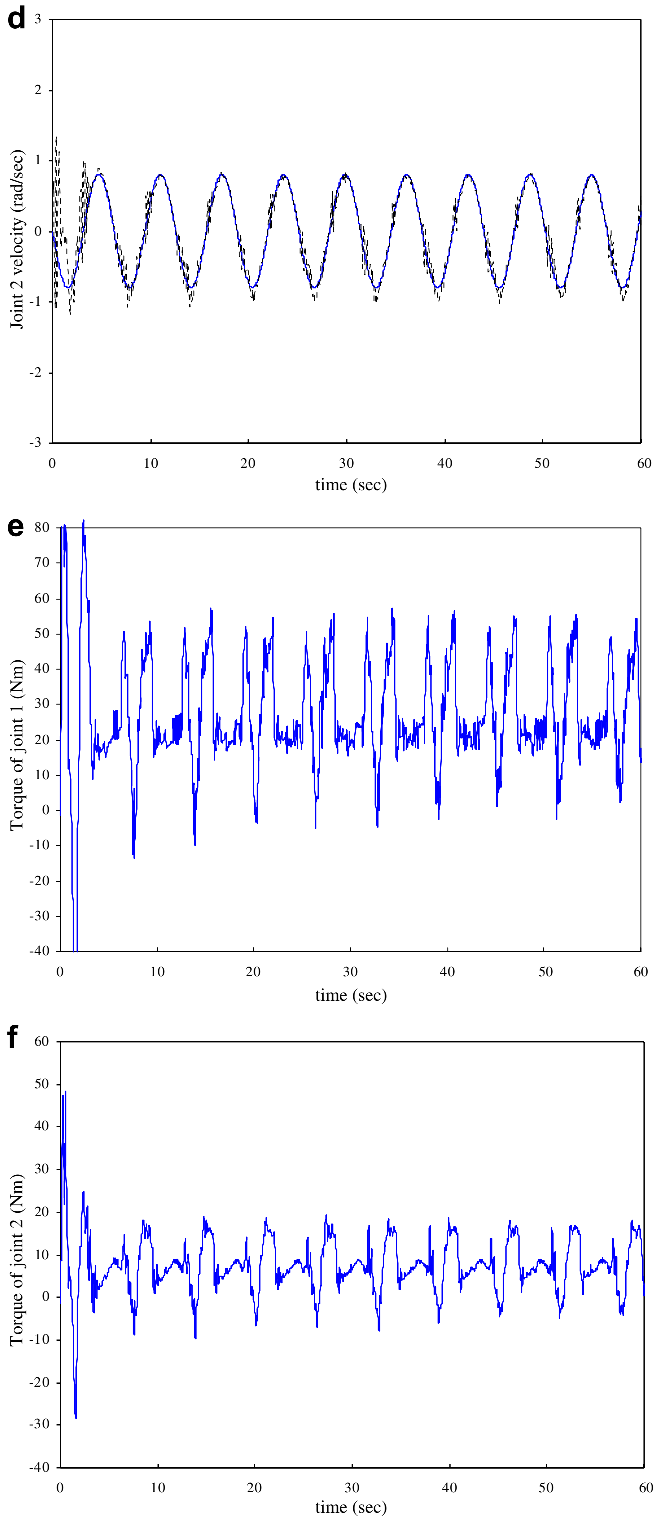

Fig. 7 (*continued*)

5. Simulation example

To verify the theoretical results, simulations were carried out in two degrees of freedom robot manipulator as shown in Fig. 2 described by [25,26]:

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \tau \tag{50}$$

where

$$M(q) = \begin{bmatrix} (m_1 + m_2)l_1^2 + m_2 l_2^2 + 2m_2 l_1 l_2 \cos(q_2) & m_2 l_2^2 + l_1 l_2 m_2 \cos(q_2) \\ m_2 l_2^2 + l_1 l_2 m_2 \cos(q_2) & m_2 l_2^2 \end{bmatrix}$$

$$C(q,\dot{q}) = \begin{bmatrix} -m_2 l_1 l_2 \sin(q_2)\dot{q}_2^2 - 2m_2 l_1 l_2 \sin(q_2)\dot{q}_1\dot{q}_2 \\ m_2 l_1 l_2 \sin(q_2)\dot{q}_1^2 \end{bmatrix}$$

$$G(q) = \begin{bmatrix} m_2 l_2 g\cos(q_1 + q_2) + (m_1 + m_2) l_1 g\cos(q_1) \\ m_2 l_2 g\cos(q_1 + q_2) \end{bmatrix}$$

The parameters of the robot used for simulation are $l_1 = 1$ m, $l_2 = 0.8$ m, $m_1 = m_2 = 1$ kg and $g = 9.8$ m/s^2. The unknown nonlinearities $f_i(q,\dot{q})$ and $m_{ij}(q)$, $i,j = 1,2$ are estimated using three triangular fuzzy sets for q and $\dot{q}$ are constructed as in Fig. 3. No prior knowledge is assumed in this simulation and the consequent parameters are initialized to zero. Select $M_{f_i} = 40$ and $M_{m_{ij}} = 80$. The controller parameters $\gamma_{f_i} = 50$, $\gamma_{m_{ij}} = 0.5$ and $\gamma_{p_i} = 0.1$. The width of the boundary layer $\Phi_i = 0.1$, $\eta_{\Delta i} = 0.01$, $i,j = 1,2$ and the sliding surface coefficient $c_1 = 1.2$, $c_1 = 0.8$, $k_1 = k_2 = 0.5$. The desired reference trajectory are chosen as $q_{d_1} = 1.0\sin(t)$, $q_{d_2} = 0.8\cos(t)$, respectively. The initial conditions $q_1(0) = 0.5$, $q_2(0) = -0.5$, $\dot{q}_1(0) = \dot{q}_2(0) = 0$ and $\hat{p}_1(0) = \hat{p}_2(0) = 0.2$. In order to verify the robustness of the controller in a pick and place environment, the mass profile were added as shown in Figs. 4 and 5. For comparison, the conventional computed torque control $\tau = M(q)(\ddot{q}_d + k_v\dot{e} + k_p e) + C(q,\dot{q})\dot{q} + G(q)$ under the same conditions is also demonstrated. The gains are chosen as $k_p = \mathrm{diag}[50,50]$, $k_v = \mathrm{diag}\,[20,20]$. Fig. 6 shows the results with computed torque control. It can be seen the controller cannot drive the joints to reach the desired positions and steady-state tracking error exist. Fig. 7 shows the results for the proposed fuzzy controller. It is observed the tracking errors go to small values after some transient, which is cause because of the initial choice of the consequent parameters. However, the tracking error decreases quickly since of the on-line learning of fuzzy logic system, and the effect of uncertainties are successfully compensated by the robust control term. The simulation results thus demonstrate the propose robust adaptive fuzzy control can effectively control the rigid robot system with uncertainties.

6. Conclusions

In this paper, we have presented a robust fuzzy control algorithm for robotic manipulators. The method is developed based on the fuzzy modeling technique with robust sliding mode control. The control scheme does not require the robot dynamics to be exactly known. With the aid of fuzzy logic system has been used to implement an adaptive feedback control strategy with the boundary layer integral sliding control, which compensate for unknown uncertainties with estimated bound. Both chattering and reaching phase problem can be avoided. The design has been proved to guarantee the closed-loop stability in the sense of Lyapunov method. Finally, the simulation results show that the proposed control algorithm is appropriate for practical control design robotic manipulator with uncertainties.

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